# Problem set 4

## Investment and phase diagrams

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# Dynamics of the model

- Consider the two dynamic equation from the lecture

$$\Delta k_{t+1} = \frac{1}{\phi}(q_t - 1)k_t \tag{1}$$

$$\Delta q_{t+1} = \frac{1-\beta}{\beta}q_t - F_{k,t+1}. \tag{2}$$

- We analyze this system of equations with a phase diagram.
- You will realize that the basic mechanism of a phase diagram we have seen in PS3 is the same.
- First we derive the zero motion lines (where $\Delta k_{t+1} = \Delta q_{t+1} = 0$).

# $\Delta k_{t+1} = 0$ and the dynamics of $k$

- Setting $\Delta k_{t+1} = 0$ in equation (1) yields

$$q_t = 1.$$

- What happens in equation (1) if we (starting from the zero motion line) increase $q_t$ a little bit?
  - $\Rightarrow$ Since $\phi \geq 0$ we have that $\Delta k_{t+1} > 0$ and vice versa.
- Keep those results in mind, we will discuss the dynamics of $q_t$ next before drawing the diagram.

# $\Delta q_{t+1} = 0$ and the dynamics of $q$

- Setting $\Delta q_{t+1} = 0$ in equation (2) yields

$$q_t = \frac{\beta}{1-\beta} F_{k,t+1}.$$

- In order to draw the zero motion line we have to remember the properties of $F(\cdot)$.
- Since $F_k > 0$ and $F_{kk} < 0$ we know that the zero motion line is downward sloping.
- You can verify this by using a specific functional form (i.e. Cobb-Douglas)
- What happens in equation (2) if we (starting from the zero motion line) increase $k_{t+1}$ a little bit?
  - $\Rightarrow$ Since $F_{kk} < 0$ we have that $\Delta q_{t+1} > 0$ and vice versa.
- On the following slide we combine the information graphically.

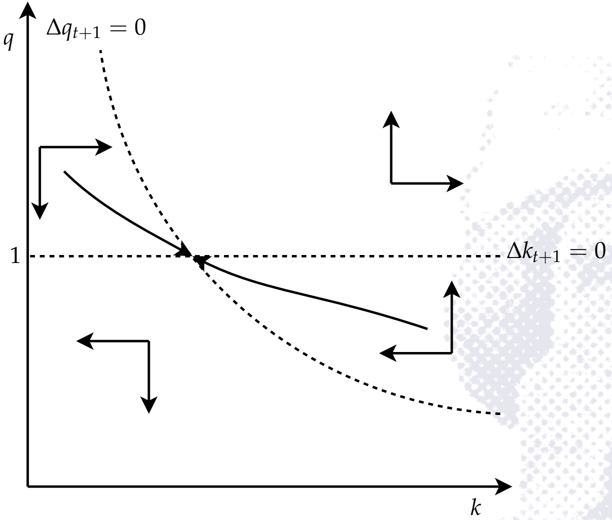


Figure: Phase diagram

# Permanent productivity shock

- You find the graphical illustration of the arguments on this slide on the next slide.
- Next, consider a permanent shock to productivity.
- From the zero motion line $\Delta q_{t+1} = 0$ we find that this equation must shift upwards (or to the right as you want).
- For the reasoning above keep in mind the properties of $F(\cdot)$.
- Denote the old (pre shocked) steady state by $A$, then when the shock occurs, we jump to point $B$.
- Point $B$ is on the new saddle path (which is stable).
- The dynamics of the stable path state that in the long run we will approach the new steady state denoted by $C$.
- Note that $k_1^* > k_0^*$ but $q_1^* = q_0^* = 1$.

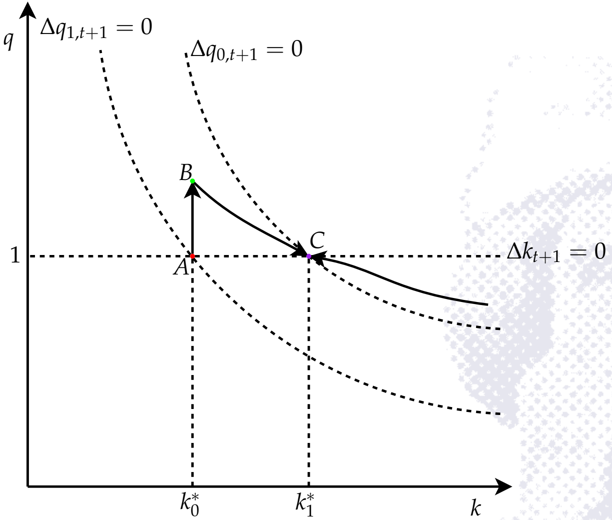


Figure: Phase diagram (shock)

# References

Wickens, M. (2008).
*Macroeconomic Theory: A Dynamic General Equilibrium Approach.*
Princeton University Press.

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