1. **Budget constraints**

   Consider a period budget constraint of the form

$$c_t + a_{t+1} = (1+r)a_t + x_t \tag{PB}$$

   and a lifetime budget constraint of the form

$$\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s c_{t+s} = (1+r)a_t + \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s x_{t+s} \tag{LB1}$$

   where c_t denotes consumption, a_t denotes assets, and x_t is income in period t.

   - Use the period budget constraint (PB) and the *no-ponzi game* condition to derive the lifetime budget constraint (LB1).
   - Use the lifetime budget constraint (LB1) to derive the period budget constraint (PB).

2. **Consumption**

   The representative household maximizes

$$\max_{\{c_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \tag{1}$$

   subject to (LB1). Household‘s period utility function is given by

$$U(c_{t+s}) = c_{t+s} - \frac{\alpha}{2}c_{t+s}^2, \tag{2}$$

   where $\alpha > 0$ is a parameter.

   - Derive the Euler equation to this problem.
   - Does expected consumption rise over time? For your answer distinguish the three cases $\beta(1+r) \lesseqgtr 1$.

3. **Permanent income hypothesis**

   Consider the objective function (1), the period utility function (2) and a budget constraint of the form

$$\mathbb{E}_t \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i c_i = (1+r)a_0 + \mathbb{E}_t \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i x_i. \tag{LB2}$$

   Assume that $\beta(1+r) = 1$.

- Use the Euler equation derived under problem 2 and the lifetime budget constraint (LB2) in order to show how actual consumption depends on lifetime wealth.
- Derive the growth rate of consumption $\Delta c_t \equiv c_t - c_{t-1}$.
- When does consumption grow/shrink, i.e. when is $\Delta c_t \neq 0$.
- Assume income follows a random walk

$$x_{t+1} = x_t + \varepsilon_{t+1}, \tag{3}$$

where $\varepsilon_{t+s} \sim \mathcal{N}(0, \sigma^2)$ for $s \geq 0$. Determine a specific expression for Δc_t.

4. **Optimal taxation**

A representative agent maximizes lifetime utility (1). The period utility function is given by

$$U(c_{t+s}) = \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma}, \tag{4}$$

which has the property of a constant relative risk aversion (CRRA). The government levies a consumption tax τ_c such that the period budget constraint is

$$(1+\tau_c)c_t + a_{t+1} = (1+r)a_t + x_t. \tag{TPB}$$

- Derive the Euler equation.
- Is the tax distortionary?