How to obtain the Nash bargaining wage equation

April 5, 2013

WageSettingEquations_.tex

The basic background is Pissarides (AER, 1985).

1 Pissarides 1985 derivation

1.1 Starting point

The equations we would start with would read (utility is now linear and we use Pissarides notation)

$$rV(y) - \dot{V}(y) = w(y) + s[V(b_1) - V(y)] \tag{(P - 7)}$$

$$rV(b_1) - \dot{V}(b_1) = b_1 + ap[V(y) - V(b_1)] \tag{(P - 6)}$$

$$rJ(y) - \dot{J}(y) = y - w(y) + s[J_0 - J(y)], \tag{(P - 4)}$$

$$rJ_0 - \dot{J}_0 = -k + aq(t)[J(y) - J_0], \tag{(P - 2)}$$

The Nash-product implies

$$(1-\beta)(V(y) - V(b_1)) = \beta[J(y) - J_0] \tag{1}$$

and its time derivative is

$$(1-\beta)\left(\dot{V}(y) - \dot{V}(b_1)\right) = \beta\left[\dot{J}(y) - \dot{J}_0\right].$$

1.2 Step 1: Replace differences and use Nash product

Replace by the differences obtained form (P-2) to (P-7), we get

$$\begin{aligned}
&(1-\beta)(rV(y) - w(y) - s[V(b_1) - V(y)] + b_1 + ap[V(y) - V(b_1)] - rV(b_1)) \\
&= \beta[rJ(y) - y + w(y) - s[J_0 - J(y)] - k + aq(t)[J(y) - J_0] - rJ_0] \Leftrightarrow \\
&(1-\beta)(-w(y) + b_1 + (ap + r + s)[V(y) - V(b_1)]) \\
&= \beta[-y + w(y) - k + (aq(t) + r + s)[J(y) - J_0]].
\end{aligned}$$

Using the Nashproduct to replace $V(y) - V(b_1)$, we get

$$(1-\beta)(-w(y)+b_1)+\beta ap[J(y)-J_0]=\beta[-y+w(y)-k+aq(t)[J(y)-J_0]].$$

We can write this as

$$\begin{aligned}-w(y)+(1-\beta)b_1&=\beta[-y-k]+\beta a(q(t)-p)[J(y)-J_0]\Leftrightarrow\\ w(y)&=(1-\beta)b_1+\beta[y+k]-\beta a(q(t)-p)[J(y)-J_0].\end{aligned}$$

1.3 Step 2: Insert arrival rate link and free-entry

Using $\theta q = p$ gives

$$w(y)=(1-\beta)b_1+\beta[y+k]-\beta aq(t)[1-\theta][J(y)-J_0].$$

With $J_0 = 0$ we know from (P - 2) that $k = aq(t)J(y)$ and therefore

$$\begin{aligned}w(y)&=\beta[y+k]+(1-\beta)b_1-\beta[1-\theta]k\\ &=\beta[y+\theta k]+(1-\beta)b_1.\end{aligned}$$

This is the wage equation 12 in Pissarides 1985.