Trade, Growth, and Macroeconomics: A Quantitative Approach

Mainz: August 2012

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1 Dornbusch, Fischer, and Samuelson (1977)

A Ricardian Model of Trade

- Ricardian model puts comparative advantage front and center.

- DFS invented a simple version with a continuum of tradable goods $j \in [0, 1]$.

- Efficiency of a worker producing good j in home country is $z(j)$.

- Efficiency of a foreign worker is $z^*(j)$.

- Order goods to create a "chain of comparative advantage" $A(j) = z(j)/z^*(j)$ so that $A'(j) < 0$.

Specialization

- Wages of workers: w and w^*, with $\omega = w/w^*$.

- Consider some arbitrary ω and associated j such that $A(j) = \omega$.

- Given the ordering, goods left of j are cheaper to produce in home and goods right of j are cheaper to produce in foreign.

- Any equilibrium of the model involves home specializing in goods to the left and foreign in goods to the right.

The Rest of the DFS Model

- Preferences (Cobb Douglas) lead you to spend the same amount on each good:
$$U = \exp\left\{ \int_0^1 \ln c(j) dj \right\}.$$

- (DFS have a more complicated formulation in which expenditure shares differ across goods. That part is not enlightening so we drop it.)

- Number of workers in home country and abroad(*): L and L^*.

- Home's GDP is simply wL.

- Perfect competition.

Equilibrium

- Equilibrium is a cutoff $\bar{j}$ and relative wage $\bar{\omega}$ such that:
  - Good j' produced in home country iff $j' \leq \bar{j}$ where
  $$A(\bar{j}) = \bar{\omega}.$$
  - Labor market clears:
  $$wL = \bar{j}(wL + w^* L^*)$$
  i.e.
  $$\bar{\omega}L = \bar{j}(\bar{\omega}L + L^*)$$

Equilibrium Diagram

- Write labor-market clearing as:

$$\omega = \left(\frac{j}{1-j}\right)\frac{L^*}{L}.$$

- Now we can plot it along with $A(j)$, with j on the horizontal axis.

- Crossing point determines relative wage $\bar{\omega}$, specialization in production, and home's share of world GDP $\bar{j}$.

2 Handy Parameterization of DFS

Let's Put DFS to Work

- Parameterize labor efficiency as:

$$z(j) = T^{1/\theta} j^{-1/\theta}$$

$$z^*(j) = T^{*1/\theta}(1-j)^{-1/\theta}.$$

- Implies a relative productivity curve:

$$A(j) = \left(\frac{T}{T^*}\right)^{1/\theta} \left(\frac{1-j}{j}\right)^{1/\theta}.$$

Home's Trade Share

- Set $A(j) = \omega$:

$$\left(\frac{w}{w*}\right)^{\theta} = \left(\frac{T}{T*}\right)\left(\frac{1-j}{j}\right).$$

- Can invert to solve for the home-country trade share:

$$j = \frac{Tw^{-\theta}}{Tw^{-\theta} + T*\left(w*\right)^{-\theta}}. \tag{1}$$

- This expression is conditional on the wage, so we're not done yet.

Equilibrium Wage

- Combine the two conditions:

$$\bar{\omega} = A(\bar{j}) = \left(\frac{T}{T^*}\right)^{1/\theta} \left(\frac{1-\bar{j}}{\bar{j}}\right)^{1/\theta}.$$

$$\bar{\omega}\frac{L}{L^*} = \frac{\bar{j}}{1-\bar{j}}.$$

- To get:

$$\bar{\omega} = (T/T^*)^{1/\theta} \left(\bar{\omega}\frac{L}{L^*}\right)^{-1/\theta}.$$

Solution

- Hence:

$$\bar{\omega} = (T/T^*)^{1/(1+\theta)}(L/L^*)^{-1/(1+\theta)}.$$

- Wage is proportional to

$$\tau = (T/L)^{1/(1+\theta)}.$$

- Home's share of world GDP is:

$$\bar{j} = \frac{Y}{Y + Y^*} = \frac{wL}{wL + w^*L^*} = \frac{\tau L}{\tau L + \tau^* L^*}. \qquad (2)$$

Price Level

- Prices typically get neglected in analysis of DFS, yet they are crucial for welfare analysis.

- Exact price index for tradables:

$$p = \exp\left(\int_0^{\bar{j}} [\ln w - \ln z(j)]dj + \int_{\bar{j}}^1 [\ln w^* - \ln z^*(j)]dj\right)$$

$$= e^{-1/\theta}\left[Tw^{-\theta} + T^* w^{*-\theta}\right]^{-1/\theta}. \tag{3}$$

Real Wage

- Substituting (3) into (1) gives:

$$\bar{j} = eT\left(\frac{w}{p}\right)^{-\theta}$$

or

$$\frac{w}{p} = e^{1/\theta}T^{1/\theta}\left(\bar{j}\right)^{-1/\theta}$$

- Home's real wage is increasing in its technology but decreasing in its purchases from local producers.

Gains from Trade

- Get Autarky real wage by setting $\bar{j} = 1$, i.e. $(w/p)^A = e^{1/\theta} T^{1/\theta}$.

- Increase in real wage due to trade:

$$\frac{w/p}{(w/p)^A} = \left(\bar{j}\right)^{-1/\theta} = \left(\frac{Y + Y^*}{Y}\right)^{1/\theta}.$$

  where GDP's are evaluated under free trade.

- Substituting in (2):

$$\frac{w}{p} = e^{1/\theta} \tau \left(\tau L + \tau^* L^*\right)^{1/\theta},$$

which shows the gains to home from foreign technology (due to trade).

3 Non-Traded Goods in DFS

What's Missing?

- Does it make sense that anything in GDP could have been imported?
- Implies that home country's import share is foreign's share of world GDP.
- Sounds like a trivial change, but assume that a fraction $1 - \alpha$ of what we spend is non-tradable.
- Non-tradable goods, of course, have to be made locally.
- Once you start to think about it, you realize α might be pretty small.

What is Tradable? (value added shares of GDP)

US INDUSTRY	1977	2005	2011
Agriculture	2.5	1.0	1.2
Mining	2.1	1.9	1.9
Construction	4.6	4.9	3.4
Manufacturing	21.6	12.1	12.2
Utilities	2.3	2.0	1.7
Wholesale	6.6	6.0	5.6
Retail	7.8	6.6	6.1
Transportation	3.8	2.8	2.8
Information	3.5	4.5	4.4
FIRE	15.0	20.4	19.9
Bus. Serv.	6.0	11.7	12.6
Educ. and Health	4.6	7.8	8.7
Entertainment	2.9	3.6	3.9
Other Serv.	2.3	2.3	2.4
Government	14.4	12.6	13.2

DFS with Non-Traded Goods

- Assume preferences lead you to spend a share α on tradable goods and $1-\alpha$ on nontradables (which you can either import or buy locally).

- Assuming trade balance, market clearing is:

$$\alpha w L = \alpha \bar{j}(wL + w^* L^*).$$

- The α's cancel so, with trade balance, we haven't changed a thing about the determination of relative GDP's.

Implications for the Price Level

- The price of tradables will be equated across countries:

$$p_T = e^{-1/\theta} \left[T w^{-\theta} + T^* w^{*-\theta} \right]^{-1/\theta}.$$

- But, the overall price level is

$$P = (p_T)^{\alpha} (p_N)^{1-\alpha}$$

- If κ is the efficiency of home (relative to foreign) in producing non-tradables, then:

$$\frac{p_N}{p_N^*} = \frac{\omega}{\kappa}$$

- Relative overall price level:

$$\frac{P}{P^*} = (\omega/\kappa)^{1-\alpha}.$$

- Deviations from PPP: price level is higher in rich country if its relative wage exceeds its relative productivity in services, as is typically the case (think of haircuts).

4 Introducing Trade Imbalances

Issues

- Can we incorporate trade deficits into this analysis?

- How would eliminating trade imbalances change the equilibrium?

- Simple idea is to think of trade imbalances as transfers.

Imbalance?

- Data for 2010 (in percent) on trade in goods and services:

	U.S.	Germany	Greece
exports/GDP	12.7	46.8	21.5
imports/GDP	16.3	41.3	30.4
deficit/GDP	3.6	-5.5	8.9

Related Literature

- The "Transfer Problem" debated by Keynes, Ohlin and others.
- Dornbusch, Fischer, and Samuelson (1977) analysis in a 2-country Ricardian model (DFS).
- Series of papers by Obstfeld and Rogoff (2000, ..., 2005).
- Dekle, Eaton, and Kortum (2007). Appendix in NBER Working Paper #13035.

Neutrality

- Say $\alpha = 1$, i.e. all goods are tradable.

- With trade balance, equating home's imports $(1-\bar{j})wL$ to home's exports $\bar{j}w^*L^*$:

$$\bar{\omega} = \frac{\bar{j}}{1-\bar{j}}\frac{L^*}{L}.$$

- Equilibrium pair $(\bar{\omega}, \bar{j})$ solves the equation above together with $A(\bar{j}) = \bar{\omega}$.

- Get same equilibrium with a home-country trade deficit of D (it cancels out):

$$(1-\bar{j})(wL + D) = \bar{j}(w^*L^* - D) + D. \qquad (4)$$

Thoughts on Neutrality

- If we cancel out the D's (which algebra allow us to do), get:

$$(1-\bar{j})wL = \bar{j}w^*L^*.$$

- Looks like imports=exports, so what happened to the trade imbalance?

- Answer: with a non-zero deficit, imports are $(1-\bar{j})(wL+D)$ not $(1-\bar{j})wL$.

- We can rewrite (4) equivalently by adding $\bar{j}(wL+D)$ to each side:

$$wL + D = \bar{j}(w^*L^* - D + wL + D) + D.$$

- Cancelling out the D's:

$$wL = \bar{j}(w^* L^* + wL)$$

which is also valid whether or not we impose $D = 0$.

Summary

- So far we've seen that:
  - Without trade deficits, non-traded goods make no difference for the determination of $\bar{\omega}$ and $\bar{j}$ in DFS.
  - Without non-traded goods, trade deficits make no difference for the determination of $\bar{\omega}$ and $\bar{j}$ in DFS.
  - So, what's the point?
- Its the combination of the two, deficits and non-traded goods, that delivers some new results for DFS.

Nontradables and the Deficit

- A source of home bias: share $\alpha < 1$ spent on tradables.

- Goods market clearing condition becomes:

$$\alpha(1 - \bar{j})(wL + D) = \alpha\bar{j}(w^*L^* - D) + D.$$

- Can also write as

$$wL = \bar{j}(wL + w^*L^*) + \frac{1-\alpha}{\alpha}D$$

- Adjustment to a reduced deficit occur via low w, which reduces the left side (lower income) and raises the right side (higher market share of home).

- To put this equation into the DFS figure, rewrite it as:

$$
\begin{aligned}
\omega &= \frac{\bar{j}}{1-\bar{j}}\frac{L^*}{L} + \frac{(1-\alpha)D}{\alpha(1-\bar{j})w^*L} \\
&= \left[\frac{\bar{j}}{1-\bar{j}} + \frac{(1-\alpha)D}{\alpha(1-\bar{j})Y^*}\right]\frac{L^*}{L}.
\end{aligned}
$$

Link from Deficit to Production

- Saw above that we get an upward sloping curve, which shifts up with D:

$$\omega = \left[\frac{\bar{j}}{1-\bar{j}} + \frac{(1-\alpha)D}{\alpha(1-\bar{j})Y*}\right]\frac{L^*}{L}.$$

- Now the curve shifts as long as $D \neq 0$ and $\alpha \neq 1$. In that case it is shifted up when $D > 0$.

- Reduction in home's deficit D results in lower relative wage ω and broader range $\bar{j}$ of tradables produced in home.

- Production of tradables as a share of home GDP rises (since ω falls):

$$\lambda = \frac{\alpha\bar{j}(wL + w^*L^*)}{wL} = \alpha\bar{j}\left(1 + \frac{L^*}{\omega L}\right).$$

4.1 Quantitative Results

What Does Model Suggest for the United States?

- GDP's $Y = 13.2$, $Y^* = 34.0$, US exports $X = 1.4$, US imports $I = 2.2$, and deficit $D = 0.8$ ($ trillions) in 2006.

- Share of US exports in total ROW spending:

$$\alpha \bar{j} = \frac{X}{Y* - D} = 0.04$$

- Share of ROW exports (US imports) in total US spending:

$$\alpha(1 - \bar{j}) = \frac{I}{Y + D} = 0.16.$$

- Logic of the model implies $\alpha = 0.2$ and $\bar{j} = 0.2$.

Parameterization

- Parameterize $A(j)$ as above:

$$A(j) = \left(\frac{T}{T^*}\right)^{1/\theta} \left(\frac{1-j}{j}\right)^{1/\theta}.$$

- (The slope of the relative productivity curve is proportional to $-1/\theta$ at $j = 1/2$.)

- Inverting the equation $A(\bar{j}) = \omega$ gives,

$$\bar{j} = \frac{T\omega^{-\theta}}{T\omega^{-\theta} + T^*}.$$

Counterfactual (1)

- Exogenous change of D to $D' = 0$. Given w^*, what happens to w? i.e to

$$\hat{w} = w'/w = \omega'/\omega = \hat{\omega}.$$

- Note that counterfactual GDP is $Y' = w'L = Y\hat{\omega}$ while $Y^{*'} = Y^*$.

- Trick to calculate counterfactual threshold good:

$$\bar{j}' = \frac{T\omega'^{-\theta}}{T\omega'^{-\theta} + T^*} = \frac{\bar{j}\hat{\omega}^{-\theta}}{\bar{j}\hat{\omega}^{-\theta} + (1 - \bar{j})}.$$

Counterfactual (2)

- Note that we didn't need to measure technology T, T^*, or the skill of a nations labor force, implicit in w.

- We then just solve for $\hat{\omega}$ in

$$\alpha(1-\bar{j}')(Y\hat{\omega}+D') = \alpha\bar{j}'(Y^*-D')+D'.$$

- Which simplifies, setting $D'=0$, to

$$(1-\bar{j}')Y\hat{\omega} = \bar{j}'Y^*$$

Counterfactual (3)

- Plugging in the expression for $\bar{j}'$ yields

$$(1-\bar{j})Y\hat{\omega} = \bar{j}\hat{\omega}^{-\theta}Y^*.$$

- Solving it out:

$$\hat{\omega} = \left(\frac{\bar{j}Y^*}{(1-\bar{j})Y}\right)^{1/(1+\theta)}.$$

- This expression gives the change in the U.S. wage, the change in the relative wage of the US, and the change in the relative GDP of the US. Use $\theta = 8.28$ from EK (2002).

Results (1)

- Change in the US wage relative to ROW (change in relative US GDP):

$$\hat{w} = \left(\frac{\bar{j} Y^*}{(1 - \bar{j}) Y} \right)^{1/(1+\theta)} = 0.954.$$

- The change in the tradables price index:

$$\frac{p'_T}{p_T} = \hat{p}_T = \left[\bar{j} \hat{\omega}^{-\theta} + (1 - \bar{j}) \right]^{-1/\theta} = 0.988.$$

Results (2)

- The change in the US overall price index:

$$\hat{P} = (\hat{p}_T)^{\alpha} (\hat{w})^{1-\alpha} = 0.961.$$

- So, change in real wage:

$$\frac{\hat{w}}{\hat{P}} = (\hat{w}/\hat{p}_T)^{\alpha} = 0.993.$$

- Bottom line: moderate change in US wage, trivial change in real wage.

Results (3)

- With labor mobility, reducing the deficit requires a huge sectoral shift:

- Range of goods produced in the US rises from 0.20 to:

$$\bar{j}' = \frac{\bar{j}\hat{\omega}^{-\theta}}{\bar{j}\hat{\omega}^{-\theta} + (1 - \bar{j})} = 0.270$$

Results (4)

- Initial tradable share of the economy is:

$$\lambda = \alpha \bar{j} \frac{Y + Y^*}{Y} = 0.143.$$

- It rises to:

$$\lambda' = \alpha \bar{j}' \frac{Y\hat{\omega} + Y^*}{Y\hat{\omega}} = 0.20$$

- No surprise that with balanced trade it equals α.

4.2 Sticky Wages and a Common Currency

Trade Adjustments in Europe

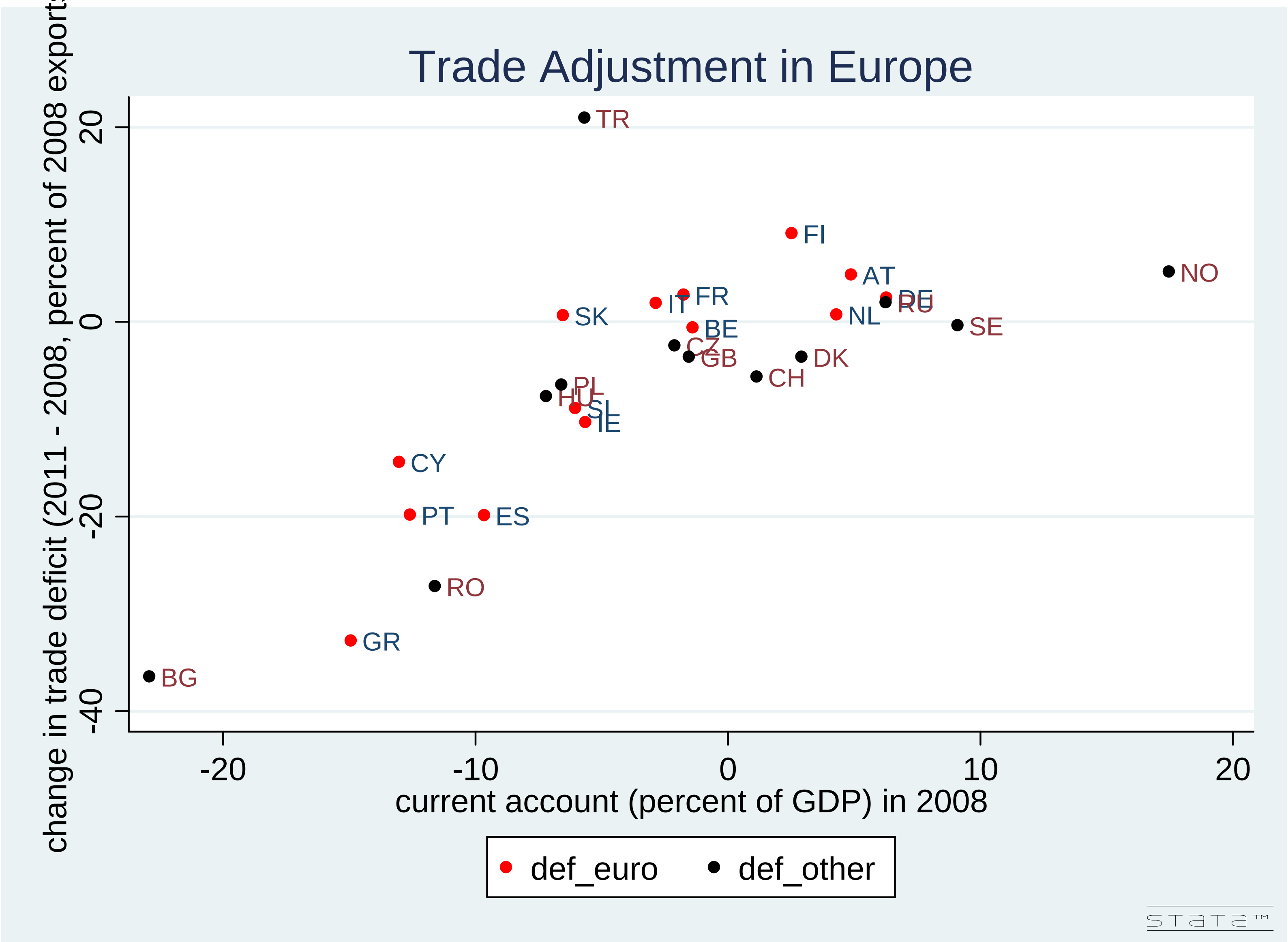
- Current account (to GDP) varied widely across Europe in 2008.

- Correlates with how well each country weathered the Great Recession.

- Confront that fact based on their use of a common currency:

  1. Deficit countries in 2008 reduced deficits in past 3 years (FIGURE 1).

  2. Model of trade and sticky wages and a common currency (based on DFS) connects deficit reduction to rising unemployment.



Trade Adjustment in Europe
change in trade deficit (2011 - 2008, percent of 2008 exports)
current account (percent of GDP) in 2008
20
0
-20
-40
-20
-10
0
10
20
TR
FI
AT
NO
FR
IT
DE
RU
SK
NL
SE
BE
CZ
GB
DK
CH
PL
HU
SI
IE
CY
PT
ES
RO
GR
BG
def_euro
def_other

Basic Model

- Whether or not the wage is flexible, home's imports of tradables must equal its exports plus its deficit:

$$\alpha(1-\bar{j})\,(wL+D) = \alpha\bar{j}\,(w^*L^*-D)+D.$$

- Can rearrange as:

$$\alpha(1-\bar{j})wL = \alpha\bar{j}w^*L^* + (1-\alpha)D$$

- Or:

$$\omega = \left[\frac{\bar{j}(\omega)}{1-\bar{j}(\omega)} + \frac{(1-\alpha)}{\alpha\left(1-\bar{j}(\omega)\right)}\frac{D}{Y*}\right]\frac{L^*}{L}.$$

- If wages are fixed in local currencies, we can still think of ω adjusting to bring about this equality via the exchange rate.

Sticky Wages

- Suppose wages fixed in the local currency *and* both countries are on the Euro.

- If D changes to D' then L changes to L' (given L^* and a fixed π) so that:

$$\left[\frac{\bar{j}}{1-\bar{j}} + \frac{(1-\alpha)}{\alpha\left(1-\bar{j}\right)}\frac{D'}{Y^*}\right]\frac{L^*}{L'} = \left[\frac{\bar{j}}{1-\bar{j}} + \frac{(1-\alpha)}{\alpha\left(1-\bar{j}\right)}\frac{D}{Y^*}\right]\frac{L^*}{L}$$

which simplifies to:

$$\left[1 + \frac{(1-\alpha)}{\alpha\bar{j}Y^*}D'\right]L = \left[1 + \frac{(1-\alpha)}{\alpha\bar{j}Y^*}D\right]L'$$

- Substitute in observations on $\alpha\bar{j} = X/(Y^* - D)$, where X is home's exports which can be obtained from national accounts data.

$$\left[1 + \frac{(1-\alpha)(Y^* - D)}{Y^*}\frac{D'}{X}\right] L = \left[1 + \frac{(1-\alpha)(Y^* - D)}{Y^*}\frac{D}{X}\right] L'$$

Convenient Approximation

- If the rest of the world GDP is large relative to the deficit in the home country, have approximately:

$$\left[1 + (1 - \alpha)\frac{D'}{X}\right] L = \left[1 + (1 - \alpha)\frac{D}{X}\right] L'$$

- Substitute in the definition of the unemployment rate $U = 1 - \frac{L}{L^F}$ (assuming the labor force is fixed):

$$\left[1 + (1 - \alpha)\frac{D'}{X}\right] (1 - U) = \left[1 + (1 - \alpha)\frac{D}{X}\right] (1 - U')$$

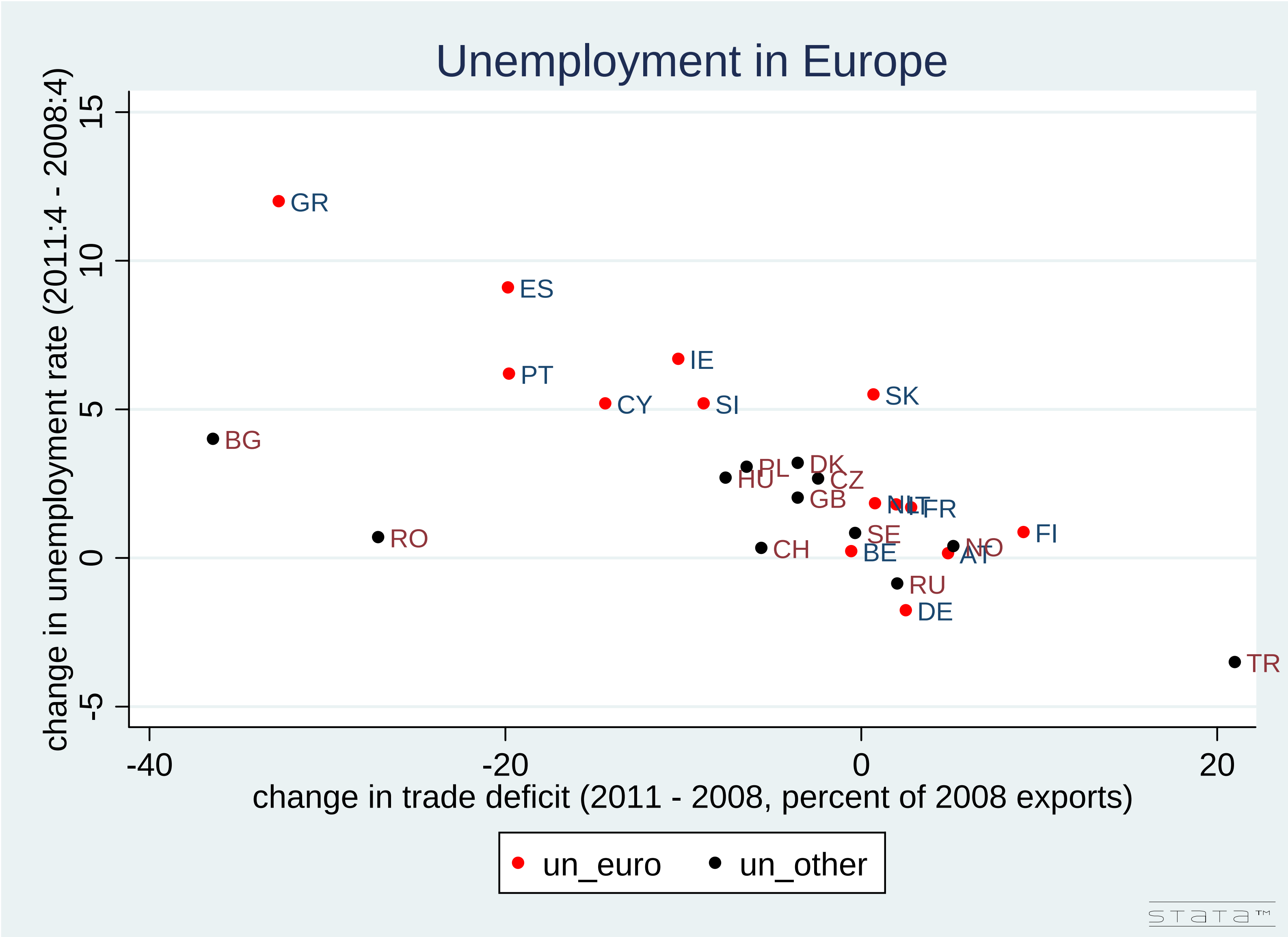
- Take logs of both sides and approximate $\ln(1+x) = x$ to get:

$$(1-\alpha)\frac{D'}{X} - U = (1-\alpha)\frac{D}{X} - U'.$$

- Rearrange as:

$$U' - U = -(1-\alpha)\frac{D' - D}{X}.$$

- Examine in Figure 2 (slope should be share of spending on non-tradables).



Unemployment in Europe
change in unemployment rate (2011:4 - 2008:4)
15
10
5
0
-5
GR
ES
IE
PT
SK
CY
SI
BG
PL
DK
HU
CZ
GB
NL
IT
FR
FI
SE
RO
CH
BE
NO
AT
RU
DE
TR
-40
-20
0
20
change in trade deficit (2011 - 2008, percent of 2008 exports)
un_euro
un_other

MULTI-COUNTRY MODELS

5 Putting Ricardo to Work (2012)

Challenge

- The basic Ricardian model has two countries and two goods.

- Express home country's comparative advantage in good 1 in two ways (in terms of labor requirements):

$$\frac{a^*(1)}{a(1)} > \frac{a^*(2)}{a(2)}$$

or

$$\frac{a^*(1)}{a^*(2)} > \frac{a(1)}{a(2)}.$$

- With many goods, only the first way works, as in DFS chain of comparative advantage.

- If home has a comparative advantage in the lower indexed goods, then $A(j)$ is decreasing in j:

$$A(j) = \frac{z(j)}{z^*(j)} = \frac{a^*(j)}{a(j)},$$

  where z is labor efficiency

- With many countries and many goods, you need to figure out some other way to express comparative advantage.

Probabilistic Approach

- Stick with a continuum of goods $j \in [0,1]$ as in DFS, but now have $i = 1, ..., I$ countries.

- In DFS we expressed all the relative labor requirements in terms of a curve $A(j)$ (rather than a set of individual values).

- Now, express in terms of a probability distribution for country i:

$$\Pr\left[a_i(j) > a\right] = e^{-(A_i a)^\theta},$$

with associate probability density function pdf:

$$f(a) = \theta a^{\theta-1} A_i^\theta e^{-(A_i a)^\theta}$$

- A country i with a higher A_i has a lower probability of a high labor requirement, i.e. its a more productive country.

- The parameter θ determines (inversely) the amount of variation in labor requirements (relative to A_i).

- The average value of the labor requirement in country i is:

$$
\begin{aligned}
E[a_i(j)] &= \int_0^\infty a f(a) da \\
&= \int_0^\infty \theta a^\theta A_i^\theta e^{-(A_i a)^\theta} da.
\end{aligned}
$$

- Let $x = (A_i a)^\theta$ so that $dx = \theta a^{\theta-1} A_i^\theta$ and $a = x^{1/\theta}/A_i$ so that:

$$
\begin{aligned}
E[a_i(j)] &= \frac{1}{A_i}\int_0^\infty x^{1/\theta} e^{-x} dx \\
&= \frac{1}{A_i}\Gamma(\frac{\theta+1}{\theta}).
\end{aligned}
$$

- Thus A_i really is average productivity.

Interpretation in Terms of Experimentation

- Let's think of a country as obtaining its distribution of labor requirements by drawing k times from $e^{-a^{\theta}}$.

- Its as if it experimented with k techniques, choosing the best one to actually use:

$$\begin{aligned}
\Pr\left[a_i(j) > a\right] &= \Pr\left[\min\left\{a_i^1(j), a_i^2(j), ..., a_i^k(j)\right\} > a\right] \\
&= \Pr\left[a_i^1(j) > a, a_i^2(j) > a, ..., a_i^k(j) > a\right] \\
&= \Pr\left[a_i^1(j) > a\right] \Pr\left[a_i^2(j) > a\right] ... \Pr\left[a_i^k(j) > a\right] \\
&= \left(e^{-a^\theta}\right)^k \\
&= e^{-ka^\theta} \\
&= e^{-\left(k^{1/\theta}a\right)^\theta}
\end{aligned}$$

- Thus, a country with a large A_i is like a country with a large k_i, i.e. a country that has experimented a lot.

- Another way to think about it is that A_i results from $k_i = A_i^\theta$ experiments.

From Labor Requirements to Costs

- Let w_i be the wage in country i.

- Let d_{ni} bet the extra cost of delivering to n from i.

- Thus, the total cost of producing good j in country i and delivering it to n is:

$$c_{ni}(j) = w_i d_{ni} a_i(j).$$

- The $c_{ni}(j)$ inherit a probability distribution from the $a_i(j)$:

$$\begin{aligned}\Pr\left[c_{ni}(j) > c\right] &= \Pr\left[a_i(j) > \frac{c}{w_i d_{ni}}\right] \\ &= e^{-(A_i \frac{c}{w_i d_{ni}})^{\theta}} \\ &= e^{-(A_{ni} c)^{\theta}},\end{aligned}$$

where $A_{ni} = A_i/(w_i d_{ni})$.

Prices

- Country n will choose the lowest cost supplier, whose cost will be the price (under perfect competition):

$$p_n(j) = c_n(j) = \min_{i=1,...,I} \{c_{ni}(j)\}.$$

- Thus, the distribution of prices is:

$$\begin{aligned}
\Pr[p_n(j) > p] &= \Pr\left[\min_{i=1,...,I} \{c_{ni}(j)\} > p\right] \\
&= \Pr[c_{n1}(j) > p] \Pr[c_{n2}(j) > p] ... \Pr[c_{nI}(j) > p] \\
&= e^{-(A_{n1}p)^\theta} e^{-(A_{n2}p)^\theta} ... e^{-(A_{nI}p)^\theta} \\
&= e^{-\left[(A_{n1})^\theta + (A_{n2})^\theta + ... + (A_{nI})^\theta\right] p^\theta} \\
&= e^{-(\overline{A}_n p)^\theta},
\end{aligned}$$

where

$$\overline{A}_n = \left[(A_{n1})^{\theta} + (A_{n2})^{\theta} + ... + (A_{nI})^{\theta} \right]^{1/\theta}.$$

- Thus, the average price is:

$$p_n = \frac{\gamma}{\overline{A}_n},$$

as countries around the world contribute to low prices in country n

Low-Cost Supplier

- Consider destination n.

- The distribution of costs if i delivered to n is the equivalent of $k_i = A_{ni}^{\theta}$ experiments.

- The total experiments for all countries is

$$\begin{aligned} k &= \overline{A}_n^{\theta} \\ &= (A_{n1})^{\theta} + (A_{n2})^{\theta} + ... + (A_{nI})^{\theta}. \end{aligned}$$

- Since each experiment is equally likely to yield the best outcome, the probability that country i is the low cost supplier to n is:

$$\begin{aligned}
\pi_{ni} &= \frac{A_{ni}^{\theta}}{(A_{n1})^{\theta} + (A_{n2})^{\theta} + ... + (A_{nI})^{\theta}} \\
&= \frac{A_i^{\theta}\left(w_i d_{ni}\right)^{-\theta}}{\overline{A}_n^{\theta}} \\
&= A_i^{\theta}\left(\frac{w_i d_{ni}}{p_n/\gamma}\right)^{-\theta}.
\end{aligned}$$

Trade Shares

- The probability above applies to a particular good j.

- But, with a unit continuum of goods, the probability becomes the fraction of goods on that continuum that country i supplies to n most cheaply.

- But, with Cobb Douglas preferences, the same amount is spent on each good, so the probability becomes the fraction of n's total spending allocated to goods from i:

$$\frac{X_{ni}}{X_n} = \pi_{ni}.$$

- Back to DFS, with no trade costs ($d_{ni} = 1$) and 2 countries:

$$
\begin{aligned}
\pi_{n1} &= \pi_1 = \frac{A_1^\theta (w_1)^{-\theta}}{A_1^\theta (w_1)^{-\theta} + A_2^\theta (w_2)^{-\theta}} \\
&= \frac{T_1 (w_1)^{-\theta}}{T_1 (w_1)^{-\theta} + T_2 (w_2)^{-\theta}} \\
&= \bar{j},
\end{aligned}
$$

where $T_i = A_i^\theta$.

Gains from Trade

- Since $d_{ii} = 1$ (no trade cost to the home market), we can write the home trade share as:

$$\pi_{ii} = A_i^{\theta} \left(\frac{w_i}{p_i/\gamma} \right)^{-\theta}$$

or

$$\frac{w_i}{p_i} = \gamma^{-1} A_i \left(\pi_{ii} \right)^{-1/\theta}. \tag{5}$$

- Of course your real wage is high when your productivity is high.

- That's all there is to it in Autarky, with $\pi_{ii} = 1$.

- More generally, buying little from domestic suppliers indicates gains from trade, i.e. a real wage higher than what you'd expect for a closed economy.

Labor Market Equilibrium

- So far we've only obtained the analog of the downward sloping curve of DFS (for a world of I countries).

- The other one is from the labor market clearing condition:

$$w_i L_i = \sum_{n=1}^{I} \pi_{ni} \left(w_n L_n + D_n \right). \tag{6}$$

- We could write it equivalently as

$$\left(\sum_{l \neq i} \pi_{il} \right) \left(w_i L_i + D_i \right) = \sum_{n \neq i} \pi_{ni} \left(w_n L_n + D_n \right) + D_i$$

which is clearly more awkward.

- Equilibrium is a set of (relative) wages and trade shares satisfying (6) and:

$$\pi_{ni} = \frac{A_i^{\theta} \left(w_i d_{ni} \right)^{-\theta}}{\sum_{l=1}^{I} A_l^{\theta} \left(w_l d_{nl} \right)^{-\theta}}.$$

- First solve in a special case, then explain how to solve it on a computer.

Frictionless Trade

- If $d_{ni} = 1$ then trade shares are $\pi_{ni} = \pi_i$ and hence (6) becomes:

$$\begin{aligned} w_i L_i &= \sum_{n=1}^{I} \pi_i \left(w_n L_n + D_n \right) \\ &= \pi_i Y^W. \end{aligned}$$

- As expected, deficits are irrelevant to the determination of relative wages when there are no trade costs (or non-traded goods).

- Divide by the equation of country l:

$$\frac{w_i L_i}{w_l L_l} = \frac{A_i^\theta \left(w_i \right)^{-\theta}}{A_l^\theta \left(w_l \right)^{-\theta}}.$$

- Solve for the relative wage:

$$\frac{w_i}{w_l} = \left(\frac{A_i^\theta / L_i}{A_l^\theta / L_l}\right)^{1/(1+\theta)} . \tag{7}$$

- As in DFS, all else equal, more labor lowers the relative wage.

- Yet, in Autarky, there would be constant returns to scale.

- With trade, being a larger country undercuts the gains from trade.

Frictionless Trade II

- Multiplying both sides of (7) by L_i/L_l gives:

$$\frac{Y_i}{Y_l} = \frac{(A_i L_i)^{\theta/(1+\theta)}}{(A_l L_l)^{\theta/(1+\theta)}}.$$

- Inverting, summing over l, and then inverting again:

$$\pi_i = \frac{Y_i}{Y^W} = \frac{(A_i L_i)^{\theta/(1+\theta)}}{\sum_l (A_l L_l)^{\theta/(1+\theta)}}.$$

- With no trade costs, a country's share of world GDP is also its trade share.

- Since $\pi_i = \pi_{ii}$ in this model, we can substitute this expression into (5) to fully solve for the real wage or welfare.

- Note that welfare in each country is increasing in the number of countries in the world.

- Higher productivity A_i raises i's real wage but lowers its gains from trade.

- The real wage no longer rises in proportion to A_i as part of the benefit of higher A_i spills over to other countries.

Computing Counterfactuals

- While we get nice expressions with frictionless trade, that assumption almost defeats the purpose of having a trade model with many countries.

- We want to solve the system of equations:

$$w_i' L_i = \sum_{n=1}^{I} \pi'_{ni} \left(w_n' L_n + D_n' \right)$$

and

$$\pi'_{ni} = \frac{A_i^{\theta} \left(w_i' d_{ni} \right)^{-\theta}}{\sum_{l=1}^{I} A_l^{\theta} \left(w_l' d_{nl} \right)^{-\theta}}$$

- Once that's solved we can compute the price level for welfare calculations:

$$p'_n = \gamma \left[\sum_{l=1}^{I} A_l^{\theta} \left(w'_l d_{nl} \right)^{-\theta} \right]^{-1/\theta}$$

- We've illustrated it here as calculating the new equilibrium for a new set of deficits, but we could have done it for a new set of trade costs, labor supplies, or technology levels.

- But, it turns out that all we're ever interested in is how the equilibrium changes relative to a baseline prior to the change.

- That insight leads to a simpler formulation.

Change Formulation

- As we saw in DFS, we can write:

$$w'L = \frac{w'}{w} wL = \hat{w}Y.$$

- What makes this approach attractive, is that Y is easily measured as the baseline value of GDP.

- Using a similar idea, we can write

$$\pi'_{ni} = \frac{\pi_{ni} (\hat{w}_i)^{-\theta}}{\sum_{l=1}^{I} \pi_{nl} (\hat{w}_l)^{-\theta}},$$

where the baseline value of π_{ni} is easily measured as X_{ni}/X_n.

- Using these results, we can rewrite the system of equations very simply as:

$$\hat{w}_i Y_i = \sum_{n=1}^{I} \frac{\pi_{ni} \left(\hat{w}_i\right)^{-\theta}}{\sum_{l=1}^{I} \pi_{nl} \left(\hat{w}_l\right)^{-\theta}} \left(\hat{w}_n Y_n + D'_n\right),$$

which we want to solve for wage changes.

- Once that's solved we can compute changes in the price level as:

$$\hat{p}_n = \left[\sum_{l=1}^{I} \pi_{nl} \left(\hat{w}_l\right)^{-\theta}\right]^{-1/\theta}.$$

Numeraire

- Convenient to take world GDP Y^W as the numeraire.

- Thus measure GDP as
$$y_i = Y_i / Y^W$$
and deficits as
$$\delta_i = D_i / Y^W$$

- Notice that any solution $\widehat{\omega}$ then has the property
$$\sum_{i=1}^{N} \widehat{\omega}_i y_i = \sum_{i=1}^{N} \frac{w_i' L_i}{Y^W} = 1.$$

- Define $\mathbf{\Delta}_{\widehat{\omega}}$ be the set of all vectors with each element positive and satisfying the adding up restriction above.

- Want an algorithm that imposes that condition.

Alvarez-Lucas Algorithm: I

- Define the excess demand function $Z(\widehat{\omega})$ to have as its i'th element:

$$\frac{1}{\widehat{\omega}_i}\left[\sum_{n=1}^{N}\pi'_{ni}(\widehat{\omega})\left[\widehat{\omega}_n y_n+\delta'_n\right]-\widehat{\omega}_i y_i\right],$$

- Thus $Z_i(\widehat{\omega})=0$ for all i when evaluated at the equilibrium relative wage changes.

- For some $\nu\in(0,1]$, define the mapping T, with i'th element

$$T(\widehat{\omega})_i=\widehat{\omega}_i\left[1+\nu Z_i(\widehat{\omega})/y_i\right],$$

so that a fixed point of T corresponds to the equilibrium condition of zero excess demand.

Alvarez-Lucas Algorithm: II

- Intuition is that wage changes should increase if there's excess demand for i's labor.

- Lucas and Alvarez (2007) give the conditions under which iterating on the mapping T will converge to the equilibrium $\widehat{\omega}$.

- A key feature is that it maps vectors in $\boldsymbol{\Delta}_{\widehat{\omega}}$ into new vectors in $\boldsymbol{\Delta}_{\widehat{\omega}}$:

$$\sum_{i=1}^{N} T(\widehat{\omega})_i y_i = 1 + \nu \sum_{i=1}^{N} \widehat{\omega}_i Z_i(\widehat{\omega}) = 1.$$

Alvarez-Lucas Algorithm III

- To show it has that feature:

$$
\begin{aligned}
& \sum_{i=1}^{N} \widehat{\omega}_i Z_i(\widehat{\omega}) = \sum_{i=1}^{N} \left[\sum_{n=1}^{N} \pi'_{ni}(\widehat{\omega}) \left[\widehat{\omega}_n y_n + \delta'_n \right] - \widehat{\omega}_i y_i \right] \\
= \;& \sum_{n=1}^{N} \left[\widehat{\omega}_n y_n + \delta'_n \right] \sum_{i=1}^{N} \pi'_{ni}(\widehat{\omega}) - \sum_{i=1}^{N} \widehat{\omega}_i y_i \\
= \;& \sum_{n=1}^{N} \left[\widehat{\omega}_n y_n + \delta'_n \right] - \sum_{i=1}^{N} \widehat{\omega}_i y_i = 0
\end{aligned}
$$