1. **Arithmetic of growth rates**

   As discussed in the Lecture Notes, **Lucas (1988)** in a key contribution to modern growth theory summarizes striking features of the **arithmetic of growth rates** as follows:

   "Rates of growth of real per capita GNP are...diverse, even over sustained periods. For 1960-80 we observe, for example, India, 1.4% per year; Egypt, 3.4%; South Korea, 7.0%; Japan, 7.1%, the United States, 2.3%, the industrial economies averaged 3.6%." (Lucas, 1988, p.3)

   Using these numbers, Lucas illustrates their implications as follows:

   "...Indian incomes will double every 50 years; Korean every 10. An Indian will, on average, be twice as well off as his grandfather; a Korean 32 times."

   Verify this statement.

   ($\rightarrow$ hint: the solution will be similar to the discussion of the half-life of growth dynamics discussed in the Lecture Notes)

2. **Solow-model and golden-rule discussion**

   Consider the central steady-state equation in $k^{\#}$ characterizing the Solow-model (as derived in the Lecture Notes):

   $$s \cdot F(k^{\#}_{So}) = (\delta + \mu_{N^{\#}}) \cdot k^{\#}_{So}$$

   (a) Find a relationship between $c^{\#}_{So}$ and $k^{\#}_{So}$.

   (b) Let $k^{\#}_{GR}$ denote the golden-rule level of the capital stock per unit of effective labor. Show that $\frac{\partial c^{\#}_{So}}{\partial s} > 0$ if $k^{\#}_{So} < k^{\#}_{GR}$.

   (c) Consider a permanent increase in the savings rate (starting out from a steady-state constellation characterized by $\frac{\partial c^{\#}_{So}}{\partial s} > 0$). Find a graphical representation of the time paths of $c^{\#}_t$ (ie consumption per unit of effective labor) and c_t (ie per capita consumption) before and after the shock occurs.

   (d) Consider the Cobb-Douglas production function

   $$F(k^{\#}) = (k^{\#})^{\alpha}$$

   Let $\alpha = 1/3$ and $s = 0.15$. Show that these values imply $\frac{\partial c^{\#}_{So}}{\partial s} > 0$.