1. **Budget constraints**

   Consider a period budget constraint of the form

$$c_t + a_{t+1} = (1+r)a_t + x_t \tag{PB}$$

   and a lifetime budget constraint of the form

$$\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s c_{t+s} = (1+r)a_t + \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s x_{t+s} \tag{LB1}$$

   where c_t denotes consumption, a_t denotes assets, and x_t is income in period t.

   (a) Use the period budget constraint (PB) and the *no-ponzi game* condition to derive the lifetime budget constraint (LB1).
   (b) Use the lifetime budget constraint (LB1) to derive the period budget constraint (PB).
   (c) Now assume that interest rates are not constant through time. How does the period budget constraint look like?
   (d) Based on the non-constant period budget constraint from the previous subquestion derive the lifetime budget constraint.

2. **Consumption**

   The representative household maximizes

$$\max_{\{c_{t+s},a_{t+s+1}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \tag{1}$$

   subject to (PB). Household‘s period utility function is given by

$$U(c_{t+s}) = c_{t+s} - \frac{\alpha}{2}c_{t+s}^2, \tag{2}$$

   where $\alpha > 0$ is a parameter.

   (a) Derive the Euler equation to this problem.
   (b) Does expected consumption rise over time? For your answer distinguish the three cases $\beta(1+r) \lesseqqgtr 1$.

3. **Permanent income hypothesis**

   Consider the objective function (1), the period utility function (2) and a budget constraint of the form

$$\mathbb{E}_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i c_i = (1+r)a_0 + \mathbb{E}_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i x_i. \tag{LB2}$$

   Assume that $\beta(1+r) = 1$.

   (a) Use the Euler equation derived under problem 2 and the lifetime budget constraint (LB2) in order to show how actual consumption depends on lifetime wealth.

   (b) Derive the growth rate of consumption $\Delta c_t \equiv c_t - c_{t-1}$.

   (c) When does consumption grow/shrink, i.e. when is $\Delta c_t \neq 0$.

   (d) Assume income follows a random walk

$$x_{t+1} = x_t + \varepsilon_{t+1}, \tag{3}$$

   where $\varepsilon_{t+s} \sim \mathcal{N}(0, \sigma^2)$ for $s \geq 0$. Determine a specific expression for Δc_t.

4. **Optimal taxation**

   A representative agent maximizes lifetime utility (1). The period utility function is given by

$$U(c_{t+s}) = \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma}, \tag{4}$$

   which has the property of a constant relative risk aversion (CRRA). The government levies a consumption tax τ_c such that the period budget constraint is

$$(1+\tau_c)c_t + a_{t+1} = (1+r)a_t + x_t. \tag{TPB}$$

   (a) Derive the Euler equation.

   (b) Is the tax distortionary?