Trade, Growth, and Macroeconomics: A Quantitative Approach

Mainz: August 2012

Samuel Kortum, Yale

Probabilistic Foundations

Model of Technological Change

- Robert Evenson and Yoav Kislev (JPE, 1976: 265-281) initiated a way of modeling technological discovery.

- Stochastic search theory had been applied earlier by Stigler (JPE, 1961:213-225) to a shopping problem.

- Application of Evenson and Kislev is more compelling.

Evenson and Kislev (1)

- Distribution of quality X_i of potential techniques:

$$\text{Pr}[X_i \leq x] = F_t(x).$$

- Applied research: draw a sample of n techniques, retaining the best

$$\begin{aligned} Z &= \max_{i=1,\dots,n} \{X_i\} \\ \text{Pr}[Z &\leq x] = [F_t(x)]^n \end{aligned}$$

- Basic research shifts the distribution F_t to make it better over time.

Evenson and Kislev (2)

- They illustrated with an exponential distribution:

$$\text{Pr}[X_i \leq x] = F_t(x) = 1 - e^{-\theta(x-\lambda_t)} \text{ for } x \geq \lambda_t.$$

- Basic research raises λ_t, the lower bound of the support.

Application to Patents

- Does this model link have implications for patent data.

- Problem: no data on x for a patent.

- Patent does indicate when z increases, it marks a *record*. Glick (*American Mathematical Monthly*, 1978: 2-26) is a good reference.

- Expected number of patents P in n random trials (with F fixed):

$$E[P|n] = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}.$$

- (Intuition, ex-ante, n'th draw has $1/n$ chance of setting a record.)

Application to Growth (1)

- Bental and Peled (IER, 1996: 687-718): Pareto distribution convenient for modeling economic growth.

- With X_i exponentially distributed, assume labor productivity $Q_i = e^{X_i}$.

- Work directly with Q_i (re-using F for its distribution):

$$\begin{aligned} F_t(q) &= \Pr[Q \leq q] = P[X \leq \ln q] \\ &= 1 - e^{-\theta(\ln q - \lambda_t)} = 1 - \left(\frac{q}{\underline{q}_t}\right)^{-\theta}, \end{aligned}$$

for $q \geq \underline{q}_t = e^{\lambda_t}$.

Application to Growth (2)

- Nice scale-free property for the distribution of proportional advances.

- Say z is initial state of the art and Q' is a new technique:

$$
\begin{aligned}
\Pr[\frac{Q'}{z} &\leq x | Q' \geq z] = \frac{\Pr[xz \geq Q' \geq z]}{\Pr[Q' \geq z]} \\
&= \frac{\Pr[Q' \geq z] - \Pr[Q' > xz]}{\Pr[Q' \geq z]} \\
&= \frac{\left(\frac{z}{\underline{q}_t}\right)^{-\theta} - \left(\frac{xz}{\underline{q}_t}\right)^{-\theta}}{\left(\frac{z}{\underline{q}_t}\right)^{-\theta}} \\
&= 1 - x^{-\theta}
\end{aligned}
$$

The Arrival of New Techniques

- In Kortum (*ECTA*, 1997: 1389-1419), I assume draws arrive as a Poisson process at rate aR_t.

- Stock of research
$$T_t = \int_0^t R_s ds.$$

- Number of draws n by date t is Poisson with parameter aT_t.

- Number n between date s and date $t > s$ is Poisson with parameter $a(T_t - T_s)$

Technological Frontier

- Fix $\underline{q}_t = \underline{q}$ so for $z \geq \underline{q}$ the state of the art Z is distributed

$$\begin{aligned}
\Pr[Z \leq z|T_t] &= \sum_{n=0}^{\infty} \frac{e^{-aT_t}[aT_t]^n}{n!} F(z)^n \\
&= e^{-aT_t[1-F(z)]} \sum_{n=0}^{\infty} \frac{e^{-aT_t F(z)}[aT_t F(z)]^n}{n!} \\
&= e^{-aT_t[1-F(z)]} = e^{-a\underline{q}^{\theta} T_t z^{-\theta}}
\end{aligned}$$

Convenient Limit

- In the derivation above, the distribution has support on $z \geq \underline{q}$.

- Convenient to set $a\underline{q}^{\theta} = 1$ while letting $\underline{q}$ approach 0 (hence $a \rightarrow \infty$) to get frontier:

$$G(z; T_t) = e^{-T_t z^{-\theta}} \tag{1}$$

  with support on $z > 0$.

- Many ideas, each quite poor (as in nature). The resulting distribution (1) is the Fréchet, one of the three types of extreme value distributions.

- Expected productivity (both the arithmetic and geometric means) are proportional to $T_t^{1/\theta}$.

Continuum of Goods

- Imagine the search process across a continuum of goods $j \in [0, 1]$.

- Fraction of goods produced with efficiency below z is

$$G(z; T_t) = e^{-T_t z^{-\theta}}$$

- Average productivity in the economy rises predictably in proportion to $T_t^{1/\theta}$.

- Progress at the level of individual goods remains nice and random.

International Trade (1)

- In EK (ECTA, 2002: $1741 - 1779$): Countries $i = 1, ..., N$ each with independent Fréchet distributions of efficiency (1).

- Wages w_i and transport costs d_{ni} so cost of i supplying a particular good j to n is $C_{ni} = \frac{w_i d_{ni}}{Z}$.

- Distribution (complement) of i's cost of supply n (let $T_{it} = T_i$):

$$\begin{aligned} \Pr[C_{ni} &\geq c] = \Pr[Z \leq \frac{w_i d_{ni}}{c}] = G(\frac{w_i d_{ni}}{c}; T_i) \\ &= e^{-T_i (w_i d_{ni})^{-\theta} c^{\theta}} = 1 - H_{ni}(c) \end{aligned}$$

International Trade (2)

- Distribution (complement) of lowest cost available to n :

$$
\begin{aligned}
1 - H_n(c) &= \Pr[C_n \geq c] = \Pr\left[\min_k \{C_{nk}\} \geq c\right] \\
&= \prod_k \Pr[C_{nk} \geq c] = e^{-\left[\sum_k T_k (w_k d_{nk})^{-\theta}\right] c^\theta} \\
&= e^{-\Phi_n c^\theta}
\end{aligned}
$$

International Trade (3)

- Probability that i is low cost in n is

$$
\begin{aligned}
\pi_{ni} &= \int_0^\infty \mathsf{Pr}\left[\min_{k\neq i}\{C_{nk}\}\geq c\right] dH_{ni}(c) \\
&= \int_0^\infty e^{-\left[\sum_{k\neq i} T_k (w_k d_{nk})^{-\theta}\right] c^\theta} T_i\left(w_i d_{ni}\right)^{-\theta} e^{-T_i (w_i d_{ni})^{-\theta} c^\theta} \theta c^{\theta - 1} dc \\
&= \frac{T_i (w_i d_{ni})^{-\theta}}{\Phi_n} \int_0^\infty e^{-\Phi_n c^\theta} \Phi_n \theta c^{\theta-1} dc \\
&= \frac{T_i (w_i d_{ni})^{-\theta}}{\Phi_n} \qquad (2)
\end{aligned}
$$

International Trade (4)

- With unit continuum of goods, π_{ni} is share of goods n buys from i.

- Delivers an N-country Ricardian model of trade with gains from trade related to trade costs and heterogeneity, θ.

Another Derivation of the Frontier (1)

- Let $H(z, s)$ be the technology frontier at date s, i.e. the probability that the state of the art is below z (it will turn out to equal $G(z; T_s)$).

- Probability that no new idea arrives in $[s, s + ds]$ to bump you above z is:

$$e^{-aR(s)\left(z/\underline{q}\right)^{-\theta} ds} = e^{-R(s)z^{-\theta} ds}$$

- Hence, you're still below z at date $s + ds$ with probability:

$$H(z, s + ds) = H(z, s)e^{-R(s)z^{-\theta} ds}.$$

Another Derivation of the Frontier (2)

- Take logs:

$$\ln H(z, s + ds) = \ln H(z, s) - R(s) z^{-\theta} ds.$$

- Thus:

$$\frac{\partial \ln H(z, s)}{\partial s} = -R(s) z^{-\theta}.$$

- Integrate from $s = 0$ to t, given $H(z, 0) = 1$:

$$H(z, t) = e^{-T_t z^{-\theta}} = G(z; T_t).$$

Back to Basics

- Original Evenson and Kislev paper had basic research shifting the distribution from which applied researchers draw.

- Lucas (*Economica*, 2009: 1-19), using results from Alvarez, Buera and Lucas (NBER WP # 14135, 2008), is a neat way to think about it in a different setting.

- For Lucas there are no blueprints. What matters is the ideas in people's heads.

- Ideas are still non-rival. When we meet and talk, we both leave with the better of our two ideas.

- More on this later ...

1 Technology, Geography, and Trade: EK (2002)

Questions

- What are the gains from trade?
  - That have already been realized, relative to autarky.
  - That could potentially be realized with costless trade.
- What is the role of trade in spreading the benefits of technology?
- What is the role of geography in determining patterns of specialization?
- How important is trade diversion following regional integration?

Assumptions

- Perfect competition and N countries.

- Unit continuum of manufactured goods, $j \in [0, 1]$.

- Manufactured goods are aggregated with a CES function (elasticity of substitution σ).

- The aggregate of manufactures is either consumed or used as an intermediate good in production.

Intermediates

- Labor and intermediates are combined to produce a composite input, with unit cost c_i (in place of the wage w_i).
  - Intermediates dominate international trade, Kei-Mu Yi, JPE (2003?).
  - Data requires matching gross production (not value added) to trade data (otherwise all sorts of puzzles emerge).
  - Intermediates introduce geography effects in production, as in Krugman and Venables, QJE (1995).
- The price index for intermediates is p_i so, given a Cobb Douglas production function,

$$c_i = w_i^{\beta} p_i^{1-\beta}.$$

Price Equations

- The exact price index for tradable goods is:

$$
\begin{aligned}
p_n &= \left[\int_0^\infty p^{-(\sigma-1)} dH_n(p)\right]^{-1/(\sigma-1)} \\
&= \gamma \left[\sum_{i=1}^{N} T_i \left(w_i^\beta p_i^{1-\beta} d_{ni}\right)^{-\theta}\right]^{-1/\theta} = \gamma \left[\Phi_n\right]^{-1/\theta}
\end{aligned}
$$

- Notice the feedback loop, via intermediates used in production, from the price level in one country to prices everywhere else.

Parameters

- Now we know how to solve it if only we had the parameters!

- Discuss the estimation approaches, ignoring intermediates ($\beta = 1$) to keep it simple.

- Intermediates are easy to append onto the estimation procedures.

- The starting point is

$$\frac{X_{ni}}{X_n} = \frac{T_i \left(w_i d_{ni}\right)^{-\theta}}{\sum_{k=1}^{N} T_k \left(w_k d_{nk}\right)^{-\theta}} = \frac{T_i \left(w_i d_{ni}\right)^{-\theta}}{\Phi_n}$$

  which holds without error according to the theory.

Original Strategy (1)

- We observe the left hand side and we observe wages on the right hand side.

- Proxy for T by a stock R of past $R\&D$

$$\ln T_i = \alpha_R \ln R_i + \tau_i$$

- Proxy for d_{ni} by vector of variables g_{ni} used in gravity equations

$$\ln d_{ni} = \alpha' g_{ni} + \delta_{ni}$$

- Plug these terms in to the bilateral trade equation, but the result would be highly non-linear in the errors (τ and δ).

Original Strategy (2)

- Choose a transformation of the dependent variable that makes the error terms enter linearly (Berry, RAND):

$$\ln \frac{X_{ni}}{X_{nn}} = \ln \frac{T_i}{T_n} - \theta \ln \frac{w_i}{w_n} - \theta \ln d_{ni}.$$

- Hence:

$$\ln \frac{X_{ni}}{X_{nn}} = \alpha_R \left(\ln R_i - \ln R_n\right) - \theta \ln \frac{w_i}{w_n} - \theta \alpha' g_{ni} + \varepsilon_{ni}$$

- This regression should yield an estimate of θ as the coefficient on the log relative wage.

Problem

- Note that the error term is

$$\varepsilon_{ni} = \tau_i - \tau_n - \theta\delta_{ni}$$

- High value of τ_i will raise i's exports at the same time it will likely raise i's relative wage (intuition from DFS).

- OLS will yield a downward biased estimate of θ. (standard endogeneity problem).

- It does deliver a very small θ of about 1.

An Instrument

- A theory-based instrument for the wage is the labor force.

- Ricardian logic has the wage declining in the labor force (intuition from DFS).

- IV estimation delivers an estimate of 3 for θ.

- Problem: A larger labor force makes it possible for the country to develop more advanced technology, so instrument may not be valid.

- Another approach is to bring in data on prices of individual goods.

Price Bounds

- Iceberg trade cost put bounds on the range of deviations from the law of one price (hit bounds when n and i trade good j):

$$d_{ni} \geq \frac{p_n(j)}{p_i(j)} \geq \frac{1}{d_{in}}$$

- From the model of trade:

$$\frac{X_{ni}}{X_n} = \frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\Phi_n}$$

$$p_n = \gamma \Phi_n^{-1/\theta}$$

Another Way to Estimate Theta (1)

- Divide by the corresponding equation for X_{ii}/X_i and substitute in price equation to get

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}. \tag{3}$$

- The left-hand side is easy to measure, but what about the right-hand side?

Another Way to Estimate Theta (2)

- From ICP get prices $p_i(j)$ for a bunch of goods j across countries i. (think of all prices in \$'s, but in the final statistic exchange rates will cancel out.)

- Define $r_{ni}(j) = \ln p_n(j) - \ln p_i(j)$.

- Notice that if we have enough good j:

$$\max_j \{r_{ni}(j)\} = \ln d_{ni}$$

and

$$mean \{r_{ni}(j)\} = \ln(p_n/p_i)$$

up to a common constant.

Another Way to Estimate Theta (3)

- Thus we have the convenient measure of the right-hand side of (3):

$$\ln\left(\frac{p_i d_{ni}}{p_n}\right) = \max_j \{r_{ni}(j)\} - mean\,\{r_{ni}(j)\}$$

(embarrassing typo in *Econometrica* paper).

- Minus the slope of a regression of $\ln\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right)$ on this measure of $\ln\left(\frac{p_i d_{ni}}{p_n}\right)$ gives an estimate of θ.

- Minimize the impact of errors in the right-hand side by imposing 0 intercept. Can also instrument for $\max_j \{r_{ni}(j)\} - mean\,\{r_{ni}(j)\}$ to reduce bias from measurement error.

Dummy Variable Approach (1)

- With an estimate of θ in hand, return to the original equation to estimate the other parameters:

$$\ln \frac{X_{ni}}{X_{nn}} = \ln S_i - \ln S_n - \theta \alpha' g_{ni} - \theta \delta_{ni}$$

- Coefficient on the country-dummy (impose constraint that they enter as a ratio):

$$\ln S_i = \ln T_i - \theta \ln w_i.$$

Dummy Variable Approach (2)

- Recover $\ln T_i$ from $\ln S_i$ given data on $\ln w_i$ and the estimate of θ.

- The estimate of α (obtained by dividing by θ) gives us an estimate of $\ln d_{ni}$.

- We can use all these parameters to perform counterfactual experiments on the model.

- But, there's an easier way, as shown in Dekle, Eaton and Kortum (2007).

2 Unbalanced Trade: DEK (2007)

Questions

- What are the consequences of US deficit reduction for:
  - Relative wages.
  - Bilateral trade patterns.
  - Real wages.

- Use the EK (2002) setup, but close the model in a more satisfactory way, following Alvarez and Lucas (2007).

The World Economy

- N countries, perfect competition.

- Two sectors, manufacturing and nonmanufacturing, with perfect labor mobility.

- Labor endowments L_i so GDP is $Y_i = w_i L_i$.

- Manufacturing value added $V_i^M = w_i L_i^M$, share β of production Y_i^M.

- Deficits: distinguish production Y_i^M from expenditure $X_i^M = Y_i^M + D_i^M$.

Income and Expenditure Accounting

- Final absorption $X_i = Y_i + D_i$ of which fraction α spent on manufactures.

- Spending on manufactures:

$$X_i^M = \alpha X_i + (1 - \beta) Y_i^M.$$

- Solve for manufacturing output in terms of the wage and deficits:

$$Y_i^M = \frac{\alpha}{\beta} \left(w_i L_i + D_i - \frac{1}{\alpha} D_i^M \right).$$

International Accounting

- The world must purchase all the manufactures that country i generates:

$$Y_i^M = \sum_{n=1}^{N} \pi_{ni}(Y_n^M + D_n^M).$$

- With the Y^M's and the π_{ni}'s endogenous, these equations become the equilibrium conditions.

Trade Shares

- Trade shares are given by

$$\pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_{k=1}^{N} T_k(c_k d_{nk})^{-\theta}}.$$

- The price index for intermediates is

$$p_n = \gamma \left[\sum_{k=1}^{N} T_k(c_k d_{nk})^{-\theta} \right]^{-1/\theta},$$

- Input costs

$$c_i = \kappa w_i^{\beta} p_i^{1-\beta}$$

Equilibrium

- Equilibrium is wages and prices satisfying:

$$
\begin{aligned}
Y_i^M &= \sum_{n=1}^{N} \pi_{ni}(Y_n^M + D_n^M), \\
Y_i^M &= \frac{\alpha}{\beta}\left(w_i L_i + D_i - \frac{1}{\alpha} D_i^M\right), \\
\pi_{ni} &= \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_{k=1}^{N} T_k(c_k d_{nk})^{-\theta}}, \\
p_n &= \gamma \left[\sum_{k=1}^{N} T_k(c_k d_{nk})^{-\theta}\right]^{-1/\theta}, \\
c_i &= \kappa w_i^{\beta} p_i^{1-\beta}.
\end{aligned}
$$

Reformulation Relative to the World (1)

- Notice that world GDP can be written as $Y = wL$ where L is the world labor force and the world average wage is

$$w = \sum_{i=1}^{N} \frac{L_i}{L} w_i.$$

- We can thus express the relative wage in i as

$$\omega_i = \frac{w_i}{w}.$$

- Reformulating the model in terms of relative wages will help in matching the model correctly to data over time expressed in current U.S. $'s.

Reformulation Relative to the World (2)

- Let $y_i = Y_i/Y = \omega_i \lambda_i$, with $\lambda_i = L_i/L$.

- It will also be helpful to define $y_i^M = Y_i^M/Y$, $\delta_i = D_i/Y$, $\delta_i^M = D_i^M/Y$.

- To go along with relative wages, we need normalized prices

$$\rho_i = p_i/w$$

Equilibrium in Relative Terms

- Equilibrium: relative wages ω and normalized prices ρ satisfying:

$$
\begin{aligned}
y_i^M &= \sum_{n=1}^{N} \pi_{ni}(y_n^M + \delta_n^M), \\
y_i^M &= \frac{\alpha}{\beta}\left(\omega_i \lambda_i + \delta_i - \frac{1}{\alpha}\delta_i^M\right), \\
\pi_{ni} &= \frac{T_i(\omega_i^{\beta}\rho_i^{1-\beta}d_{ni})^{-\theta}}{\sum_{k=1}^{N} T_k(\omega_k^{\beta}\rho_k^{1-\beta}d_{nk})^{-\theta}}, \\
\rho_n &= \gamma\left[\sum_{k=1}^{N} T_k(\kappa\omega_k^{\beta}\rho_k^{1-\beta}d_{nk})^{-\theta}\right]^{-1/\theta}.
\end{aligned}
$$

Changes

- Suppose deficits, relative to world GDP, change to δ_i' and $\delta_i^{M'}$ (for $i = 1, \ldots, N$) in the following year.

- In the DFS model we set deficits to zero, but don't have to.

- We want to calculate the implications for changes in relative GDP in each country, holding fixed all structural parameters in the model.

- Note how our formulation of next year's deficit relative to next years GDP eliminates any issues with $ inflation.

Equilibrium Changes

- For variable x, counterfactual value is x' and change is $\hat{x} = x'/x$. Equilibrium is relative wage changes and normalized price changes satisfying:

$$
\begin{aligned}
y_i^{M'} &= \sum_{n=1}^{N} \pi'_{ni}(y_n^{M'} + \delta_n^{M'}), \\
y_i^{M'} &= \frac{\alpha}{\beta}\left(\widehat{\omega}_i y_i + \delta_i' - \frac{1}{\alpha}\delta_i^{M\prime}\right), \\
\pi'_{ni} &= \frac{\pi_{ni}(\widehat{\omega}_i^{\beta}\widehat{\rho}_i^{1-\beta})^{-\theta}}{\sum_{k=1}^{N}\pi_{nk}(\widehat{\omega}_k^{\beta}\widehat{\rho}_k^{1-\beta})^{-\theta}}, \\
\widehat{\rho}_n &= \left[\sum_{k=1}^{N}\pi_{nk}(\widehat{\omega}_k^{\beta}\widehat{\rho}_k^{1-\beta})^{-\theta}\right]^{-1/\theta}.
\end{aligned}
$$

Implementation

- Calculate change in manufacturing share in country i as

$$\frac{\widehat{y}_i^M}{\widehat{y}_i} = \frac{\widehat{y}_i^M}{\widehat{\omega}_i}.$$

- Calculate change in real GDP in country i as

$$\frac{\widehat{Y}_i}{\widehat{p}_i^{\alpha}\widehat{w}_i^{1-\alpha}} = \left(\frac{\widehat{\omega}_i}{\widehat{\rho}_i}\right)^{\alpha}.$$

Numeraire

- Notice that any solution $\widehat{\omega}$ has the property

$$\sum_{i=1}^{N} \widehat{\omega}_i y_i = \sum_{i=1}^{N} \frac{w_i'}{w'} \frac{L_i}{L} = 1.$$

- Define $\Delta_{\widehat{\omega}}$ be the set of all vectors with each element positive and satisfying the adding up restriction above.

- Want an algorithm that imposes that condition.

Alvarez-Lucas Algorithm for Prices (1)

- Start with the price equation. Let $\widetilde{\rho}_n = \ln\,\widehat{\rho}_n$ and $\widetilde{\omega}_n = \ln\,\widehat{\omega}_n$ so that

$$\widetilde{\rho}_n = \frac{-1}{\theta} \ln \left[\sum_{k=1}^{N} \pi_{nk} \exp\left\{-\theta \left[\beta \widetilde{\omega}_k + (1-\beta)\widetilde{\rho}_k\right]\right\} \right].$$

- We can put all the price equations together as $\widetilde{\rho} = h(\widetilde{\rho}, \widetilde{\omega})$.

- If we fix $\widetilde{\omega}$ then $h(., \widetilde{\omega})$ maps $g(\widetilde{\omega})$ into $(Tg)(\widetilde{\omega}) = h(g(\widetilde{\omega}), \widetilde{\omega})$, with a fixed point $\widetilde{\rho}(\widetilde{\omega}) = (T\widetilde{\rho})(\widetilde{\omega})$.

Alvarez-Lucas Algorithm for Prices (2)

- The mapping T is a contraction since, as is easy to verify, (i) $f(\tilde{\omega}) \leq g(\tilde{\omega})$ implies $(Tf)(\tilde{\omega}) \leq (Tg)(\tilde{\omega})$ and (ii) for constant $a \geq 0$,

$$
\begin{aligned}
& [T(f+a)](\tilde{\omega}) \\
= \; & \frac{-1}{\theta} \ln \left[\exp(-\theta(1-\beta)a) \sum_{k=1}^{N} \pi_{nk} \exp\left\{-\theta\left[\beta\tilde{\omega}_k + (1-\beta)f_k(\tilde{\omega})\right]\right\} \right] \\
= \; & (Tf)(\tilde{\omega}) + (1-\beta)a
\end{aligned}
$$

where $(1-\beta) \in (0,1)$.

Alvarez-Lucas Algorithm for Prices (3)

- Thus, to find the vector of normalized price changes, given a vector of relative wage changes, simply iterate on the mapping T.

- Having solved for $\widehat{\rho}_n(\widehat{\omega})$, we can write

$$\pi'_{ni}(\widehat{\omega}) = \frac{\pi_{ni}\left[\widehat{\omega}_i^{\beta}\widehat{\rho}_i(\widehat{\omega})^{1-\beta}\right]^{-\theta}}{\sum_{k=1}^{N}\pi_{nk}\left[\widehat{\omega}_k^{\beta}\widehat{\rho}_k(\widehat{\omega})^{1-\beta}\right]^{-\theta}}$$

Alvarez-Lucas Algorithm for Wages (1)

- The excess demand function $Z(\widehat{\omega})$ has i'th element:

$$\frac{1}{\widehat{\omega}_i}\left[\sum_{n=1}^{N}\pi'_{ni}(\widehat{\omega})\left[\widehat{\omega}_n y_n+\delta'_n-\frac{1-\beta}{\alpha}\delta_n^{M\prime}\right]-\left[\widehat{\omega}_i y_i+\delta'_i-\frac{1}{\alpha}\delta_i^{M\prime}\right]\right],$$

  with $Z_i(\widehat{\omega})=0$ for all i when evaluated at the equilibrium relative wage changes.

- For some $\nu\in(0,1]$, define the mapping T by

$$T(\widehat{\omega})_i=\widehat{\omega}_i\left[1+\nu Z_i(\widehat{\omega})/y_i\right],$$

  so that a fixed point of T satisfies the equilibrium condition of zero excess demand.

Alvarez-Lucas Algorithm for Wages (2)

- Lucas and Alvarez (2007) give the conditions under which iterating on the mapping T will converge to the equilibrium $\widehat{\omega}$.

- A key feature is that it maps vectors in $\mathbf{\Delta}_{\widehat{\omega}}$ into new vectors in $\mathbf{\Delta}_{\widehat{\omega}}$:

$$\sum_{i=1}^{N} T(\widehat{\omega})_i y_i = 1 + \nu \sum_{i=1}^{N} \widehat{\omega}_i Z_i(\widehat{\omega}) = 1.$$

Alvarez-Lucas Algorithm for Wages (3)

- To show it has that feature:

$$\sum_{i=1}^{N}\left[\begin{array}{c}\sum_{n=1}^{N}\pi'_{ni}(\widehat{\omega})\left[\widehat{\omega}_n y_n+\delta'_n-\frac{1-\beta}{\alpha}\delta_n^{M\prime}\right]\\ -\left[\widehat{\omega}_i y_i+\delta'_i-\frac{1}{\alpha}\delta_i^{M\prime}\right]\end{array}\right]$$

$$\begin{aligned}
&= \sum_{n=1}^{N}\left[\widehat{\omega}_n y_n+\delta'_n-\frac{1-\beta}{\alpha}\delta_n^{M\prime}\right]\sum_{i=1}^{N}\pi'_{ni}(\widehat{\omega})\\
&\quad -\sum_{i=1}^{N}\left[\widehat{\omega}_i y_i+\delta'_i-\frac{1}{\alpha}\delta_i^{M\prime}\right]\\
&= \sum_{n=1}^{N}\left[\widehat{\omega}_n y_n+\delta'_n-\frac{1-\beta}{\alpha}\delta_n^{M\prime}\right]-\sum_{i=1}^{N}\left[\widehat{\omega}_i y_i+\delta'_i-\frac{1}{\alpha}\delta_i^{M\prime}\right]\\
&= -\left[\frac{1-\beta}{\alpha}+\frac{1}{\alpha}\right]\sum_{i=1}^{N}\delta_i^{M\prime}=0
\end{aligned}$$

WEEK #5: ANALYSIS of PRODUCER-LEVEL BEHAVIOR

Motivation (1)

- The majority of manufacturing exports are shipped directly from individual firms (at least firms are aware of where their production is going).

- In the 1990's, Bernard and Jensen (BPEA, 1995) and others began to analyze export data collected directly from individual producers.

- Bernard and Jensen explored the finding that exporting establishments are more productive.

- The most striking facts: (i) only about 20% export at all and (ii) those that do don't export a large fraction of sales (histogram of export intensity).

Motivation (2)

- Producer level facts seemed consistent with a Ricardian model of productivity differences in a world with trade costs.

- This combination could explain why some firms don't export, even with no fixed costs of exporting.

- Could the same model work at the micro and macro level? Would the micro data pin down parameters that we couldn't identify from macro data on bilateral trade?

- Distinction between *efficiency* as formulated in the Ricardian model and *productivity* as measured added a wrinkle we didn't expect.

Definitions

- A *plant* or *establishment* refers to an individual location of production, i.e. a factory.

- A *firm* or *enterprise* may consist of one or more plants. Most firms have only one plant, but big firms typically have many.

- The micro data can come either way.

- Most of the recent data on exports, from Customs declarations, are at the firm level.

3 Bernard, Eaton, Jensen, and Kortum (BEJK)

Firm-Level Ricardian Model

- Model the behavior of US exporting plants: explain why they are in the minority, more productive, much larger, and only dabble in the export market.

- Stick to the Ricardian model as closely as possible: goods $j \in [0, 1]$, countries $1, 2, ..., N$, CES demand, ...

- Think of a plant producing a single good j: We're just interpreting, as a plant, an object that was already in the aggregate model.

- Observe the plant producing only if it can deliver j somewhere at a lower cost than any other domestic or foreign plant.

Assumptions

- Bertrand Competition (between all potential suppliers of good j to market n) replaces Perfect Competition.

- Assumed because it captures a productivity effect (as we'll see below).

- Otherwise the same as our Macro Applications.

- With Bertrand Competition, need to consider the joint distribtion of the second-lowest cost together with the lowest cost producer in each destination n.

- (For today) L_i is the only input, i.e. ignore intermediates.

Bertrand Competition

- Sales of good j in market n:

$$X_n(j) = \left(\frac{p_n(j)}{P_n}\right)^{-(\sigma-1)} X_n$$

- Bertrand pricing:

$$p_n(j) = \min\left\{C_n^{(2)}(j), \overline{m}C_n^{(1)}\right\},$$

where $\overline{m} = \infty$ if $\sigma \leq 1$ otherwise $\overline{m} = \sigma/(\sigma - 1)$.

- The lowest cost is just like in the Ricardian model:

$$C_n^{(1)}(j) = \min_i \left\{ \frac{w_i d_{ni}}{Z_i^{(1)}(j)} \right\}.$$

- The lowest cost producer will end up being the only supplier (exactly as in perfect competition).

- Results in the aggregate should look very similar to the original model, but we need to see how the pricing will work.

Second Lowest Cost Producer

- If the low cost supplier is from i, the second lowest cost producer's cost is:

$$C_n^{(2)}(j) = \min\left\{\frac{w_i d_{ni}}{Z_i^{(2)}(j)}, \min_{l \neq i}\left\{\frac{w_l d_{nl}}{Z_l^{(1)}(j)}\right\}\right\}.$$

- If we didn't consider second most efficient $Z_i^{(2)}(j)$, there would be a bias, for example, toward the competitor of a domestic producer necessarily being a foreign producer.

- To go farther quantitatively, we need to say something about the joint distribution of $(Z_i^{(1)}(j), Z_i^{(2)}(j))$.

- Can still assess the model at a qualitative level.

Is the Model Plausible?

- Correlation between exporting and a low cost draw, and hence size.

- Correlation between a low cost and a large cost gap to second-best firm.

- Measured productivity depends on the cost gap, which generates price markups.

- Can also use model to think about pass-through of costs into prices (see Atkeson and Burstein)

Technology

- The natural generalization of the Frechet distribution is:

$$\Pr\left[Z_i^{(1)} \leq z_1, Z_i^{(2)} \leq z_2\right] = \left[1 + T_i\left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-T_i z_2^{-\theta}}.$$

- In the on-line appendix to BEJK, you can work with it that way.

- Its much easier, however, to go back to our dynamic setting. (We'll do that later)

Model as a Computer Algorithm

- In the published paper we develop the theory through analytic results.

- But some results, such as the probability of exporting, need to be simulated.

- Here, we'll simply introduce the model via the simulation algorithm.

- The analytical connection to the previous trade models is described in Lecture Notes 2.

Simulating the Model (1)

- For some hypothetical good j and each source $i = 1, .., I$, draw V_{1i}, V_{2i} i.i.d. exponential 1.

- Construct

$$
\begin{aligned}
U_i^{(1)} &= V_{1i} \\
U_i^{(2)} &= V_{1i} + V_{2i}
\end{aligned}
$$

Simulating the Model (2)

- Note that we can easily map the U's into the relevant costs:

$$
\begin{aligned}
C_{ni}^{(1)} &= \left(\frac{U_i^{(1)}}{\pi_{ni}\Phi_n}\right)^{1/\theta} \\
C_{ni}^{(2)} &= \left(\frac{U_i^{(2)}}{\pi_{ni}\Phi_n}\right)^{1/\theta} .
\end{aligned}
$$

Necessary Parameters

- Suppose we have values for θ, σ. It turns out that we don't need Φ_n's.

- Low cost supplier of good j to n is:

$$i^* = \arg\min_i \{C_{ni}^{(1)}\} = \arg\min_i \left\{ \frac{U_i^{(1)}}{\pi_{ni}} \right\}.$$

- Notice that for this prediction we don't even need to know θ or σ.

- Notice the role of the trade share $\pi_{ni} = X_{ni}/X_n$. If i has a smaller trade share in n it is less likely to turn out to be the minimum cost i^*.

The Price Markup (1)

- Markup of price over unit cost for good j in market n

$$
\begin{aligned}
M_n &= \min\left\{\frac{C_n^{(2)}}{C_n^{(1)}}, \frac{\sigma}{\sigma-1}\right\} \\
&= \min\left\{\frac{\min\left\{C_{ni^*}^{(2)}, \min\limits_{i\neq i^*}\left\{C_{ni}^{(1)}\right\}\right\}}{C_{ni^*}^{(1)}}, \frac{\sigma}{\sigma-1}\right\}.
\end{aligned}
$$

The Price Markup (2)

- In terms of normalized cost we get

$$M_n = \min\left\{\left[\frac{\min\left\{\frac{U_{i^*}^{(2)}}{\pi_{ni^*}}, \min_{i \neq i^*}\left\{\frac{U_i^{(1)}}{\pi_{ni}}\right\}\right\}}{\frac{U_{i^*}^{(1)}}{\pi_{ni^*}}}\right]^{1/\theta}, \frac{\sigma}{\sigma - 1}\right\}.$$

Size in Domestic and Export Markets

- Size of a producer in a particular market (yields export intensity):

$$
\begin{aligned}
X_n(j) &= X_n\left(\frac{P_n(j)}{P_n}\right)^{1-\sigma} = X_n\left(\frac{M_n C_n^{(1)}}{\widetilde{\gamma}\Phi_n^{-1/\theta}}\right)^{1-\sigma} \\
&= X_n\left(\frac{M_n}{\widetilde{\gamma}}\right)^{1-\sigma}\left[\frac{\left(\frac{U_{i*}^{(1)}}{\pi_{ni*}\Phi_n}\right)^{1/\theta}}{\Phi_n^{-1/\theta}}\right]^{1-\sigma} \\
&= X_n\left(\frac{M_n}{\widetilde{\gamma}}\right)^{1-\sigma}\left(\frac{U_{i*}^{(1)}}{\pi_{ni*}}\right)^{\frac{1-\sigma}{\theta}}.
\end{aligned}
$$

Apply to US-Based Producers (1)

- For a particular j, let $\Omega(j)$ be the set of countries n for which i^*turns out to be the United States.

- Every non-empty $\Omega(j)$ represents a simulation of a U.S. producer.

- Producer is an exporter if $\Omega(j)$ contains countries other than the United States.

Apply to US-Based Producers (2)

- Total sales of producers:

$$X(j) = \sum_{n \in \Omega(j)} X_n(j)$$

- Total exports:

$$\sum_{n \neq US, n \in \Omega(j)} X_n(j)$$

- Total labor:

$$L(j) = \sum_{n \in \Omega(j)} \left[\frac{X_n(j)}{M_n(j)} \right] / w = \sum_{n \in \Omega(j)} \left[Y_n(j) C_n^{(1)}(j) \right] / w.$$

Connections to Productivity

- Productivity $\frac{X(j)}{L(j)}$.

- Productivity is high iff markup is high (as we saw in Notes 2).

- But high markup is associated with high technology draw for the producer.

- High technology draw is associated with being competitive in export markets.

- High technology draw is associated with low cost, low price and hence high sales.

Results: Parameters

- Data from the 1992 US Census of Manufactures.
- Rather than fitting the distribution of sales and productivity across US plants, fit the shift in distribution between exporters and non-exporters.
- Productivity advantage (value added per worker) of U.S. exporting plants is 33%.
- Size (domestic shipments) advantage: 4.8 times larger relative to non-exporters.
- Parameter $\theta = 3.60$ and $\sigma = 3.20$ are just identified by these moments.

Results: Fitting the Facts

- With these parameters, the model explains well the distribution of export intensity across exporting plants.

- A problem: model predicts too high a fraction of US plants export (data=21%, model=51%). Given bilateral trade data, no parameter influences this prediction.

Results: Export Intensity

% exported	model	data
0-10	76	66
10-20	19	16
20-30	4.2	7.7
30-40	0.0	4.4
40-50	0.0	2.4
50-60	0.0	1.5
60-70	0.0	1.0
70-80	0.0	0.6
80-90	0.0	0.5
90-100	0.0	0.7

A Tougher Test

- In BEJK need to simulate competition around the world to calculate statistics on the exports of US plants.

- Thus, model makes predictions about US plant exports to any given destination, but data not collected by the US Census.

- French Customs has this detail for French firms.

- BEJK does fairly well at predicting how many firms sell to k or more markets (for $k = 1, ..., 113$) and how sales in France rise with k.

Challenge

- More French firms export to larger markets. BEJK predicts the number of French firms exporting to market n should be proportional to French market share in n (not on the size of market n).

- More French firms do export to countries in which France has a larger market share, but need some mechanism that makes larger markets more attractive.

- First idea: A market-specific cost of entry that doesn't scale with the size of the market.

From BEJK to Melitz and Chaney (2008)

- With fixed cost of entering a market, Bertrand competition will reduce to monopolistic competition (to simplify strategies, assume low cost supplier moves first).

- Consider a range of goods $j \in [0, J]$, for any finite value of $J \geq 1$. Firm must pass 2 hurdles to enter a market: (i) it must be the low cost supplier and (ii) it must be able to cover its entry cost.

- In either case, its only the most efficient firms that are relevant.

Limiting Result

- If firm efficiencies are drawn from the Fréchet, the measure of firms from i with efficiency greater than z is

$$\mu_i^Z(Z \geq z; J) = J\left\{1 - \exp\left[-(T_i/J)z^{-\theta}\right]\right\}.$$

- As we let J get arbitrarily large,

$$\lim_{J\to\infty} \mu_i^Z(Z \geq z; J) = T_i z^{-\theta}.$$

- Furthermore, these measures become additive as they apply to costs of supplying any market n. Thus the first hurdle drops out and only the second is relevant.

Arkolakis (2011)

- Second idea: develop a deeper theory of entry costs.
- Want to explain why you might enter a market yet sell very little.
- Want to account for very low export intensity among exporters.
- We'll develop the empirical specification with this marketing technology from the start.

4 Eaton, Kortum, and Kramarz (EKK)

A Look at the Data

- Cross-section of 230,000 French manufacturing firms, in 1986.
- Approximately 35,000 of them export somewhere.
- Observe exports to each of 112 destinations plus sales in France.
- Tables and Figures reveal some striking regularities ...

Table 1 - French Firms Exporting to the Seven Most Popular Destinations

Country	Number of Exporters	Fraction of Exporters
Belgium* (BE)	17,699	0.520
Germany (DE)	14,579	0.428
Switzerland (CH	14,173	0.416
Italy (IT)	10,643	0.313
United Kingdom (UK)	9,752	0.287
Netherlands (NL)	8,294	0.244
United States (US)	7,608	0.224
Total Exporters	34,035	

* Belgium includes Luxembourg

Table 2 - French Firms Selling to Strings of Top Seven Countries

	Number of French Exporters		
Export String	Data	Under Independence	Model
BE*	3,988	1,700	4,417
BE-DE	863	1,274	912
BE-DE-CH	579	909	402
BE-DE-CH-IT	330	414	275
BE-DE-CH-IT-UK	313	166	297
BE-DE-CH-IT-UK-NL	781	54	505
BE-DE-CH-IT-UK-NL-US	2,406	15	2,840
Total	9,260	4,532	9,648

* The string "BE" means selling to Belgium but no other among the top 7, "BE-DE" means selling to Belgium and Germany but no other, etc.

Figure 1: Entry and Sales by Market Size

Figure 2
Sales Distributions of French Firms

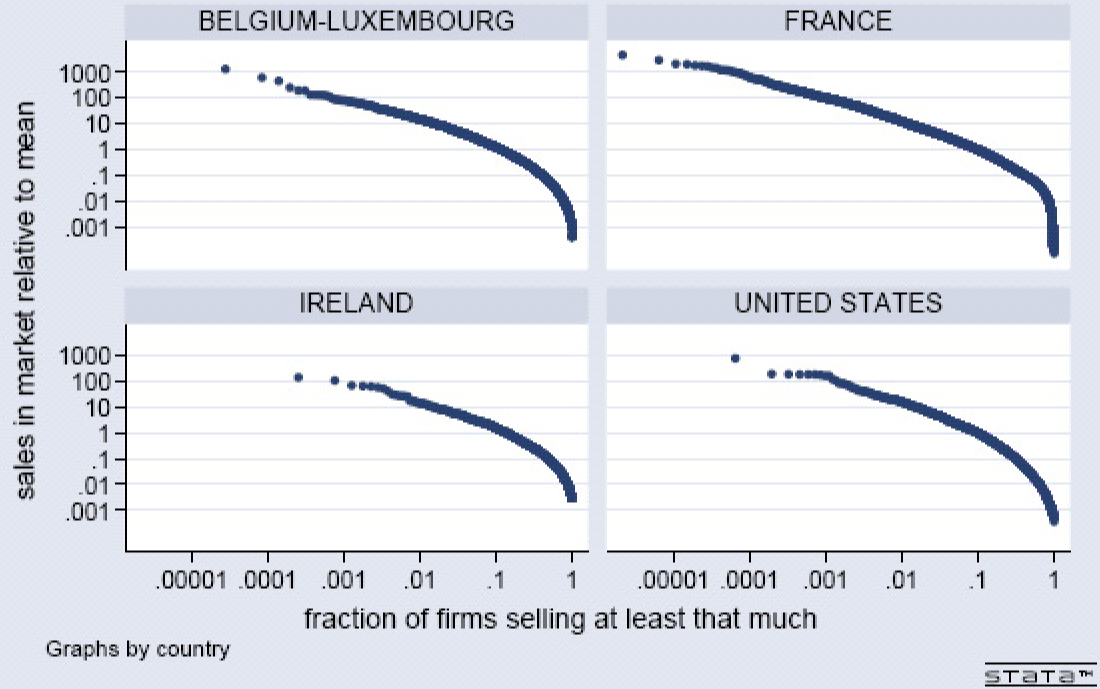
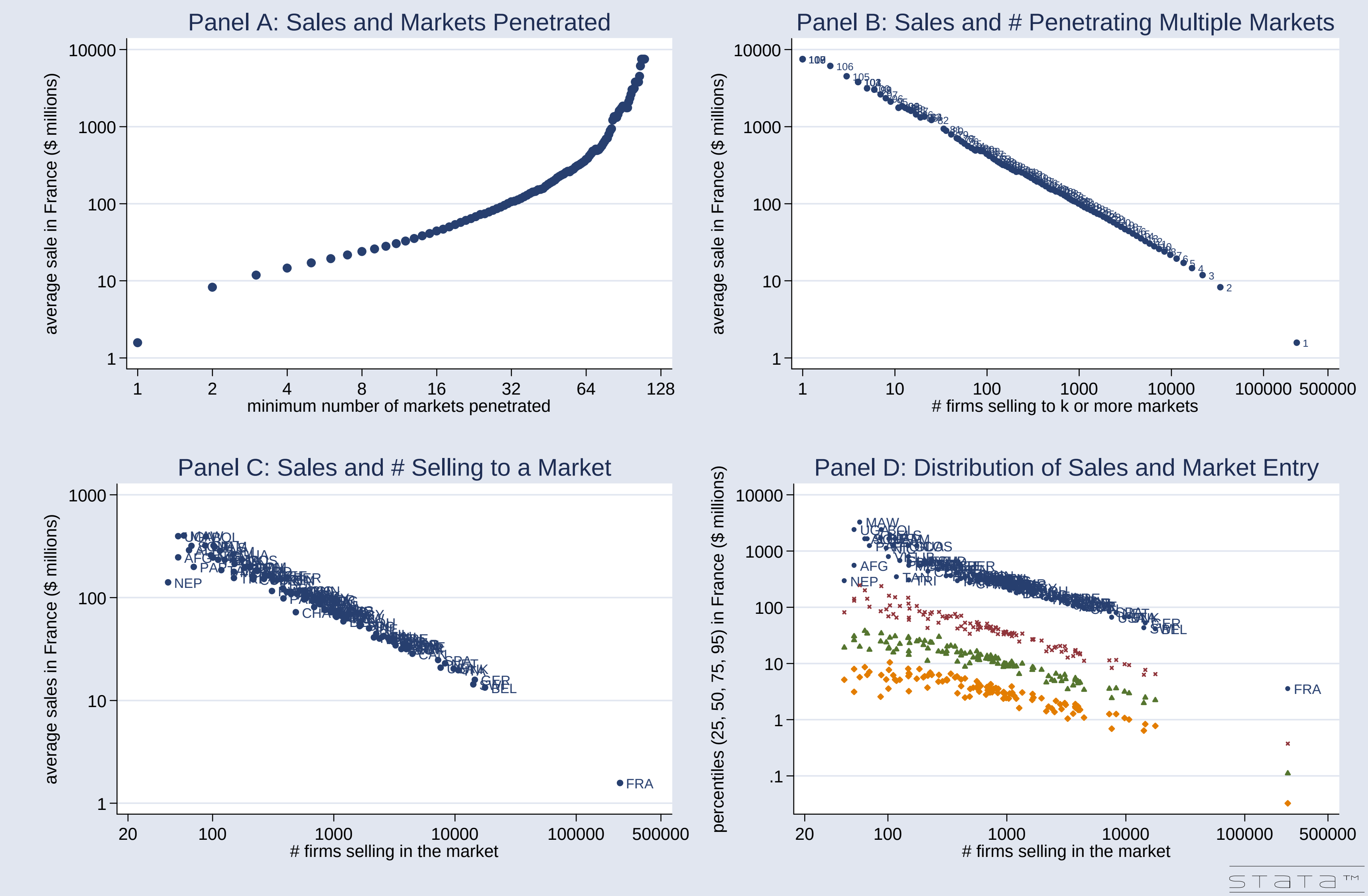


Figure 3: Sales in France and Market Entry



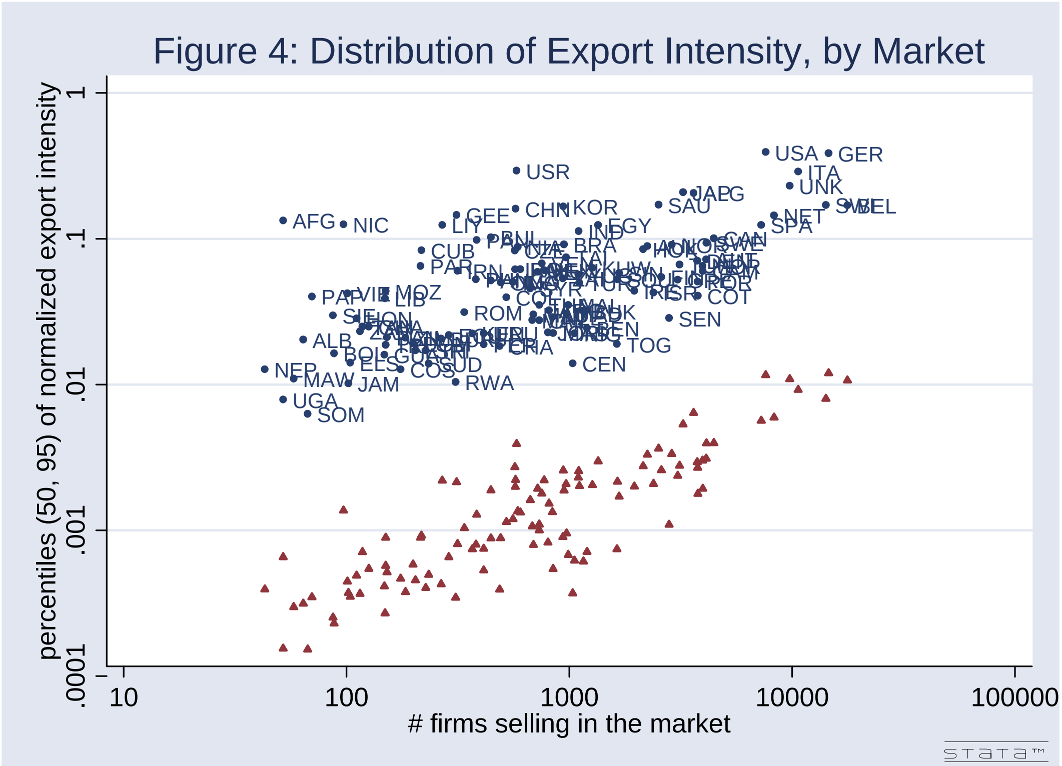


Figure 4: Distribution of Export Intensity, by Market

Lessons from the Data

- Entry patterns suggest market-specific fixed costs.
- Sales in France vs. export penetration suggests Pareto firm-level factor.
- Imperfect destination hierarchy suggests entry shocks.
- Sales distribution suggests small entry cost for some.
- Export intensity suggests market specific shocks.

Elements of the Model: I

- N countries; continuum of goods j (one good per firm).

- Measure of firms in i who can produce some good j with efficiency $> z$:

$$\mu_i^z(z) = T_i z^{-\theta} \quad z > 0.$$

- Unit cost of producing j in i for delivery to n is $c_{ni}(j) = w_i d_{ni}/z_i(j)$.

- Measure of goods that i can deliver to n at cost below c:

$$\mu_{ni}(c) = \mu_i^z(\frac{w_i d_{ni}}{c}) = T_i(w_i d_{ni})^{-\theta} = \Phi_{ni} c^{\theta}$$

Elements of the Model: II

- Firm spends $\varepsilon_n(j)E_{ni}M(f)$ to reach a fraction f of consumers in n:

$$M(f) = \frac{1 - (1-f)^{1-1/\lambda}}{1 - 1/\lambda}.$$

- If it reaches f, charging price p, its sales are:

$$X_n(j) = \alpha_n(j) f X_n \left(\frac{p}{P_n}\right)^{1-\sigma} \quad \sigma < \theta + 1.$$

- Firm from i chooses $f = 0$ if $c_{ni}(j)$ exceeds a threshold, $\bar{c}_{ni}(\eta_n(j))$.

- Entry shock $\eta_n(j) = \alpha_n(j)/\varepsilon_n(j)$; joint density $g(\alpha, \eta)$.

Analytical Solutions

- Cost threshold for entry:

$$\bar{c}_{ni}(\eta) = \left(\eta \frac{X_n}{\sigma E_{ni}}\right)^{1/(\sigma-1)} \frac{P_n}{\sigma/(\sigma-1)}.$$

- Aggregate price index:

$$P_n = \frac{\sigma}{\sigma-1}(\kappa_1 \Psi_n)^{-1/\theta} X_n^{(1/\theta)-1/(\sigma-1)},$$

where

$$\kappa_1 = \left[\frac{\theta}{\theta-(\sigma-1)} - \frac{\theta}{\theta-(\sigma-1)(1-\lambda)}\right] E[\alpha\eta^{[\theta-(\sigma-1)]/(\sigma-1)}]$$

$$\Psi_n = \sum_{i=1}^{N} \Phi_{ni}(\sigma E_{ni})^{-[\theta-(\sigma-1)]/(\sigma-1)}$$

Key Implications

- Firm from i will sell in n if its cost $c_{ni}(j)$ is below:

$$\bar{c}_{ni}(\eta_n(j)) = (\eta_n(j))^{1/(\sigma-1)} \left(\frac{X_n}{\kappa_1 \Psi_n}\right)^{1/\theta} (\sigma E_{ni})^{-1/(\sigma-1)}.$$

- If it enters, it will sell:

$$X_{ni}(j) = \varepsilon_n(j) \left[1 - \left(\frac{c_{ni}(j)}{\bar{c}_{ni}(\eta_n(j))}\right)^{\lambda(\sigma-1)}\right] \left(\frac{c_{ni}(j)}{\bar{c}_{ni}(\eta_n(j))}\right)^{-(\sigma-1)} \sigma E_{ni}.$$

- Resulting trade share of all firms from i:

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{\Phi_{ni}(\sigma E_{ni})^{-[\theta-(\sigma-1)]/(\sigma-1)}}{\Psi_n}. \qquad (1)$$

Reformulation for French Exporters

- French firm's *standardized unit cost*:

$$u(j) = T_F z_F(j)^{-\theta}.$$

- Hence, measure of French firms with $u(j) \leq u$ is simply u.

- Cost of j supplying market n:

$$c_{nF}(j) = \frac{w_F d_{nF}}{z_F(j)} = \left(\frac{u(j)}{\Phi_{nF}}\right)^{1/\theta}$$

- Note standardized unit cost, like efficiency, is common across all markets.

French Firm Entry

- The hurdle for normalized unit cost satisfies:

$$\overline{c}_{nF}(\eta_n(j)) = \left(\frac{\overline{u}_{nF}(\eta_n(j))}{\Phi_{nF}}\right)^{1/\theta}$$

- Solving it out gives:

$$\overline{u}_{nF}(\eta_n(j)) = \left(\frac{\pi_{nF}X_n}{\kappa_1 \sigma E_{nF}}\right)\eta_n(j)^{\widetilde{\theta}}.$$

- Enter market n if $u(j) \leq \overline{u}_{nF}(\eta_n(j))$.

- Key parameter $\widetilde{\theta} = \theta/(\sigma - 1)$.

French Firm Sales

- Given entry, sales in market n:

$$X_{nF}(j) = \varepsilon_n(j) \left[1 - \left(\frac{u(j)}{\overline{u}_{nF}(\eta_n(j))} \right)^{\lambda/\widetilde{\theta}} \right] \left(\frac{u(j)}{\overline{u}_{nF}(\eta_n(j))} \right)^{-1/\widetilde{\theta}} \sigma E_{nF}.$$

- Note how $u(j)$ connects entry and sales across all markets, "backbone of the firm".

- The market-specific shocks α_n and η_n scramble it up a bit.

Convenient Substitution

- Equating the integer number of French firms with the model's continuum:

$$N_{nF} = \int \overline{u}_{nF}(\eta_n) g_2(\eta_n) d\eta_n = \left(\frac{\pi_{nF} X_n}{\kappa_1 \sigma E_{nF}} \right) \kappa_2,$$

where $\kappa_2 = E[\eta^{\widetilde{\theta}}]$.

- Two useful expressions follow:

$$\sigma E_{nF} = \frac{\kappa_2}{\kappa_1} \overline{X}_{nF} \tag{2}$$

$$\overline{u}_{nF}(\eta_n(j)) = \frac{N_{nF}}{\kappa_2} \eta_n(j)^{\widetilde{\theta}} \tag{3}$$

Matching the Pictures I

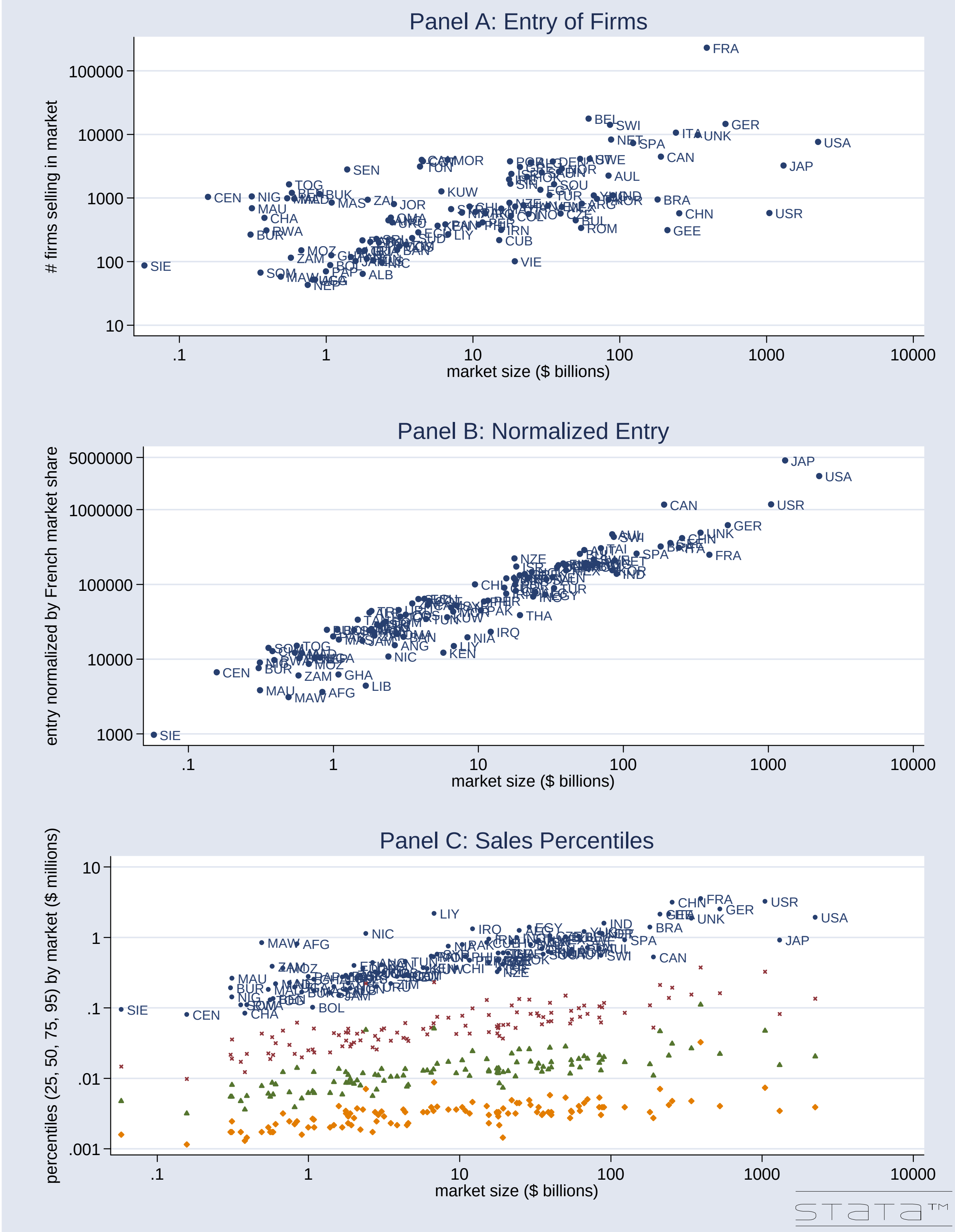
- From (2), covariates of E_{ni} [for $i =$ France, Denmark (Pedersen), Uruguay (Sampognaro)].

- Gives nice interpretation of Figure 1.

$$\frac{N_{nF}}{\pi_{nF}} = \frac{\kappa_2}{\kappa_1} \frac{X_n}{\sigma E_{nF}}$$

- In the end, just let $\overline{X}_{nF}$ absorb E_{nF} and condition on N_{nF} (don't model determinants of π_{nF}).

Figure 1: Entry and Sales by Market Size



Matching the Pictures II

- From (3), entry condition for a firm becomes

$$u(j) \leq \frac{N_{nF}}{\kappa_2} \eta_n(j)^{\widetilde{\theta}}$$

- Notice how variation in $\eta_n(j)$ regulates strength of market hierarchy.

Matching the Pictures III

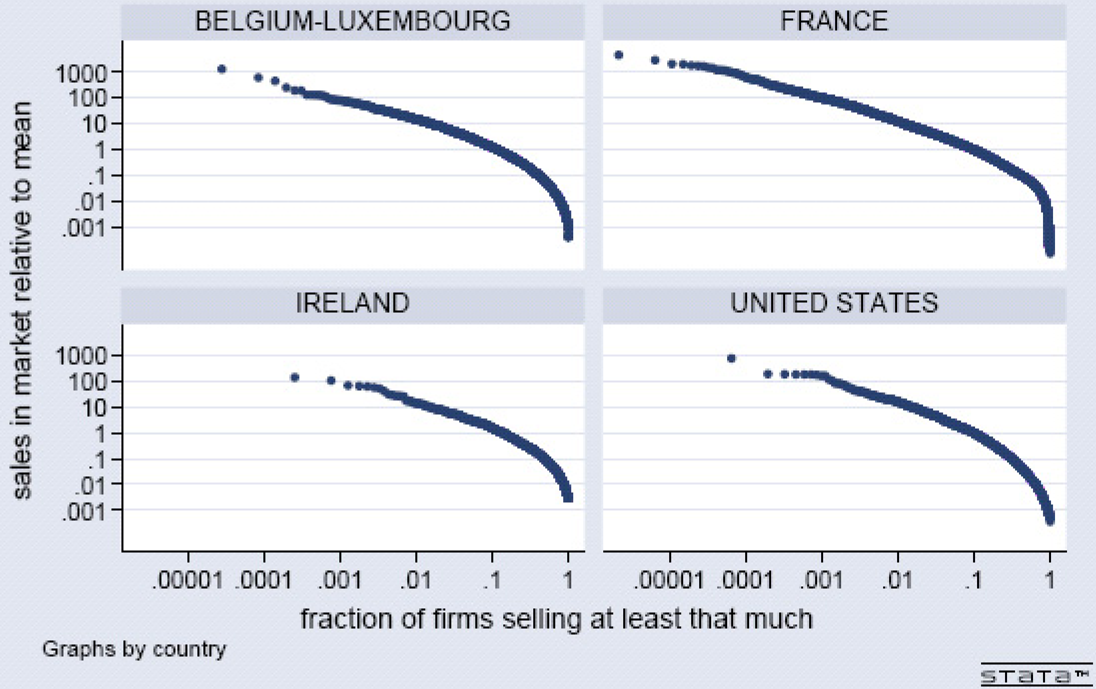
- Note that $v_{nF}(j) = u(j)/\overline{u}_{nF}(\eta_n(j))$ is uniform on $[0,1]$ across firms that sell in n.

- While $v_{nF}(j)$, unlike $u(j)$, varies by country, its distribution is common across countries.

- Sales distributions governed by:

$$X_{nF}(j) = \varepsilon_n(j)\left[1 - v_{nF}(j)^{\lambda/\widetilde{\theta}}\right] v_{nF}(j)^{-1/\widetilde{\theta}} \frac{\kappa_2}{\kappa_1} \overline{X}_{nF}.$$

Figure 2
Sales Distributions of French Firms



Matching the Pictures IV

- We know that $v_{nF}(j)$ is uniform on $[0,1]$ for firms selling in n. What about the sales in France of these firms?

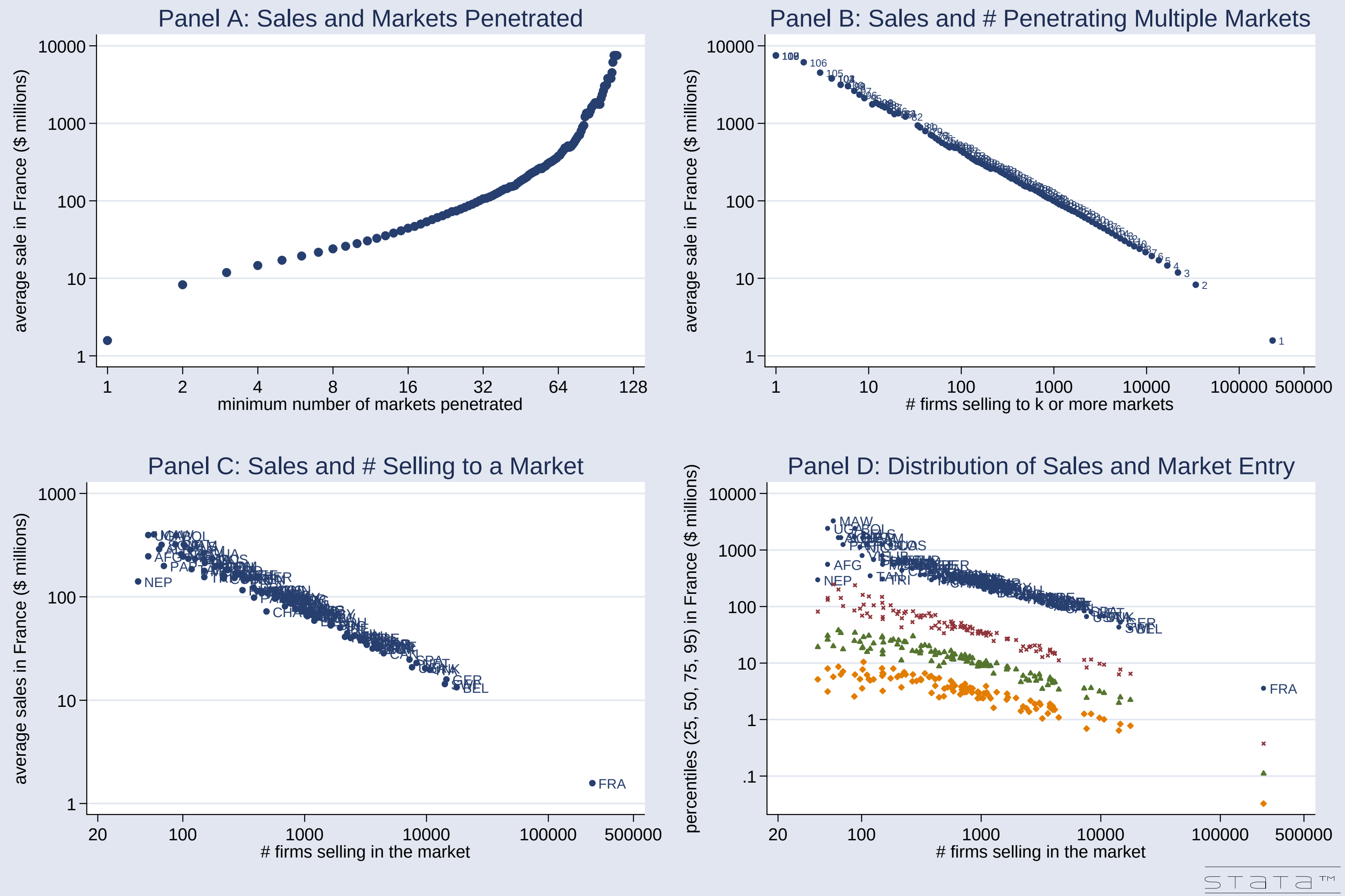
- Exploit the fact that $v_{nF}(j)/v_{FF}(j) = (N_{nF}/N_{FF})\,[\eta_n(j)/\eta_F(j)]^{\widetilde{\theta}}$

- French sales, given n ($f_{nF} \approx 1$ for $n \neq F$):

$$X_{FF}(j)|_n = \frac{\alpha_F(j)}{\eta_n(j)}\,[f_{nF}]\,v_{nF}(j)^{-1/\widetilde{\theta}}\left(\frac{N_{nF}}{N_{FF}}\right)^{-1/\widetilde{\theta}}\frac{\kappa_2}{\kappa_1}\overline{X}_{FF},$$

$$f_{nF} = 1 - v_{nF}(j)^{\lambda/\widetilde{\theta}}\left(\frac{N_{nF}}{N_{FF}}\right)^{\lambda/\widetilde{\theta}}\left(\frac{\eta_n(j)}{\eta_F(j)}\right)^{\lambda}$$

Figure 3: Sales in France and Market Entry



Completing the Specification

- Assume bivariate normal shocks: $(\ln \alpha_n, \ln \eta_n)$.

- Five parameters:

$$\Theta = (\widetilde{\theta},\ \sigma_a^2,\ \sigma_h^2,\ \rho_{ah},\ \lambda)$$

- Analytical expressions for κ_1 and κ_2 in terms of these parameters.

- Calibrate σE_{nF} and $\overline{u}_{nF}(.)$ (given other 5 parameters) using data on $\overline{X}_{nF}$ and N_{nF}.

Simulation Algorithm I

- Once and for all, draw $s = 1, ..., S$ shock vectors: standard normals (2 for each market) and uniforms $v(s)$ (to create firm backbone).

- Given a proposed Θ: calculate κ_1, κ_2, σE_{nF}, and $(\ln \alpha_n(s), \ln \eta_n(s))$.

- Construct hurdles (streamlined notation):

$$\overline{u}_n(s) = \frac{N_{nF}}{\kappa_2} \eta_n(s)^{\widetilde{\theta}}$$

- Calculate hurdle for selling in France and exporting somewhere:

$$\overline{u}(s) = \min\left(\overline{u}_F(s), \max_{n \neq F} \{\overline{u}_n(s)\}\right).$$

Simulation Algorithm II

- To simulate only exporters selling in France: $u(s) = v(s)\overline{u}(s)$.

- Importance weight $\overline{u}(s)$.

- Sells in n iff $u(s) \leq \overline{u}_n(s)$.

- Sales given entry:

$$X_{nF}(s) = \frac{\alpha_n(j)}{\eta_n(j)} \left[1 - \left(\frac{u(s)}{\overline{u}_n(s)} \right)^{\lambda/\widetilde{\theta}} \right] \left(\frac{u(s)}{\overline{u}_n(s)} \right)^{-1/\widetilde{\theta}} \sigma E_{nF}.$$

Simulation Algorithm III

- Let $\delta^k(s)$ indicate a firm achieving some outcome k.

- Simulate the number of such firms by:

$$\widehat{N}^k = \frac{1}{S} \sum_{s=1}^{S} \overline{u}(s) \delta^k(s).$$

Estimate by Simulated Method of Moments

- Fit to 4 sets of moments defined by acheiving an observable outcome:

- Firms selling to each of the 2^7 strings of top-7 destinations.

- Firms selling in n with sales there falling into a bin formed by 50th, 75th, and 95th percentiles.

- Firms selling in n with sales in France falling into a bin formed by 50th, 75th, and 95th percentiles.

- Firms selling in n with normalize export intensity falling into a bin defined by 50th and 75th percentiles.

Results

- Parameter estimates $\widehat{\Theta}$:

$\widetilde{\theta}$	λ	σ_a	σ_h	ρ
2.46	0.91	1.69	0.34	-0.65
(0.10)	(0.12)	(0.03)	(0.01)	(0.03)

- Standandard errors from resampling the data and reestimating the parameters 25 times.

- Fair amount of uncertainty about $\widetilde{\theta}$ and λ.

- Huge implied variance of $\ln \varepsilon$ is $\sigma_e^2 = \sigma_a^2 + \sigma_h^2 - \rho_{ah}\sigma_a\sigma_h = 2.52$.

Model Fit

- Sales distributions by market.
- Sales distributions in France, given entry in n.
- Market-specific normalized export intensity.
- Sales to strings of 7 most popular destinations (see earlier table).

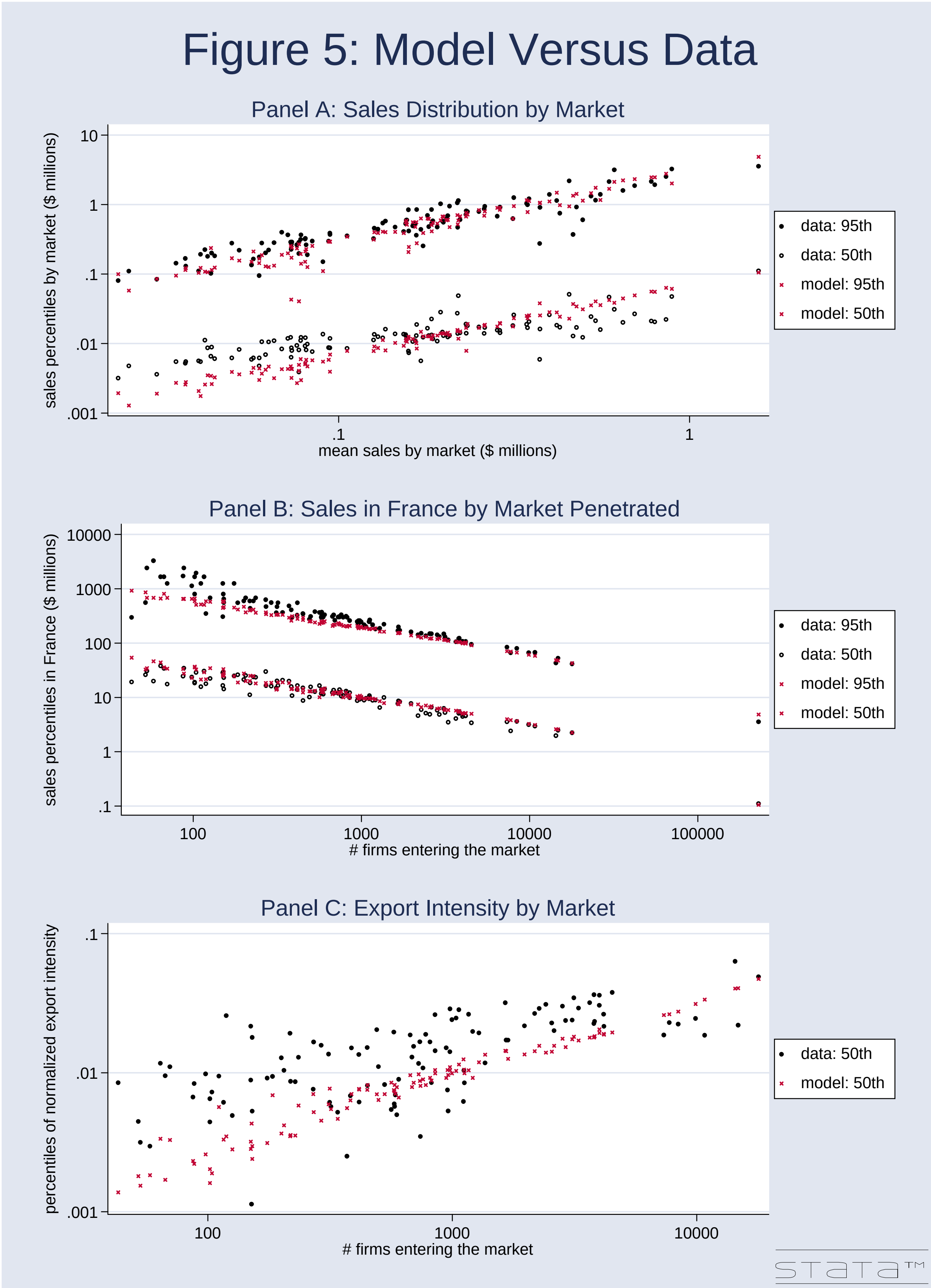


Figure 5: Model Versus Data
Panel A: Sales Distribution by Market
sales percentiles by market ($ millions)
10
1
.1
.01
.001
.1
1
mean sales by market ($ millions)
data: 95th
data: 50th
model: 95th
model: 50th
Panel B: Sales in France by Market Penetrated
sales percentiles in France ($ millions)
10000
1000
100
10
1
.1
100
1000
10000
100000
firms entering the market
data: 95th
data: 50th
model: 95th
model: 50th
Panel C: Export Intensity by Market
percentiles of normalized export intensity
.1
.01
.001
100
1000
10000
firms entering the market
data: 50th
model: 50th

Implications

- Given the backbone of the firm, $u(j)$, can explain nearly 60% of variation in entry.

- But, only 5 to 40% of the variation in sales in a market (depending on treatment of interaction with η_n).

- Sales variation is mostly due to a market-specific shocks.

Counterfactual

- Model aggregates nicely for general equilbrium analysis.
- Use methodology from Dekle, Eaton, and Kortum (2008).
- Simulate the consequences of a 10% decline in all trade costs.
- Effects on entry, exit, and size for French firms.

Table 3 - Aggregate Outcomes of Counterfactual Experiment (first of two panels)

		Counterfactual Changes (ratio of counterfactual to baseline)			
Country	Code	Real Wage	Relative Wage	Sales of French Firms	Number of French Firms
AFGHANISTAN	AFG	1.01	0.92	1.22	1.23
ALBANIA	ALB	1.01	0.94	1.35	1.34
ALGERIA	ALG	1.00	0.90	1.09	1.12
ANGOLA	ANG	1.00	0.90	1.08	1.10
ARGENTINA	ARG	1.01	0.96	1.57	1.52
AUSTRALIA	AUL	1.02	0.96	1.35	1.29
AUSTRIA	AUT	1.04	1.04	1.49	1.32
BANGLADESH	BAN	1.01	0.95	1.37	1.33
BELGIUM*	BEL	1.09	1.11	1.44	1.19
BENIN	BEN	1.02	0.94	1.12	1.09
BOLIVIA	BOL	1.02	0.94	1.21	1.18
BRAZIL	BRA	1.01	0.96	1.64	1.57
BULGARIA	BUL	1.02	0.95	1.38	1.34
BURKINA FASO	BUK	1.01	0.93	1.17	1.17
BURUNDI	BUR	1.01	0.92	1.21	1.21
CAMEROON	CAM	1.01	0.92	1.19	1.19
CANADA	CAN	1.04	1.05	1.43	1.26
CENTRAL AFRICAN REPUBLIC	CEN	1.02	1.03	1.33	1.20
CHAD	CHA	1.01	0.90	1.07	1.10
CHILE	CHI	1.03	1.02	1.53	1.38
CHINA	CHN	1.01	0.94	1.38	1.36
COLOMBIA	COL	1.01	0.92	1.22	1.23
COSTA RICA	COS	1.02	0.94	1.22	1.20
COTE D'IVOIRE	COT	1.03	0.98	1.36	1.28
CUBA	CUB	1.01	0.93	1.26	1.24
CZECHOSLOVAKIA	CZE	1.03	1.01	1.52	1.38
DENMARK	DEN	1.04	1.06	1.46	1.27
DOMINICAN REPUBLIC	DOM	1.04	0.99	1.36	1.28
ECUADOR	ECU	1.02	0.96	1.33	1.28
EGYPT	EGY	1.02	0.92	1.12	1.12
EL SALVADOR	ELS	1.02	0.93	1.10	1.10
ETHIOPIA	ETH	1.01	0.92	1.08	1.09
FINLAND	FIN	1.03	1.02	1.53	1.38
FRANCE	FRA	1.02	1.00	0.95	0.88
GERMANY, EAST	GEE	1.01	0.96	1.58	1.52
GERMANY, WEST	GER	1.03	1.02	1.60	1.45
GHANA	GHA	1.02	0.99	1.38	1.28
GREECE	GRE	1.02	0.97	1.37	1.30
GUATEMALA	GUA	1.01	0.92	1.16	1.17
HONDURAS	HON	1.02	0.95	1.19	1.16
HONG KONG	HOK	1.14	1.20	1.33	1.02
HUNGARY	HUN	1.05	1.04	1.41	1.25
INDIA	IND	1.01	0.95	1.40	1.37
INDONESIA	INO	1.02	0.96	1.44	1.38
IRAN	IRN	1.01	0.93	1.16	1.16
IRAQ	IRQ	1.04	0.94	1.08	1.06
IRELAND	IRE	1.07	1.09	1.43	1.21
ISRAEL	ISR	1.04	1.01	1.44	1.31
ITALY	ITA	1.02	0.99	1.57	1.46
JAMAICA	JAM	1.05	1.02	1.35	1.22
JAPAN	JAP	1.01	0.98	1.80	1.69
JORDAN	JOR	1.03	0.95	1.16	1.13
KENYA	KEN	1.01	0.93	1.18	1.18
KOREA, SOUTH	KOR	1.04	1.04	1.58	1.40
KUWAIT	KUW	1.02	0.94	1.14	1.12

* Belgium includes Luxembourg

Country	Code	Counterfactual Changes (ratio of counterfactual to baseline) Real Wage	Relative Wage	Sales of French Firms	Number of French Firms
LIBERIA	LIB	1.49	1.03	1.27	1.14
LIBYA	LIY	1.02	0.95	1.10	1.07
MADAGASCAR	MAD	1.01	0.94	1.22	1.20
MALAWI	MAW	1.01	0.92	1.17	1.17
MALAYSIA	MAY	1.07	1.08	1.45	1.23
MALI	MAL	1.02	0.95	1.15	1.12
MAURITANIA	MAU	1.08	1.19	1.36	1.05
MAURITIUS	MAS	1.07	1.05	1.47	1.29
MEXICO	MEX	1.01	0.94	1.32	1.30
MOROCCO	MOR	1.02	0.98	1.37	1.30
MOZAMBIQUE	MOZ	1.01	0.94	1.29	1.26
NEPAL	NEP	1.01	1.01	1.35	1.24
NETHERLANDS	NET	1.06	1.14	1.41	1.14
NEW ZEALAND	NZE	1.03	1.00	1.45	1.33
NICARAGUA	NIC	1.01	0.89	1.06	1.09
NIGER	NIG	1.02	1.09	1.47	1.25
NIGERIA	NIA	1.00	0.89	1.07	1.12
NORWAY	NOR	1.04	1.03	1.38	1.23
OMAN	OMA	1.04	0.99	1.11	1.03
PAKISTAN	PAK	1.02	0.97	1.41	1.34
PANAMA	PAN	1.09	0.96	1.15	1.10
PAPUA NEW GUINEA	PAP	1.07	1.09	1.33	1.13
PARAGUAY	PAR	1.01	0.93	1.21	1.20
PERU	PER	1.02	0.97	1.39	1.32
PHILIPPIPINES	PHI	1.02	0.97	1.50	1.43
PORTUGAL	POR	1.03	1.03	1.47	1.32
ROMANIA	ROM	1.01	0.97	1.68	1.61
RWANDA	RWA	1.00	0.90	1.14	1.17
SAUDI ARABIA	SAU	1.02	0.95	1.15	1.11
SENEGAL	SEN	1.03	1.01	1.36	1.24
SIERRA LEONE	SIE	1.03	1.17	1.36	1.08
SINGAPORE	SIN	1.24	1.15	1.37	1.10
SOMALIA	SOM	1.03	0.96	1.09	1.05
SOUTH AFRICA	SOU	1.03	1.01	1.56	1.43
SPAIN	SPA	1.02	0.97	1.49	1.42
SRI LANKA	SRI	1.03	0.99	1.34	1.24
SUDAN	SUD	1.00	0.91	1.13	1.15
SWEDEN	SWE	1.04	1.05	1.51	1.33
SWITZERLAND	SWI	1.05	1.05	1.48	1.31
SYRIA	SYR	1.02	0.96	1.20	1.15
TAIWAN	TAI	1.04	1.05	1.64	1.44
TANZANIA	TAN	1.01	0.94	1.15	1.13
THAILAND	THA	1.03	0.99	1.50	1.40
TOGO	TOG	1.03	0.96	1.11	1.07
TRINIDAD AND TOBOGO	TRI	1.04	1.01	1.22	1.12
TUNISIA	TUN	1.04	1.00	1.36	1.26
TURKEY	TUR	1.01	0.95	1.37	1.33
UGANDA	UGA	1.00	0.90	1.06	1.08
UNITED KINGDOM	UNK	1.03	1.00	1.46	1.35
UNITED STATES	USA	1.01	0.96	1.45	1.40
URUGUAY	URU	1.02	1.00	1.65	1.53
USSR	USR	1.00	0.92	1.32	1.33
VENEZUELA	VEN	1.01	0.91	1.18	1.20
VIETNAM	VIE	1.01	0.95	1.37	1.33
YUGOSLAVIA	YUG	1.02	0.97	1.48	1.41
ZAIRE	ZAI	1.06	1.21	1.37	1.04
ZAMBIA	ZAM	1.03	1.12	1.49	1.22
ZIMBABWE	ZIM	1.02	0.97	1.43	1.36

Table 4 - Counterfactuals: Firm Totals

		Counterfactual	
	Baseline	Change from Baseline	Percentage Change
Number:			
All Firms	231,402	-26,589	-11.5
Exporting	32,969	10,716	32.5
Values ($ millions):			
Total Sales	436,144	16,442	3.8
Domestic Sales	362,386	-18,093	-5.0
Exports	73,758	34,534	46.8

Counterfactual simulation of a 10% decline in trade costs.

Table 5 - Counterfactuals: Firm Entry and Exit by Initial Size

	All Firms			Exporters		
		Counterfactual			Counterfactual	
Initial Size Interval (percentile)	Baseline # of Firms	Change from Baseline	Change in %	Baseline # of Firms	Change from Baseline	Change in %
not active	0	1,118	---	0	1,118	---
0 to 10	23,140	-11,551	-49.9	767	15	2.0
10 to 20	23,140	-5,702	-24.6	141	78	55.1
20 to 30	23,140	-3,759	-16.2	181	192	106.1
30 to 40	23,140	-2,486	-10.7	357	357	100.0
40 to 50	23,140	-1,704	-7.4	742	614	82.8
50 to 60	23,138	-1,141	-4.9	1,392	904	65.0
60 to 70	23,142	-726	-3.1	2,450	1,343	54.8
70 to 80	23,140	-405	-1.8	4,286	1,829	42.7
80 to 90	23,140	-195	-0.8	7,677	2,290	29.8
90 to 99	20,826	-38	-0.2	12,807	1,915	15.0
99 to 100	2,314	0	0.0	2,169	62	2.8
Totals	231,402	-26,589		32,969	10,716	

Table 6 - Counterfactuals: Firm Growth by Initial Size

	Total Sales			Exports		
		Counterfactual			Counterfactual	
Initial Size Interval (percentile)	Baseline in $millions	Change from Baseline	Change in %	Baseline in $millions	Change from Baseline	Change in %
not active	0	3	---	0	3	---
0 to 10	41	-24	-58.0	1	2	345.4
10 to 20	190	-91	-47.7	1	2	260.3
20 to 30	469	-183	-39.0	1	3	266.7
30 to 40	953	-308	-32.3	2	7	391.9
40 to 50	1,793	-476	-26.6	6	18	307.8
50 to 60	3,299	-712	-21.6	18	48	269.7
60 to 70	6,188	-1,043	-16.9	58	130	223.0
70 to 80	12,548	-1,506	-12.0	206	391	189.5
80 to 90	31,268	-1,951	-6.2	1,085	1,501	138.4
90 to 99	148,676	4,029	2.7	16,080	11,943	74.3
99 to 100	230,718	18,703	8.1	56,301	20,486	36.4
Totals	436,144	16,442		73,758	34,534	

Conclusion

- Basic trade model with heterogeneity goes a long way in explaining producer-level behavior.

- Strong evidence for the convenient Pareto distribution of heterogeneity.

- Yet, sales in a given market is largely the result of idiosyncratic shocks.

- In general equilbrium, reducing trade costs has very skewed effects across firms.