Trade, Growth, and Macroeconomics: A Quantitative Approach

Mainz: August 2012

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TRADE AND GROWTH

Seven Facts

1. A few R&D-intensive countries specialize in producing equipment.

2. Developing countries import most of their equipment.

3. Equipment is highly traded, but still a home bias.

4. In local prices, equipment spending over GDP is unrelated to development.

5. But developing countries have low real equipment investment rates.

6. Relative price of equipment has fallen rapidly over time.

7. The price of equipment is slightly lower in developing countries.

“Trade in Capital Goods” EER (2001)

- Explanation of 1-6 (with 7 remaining a puzzle).

- Combines Ricardian model of trade and Solow growth model.

- Interpretation of Mankiw, Romer, and Weil paper.

- Hsieh and Klenow follow up with better data (dropping the trade part).

- Can it be improved?

Ricardian Trade Model

- Two countries (i): North (N) and South (S).

- Two goods (l): capital goods (K) and consumption goods (C).

- *Labor Productivity*: A_i^l e.g. A_N^K.

- North has comparative advantage in capital goods:

$$\frac{A_S^K}{A_S^C} < \frac{A_N^K}{A_N^C}$$

- From an engineering perspective, more logical to write it as:

$$\frac{A_N^C}{A_S^C} < \frac{A_N^K}{A_S^K}$$

- *Constant Returns to Scale* (CRS): $Q_N^K = A_N^K L_N^K$.

- Endowment of labor (mobile across sectors): $L_N^C + L_N^K = L_N$.

Ricardian Equilibria

- Assume perfect competition and preferences such that, at any relative price, some of each good is demanded.

- With costless trade, relative price of capital goods: $P = P^K / P^C$.

- In any equilibrium $P \in \left[\frac{A_N^C}{A_N^K}, \frac{A_S^C}{A_S^K}\right]$, else one or the other good won't be supplied.

- If $P \in \left(\frac{A_N^C}{A_N^K}, \frac{A_S^C}{A_S^K}\right)$ then S specializes in consumption goods and N in capital goods.

North not Specialized

- If $P = \frac{A_N^C}{A_N^K}$ then S specializes in consumption goods and N produces capital goods (and could produce both goods).

- In this case, relative wage in N is $W_N/W_S = A_N^C/A_S^C$, which equates unit labor costs in producing consumption goods:

$$\frac{W_N}{A_N^C} = \frac{W_S}{A_S^C}.$$

- We'll consider situations in which this equilibrium holds: North exports capital goods in return for consumption goods imported from South (with some produced in the North).

- Draw production possibility frontier (PPF) with Q^K on the horizontal and Q^C on the vertical.

- Budget constraint has slope of P as does ppf of North.

- PPF of South is steeper (slope of A_S^C/A_S^K).

Trade Costs

- Iceberg costs: $d^C \geq 1$ and $d^K \geq 1$.

- Let consumption good in S be the numeraire: $P_S^C = 1$.

- North must import C so $P_N^C = d^C$.

- North produces both goods so:

$$P_N^K = \frac{A_N^C}{A_N^K} P_N^C = \frac{A_N^C}{A_N^K} d^C.$$

- Finally, capital goods imported from the North cost $P_S^C = d^K P_N^K = \frac{A_N^C}{A_N^K} d^K d^C$.

- To keep our equilibrium configuration requires:

$$\frac{A_S^K}{A_S^C} d^K d^C < \frac{A_N^K}{A_N^C}.$$

- Let $P = P_N^K / P_N^C$ so that $P = A_N^C / A_N^K$ as in the model without trade costs.

- For South to specialize in consumption good, need PPF of South to be flatter than $P/(d^K d^C)$.

Solow Growth Model

- Closed economy producing a single good, with price 1.

- CRS production function:

$$Y = F(K, AL) = ALf(k).$$

- Here A is labor-augmenting technology, assumed to grow at rate g.

- Output per effective unit of labor is:

$$y = \frac{Y}{AL} = \frac{F(K, AL)}{AL} = \frac{ALf(k)}{AL} = f(k),$$

where $k = K/(AL)$ capital per effective unit of labor.

- Here ignore population growth (easily handled like g).

- The heart of the model is dynamics of K and hence k.

Capital Accumulation

- Work in continuous time:

$$\dot{K} = I - \delta K.$$

- Investment made possible from foregone consumption:

$$I = S = sY.$$

- In the Solow model, investment is just consumption goods you don't consume.

- Evolution of capital per effective unit of labor:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - g.$$

Dynamics

- Can assemble the pieces into a differential equation in k:

$$\begin{aligned}\dot{k} &= \frac{\dot{K}}{AL} - gk \\ &= \frac{sY - \delta K}{AL} - gk \\ &= sf(k) - (\delta + g)k.\end{aligned}$$

- Phase diagram: plot k on horizontal axis, $f(k)$ and $(\delta+g)k$ on the vertical.

- With mild assumption on f, converge to a steady state value of $k = \bar{k}$.

- Puzzle: why does higher g lower $\bar{k}$?

Market Equilibrium Interpretation

- Household owns the capital K (set aside from foregone consumption) and rents it to firms.

- Equilibrium is a wage w and rental rate q such that.

- Firm with 1 worker rents capital to the point at which its marginal product (which is its marginal revenue product since the price is 1) is:

$$F_K(k, 1) = f'(k) = q$$

- Since all choose the same k, it must equal the aggregate $k = K/L$. (If a firm had 2 workers, it would have rented $2k$ units of capital.)

- Equating the marginal product of labor and the wage is correct but redundant.

- What is needed is an additional condition that firm doesn't make more by expanding or contracting (by hiring 2 or 1/2 while keeping k fixed):

$$(y - qk) = w$$

- Note that this result is consistent, in equilibrium, with equating marginal product of labor and wage:

$$\begin{aligned} F_L(K, L) &= f(k) - L f'(k) \frac{K}{L^2} \\ &= f(k) - k f'(k) \\ &= y - kq. \end{aligned}$$

- Bottom line: for any arbitrary w and q the firm cannot equate both the marginal product of labor to w and the marginal product of capital to q.

- In equilibrium it can, which implies its profit is zero.

- Hence, we have shown that in equilibrium the revenue of all the firms is the gross income of households:

$$Y = wL + qK.$$

- Consumption of households:

$$C = (1 - s)Y.$$

- The rest is set aside for investment to build up K.

- But, remember K also depreciates as the household owns it. Part of q is just offsetting this cost of depreciation.

- Thus the interest rate (return on investment) is $r = q - \delta$.

- Net income of the household (Net Domestic product rather than GDP) is:

$$Y - \delta K = wL + qK - \delta K = wL + rK$$

- Bottom line, given K and L and hence $k = K/L$ and $y = f(k)$, we have the equilibrium "prices":

$$
\begin{aligned}
q &= f'(k) \\
w &= f(k) - kf'(k) \\
r &= f'(k) - \delta.
\end{aligned}
$$

Steady State

- At the steady state, capital and capital per worker grow at constant rates:

$$\frac{\dot{K}}{K} = g$$

- Capital output ratio is contant:

$$\frac{K}{Y} = \frac{k}{y} = \frac{k}{f(k)}$$

- Thus output also grows at rate g.

- Capital accumulation just keeps up with labor augmenting technology.

Cross-Country Implications

- Mankiw, Romer, and Weil assume a bunch of countries in autarky.
- Say each country i has its own savings rate s_i.
- Assume Cobb Douglas production function to make it simple $f(k) = k^\alpha$:

$$s_i \left(\bar{k}_i\right)^\alpha = (\delta + g)\bar{k}_i$$

or

$$\bar{k}_i = \left(\frac{s_i}{\delta + g}\right)^{1/(1-\alpha)}$$

- Steady state output per efficiency unit of labor

$$\bar{y}_i = \left(\frac{s_i}{\delta + g}\right)^{\alpha/(1-\alpha)}$$

- Assume all countries share the same technology (up to a random bit ε_i):

$$A_i = Ae^{\varepsilon_i}$$

- Cross-country regression equation:

$$\ln(Y_i/L_i) = \ln A + \frac{\alpha}{1-\alpha} \ln\left(\frac{s_i}{\delta + g}\right) + \varepsilon_i.$$

Discrete Dynamics

- For computing the model, its helpful and intuitive to work in discrete time.

- Consider time period of length h, with time measured in years.

- Hence δ and g are still thought about as a rate per year.

- Capital accumulates as

$$K(t+h) = hI(t) + K(t) - \delta h K(t)$$

so that

$$\frac{K(t+h) - K(t)}{h} = I(t) - \delta K(t).$$

- Continuous time lets $h \longrightarrow 0$, but $h = 1$ is convenient for computation.

- We'd get the same thing, as $h \longrightarrow 0$, if we had thought of a hazard of depreciation (Poisson arrival at rate δ of something that destroys a piece of it):

$$K(t+h) = hI(t) + e^{\delta h} K(t)$$

- Rest of the model is output:

$$Y(t) = K(t)^{\alpha}(A(t)L)^{1-\alpha}.$$

- Savings:

$$I(t) = sY(t).$$

- Technological change:

$$A(t) = e^{gt}.$$

- Can solve the model very easily using a spreadsheet.

Trade in Capital Goods and Growth

Weaving Together Ricardo and Solow

- Augment Ricardo: replace L with $F(K, L)$ which grows.

- Augment Solow: be explicit about two sectors of consumption and investment goods.

- Keep it simple by having F be the same in both sectors.

- Keep it simple by having technological change only in North's capital good production.

- Keep it simple by fixing the form of the Ricardian equilibrium.

Cost Minimization

- Think of some firm j facing a wage w and rental price of capital q.

- A necessary condition for optimality is that the firm solves

$$\min \left\{ wL_j + qK_j \right\}$$
$$\text{subject to}$$
$$F(K_j, L_j) = 1$$

- Note that this same problem applies to a firm in either sector since capital-good and consumption-good producers combine capital and labor according to the same function.

- The first order conditions of the Lagrangian are:

$$q = \lambda F_K(K_j, L_j)$$
$$w = \lambda F_L(K_j, L_j)$$

- Taking the ratio:

$$\frac{q}{w} = \frac{f'(k_j)}{f(k_j) - k_j f'(k_j)}$$

- So, a firm in either sector facing the same q/w chooses the same $k_j = k$.

National Income Accounting

- South makes only the consumption good, whose price there is 1:

$$Y_S = F(K_S, L_S) = L_S f(k_S).$$

- Both sectors operate in the North, hiring labor and renting capital (to minimize cost) in the same proportion $k_N = \frac{K_N}{L_N}$:

$$Y_N = P_N^K A_N L_N^K f(k_N) + P_N^C L_N^C f(k_N)$$

- Since $A_N^K = A_N$ and $A_N^C = 1$ we have $P_N^K A_N = P_N^C = d^C$ so that

$$Y_N = d^C L_N f(k_N).$$

- Thus, comparing GDP per capita by using exchange rates to translate into common currency (as in one of the empirical exercises on the problem set):

$$\frac{Y_N/L_N}{Y_S/L_S} = \frac{d^C f(k_N)}{f(k_S)}.$$

- Measuring output in units of the consumption good (rather than its value at a given location), more in line with Penn World Tables, and per capita $y = (Y/P^C)/L$:

$$\frac{y_N}{y_S} = \frac{f(k_N)}{f(k_S)}.$$

- Assuming Cobb Douglas production function, as in MRW:

$$\frac{y_N}{y_S} = \left(\frac{k_N}{k_S}\right)^{\alpha}.$$

- Note how we can represent a two-sector model as if there was simply an aggregate production function.

- But, now we have to think harder about prices. If we compare per capita income in terms of how much of the consumption good you can consume, we drop d^C from the equation.

Capital Accumulation

- Need a concept of real investment rate:

$$\iota = \frac{I}{Y/P^C}.$$

- Then, following the Solow Growth model:

$$\begin{aligned}
\dot{k} &= \frac{\dot{K}}{L} \\
&= \frac{\iota\left(Y/p^C\right) - \delta K}{L} \\
&= \iota f(k) - \delta k.
\end{aligned}$$

- Here ι stands in for the savings rate, but differs from the concept of the savings rate as the faction of income set aside, all in a common currency:

$$s = \frac{P^K I}{Y} = \frac{P^K}{P^C}\iota.$$

- We're going to assume that the savings rate s is a constant, and maybe even the same across countries.

- With the savings rate constant, the real investment rate can rise forever if the relative price of capital keeps falling:

$$\iota = \frac{s}{(P^K/P^C)}.$$

- From the Ricardian model, the real investment rate is driven by the technology for producing capital goods in the North:

$$\iota_N = sA_N^K$$

$$\iota_S = s\frac{A_N^K}{d^C d^K}.$$

- With technological change in production of capital goods at rate g , the real investment rate in both countries grows at rate g forever.

- With trade costs, at any point in time the real investment rate is higher in the North by the factor $d^C d^K$.

State Variable

- The Solow equation:

$$\dot{k} = \iota k^{\alpha} - \delta k,$$

  while valid, only captures one part of the growth process since ι is also growing.

- Essentially k fails as a state variable for this problem. Let's muddle towards a better one.

- Consider growth in the North (later make minor adaption to the South):

$$\dot{k} = sA_N^K k^{\alpha} - \delta k$$

or

$$\begin{aligned}\frac{\dot{k}}{k} &= sA_N^K k^{\alpha-1} - \delta \\ &= s\kappa^{\alpha-1} - \delta.\end{aligned}$$

- It looks like

$$\kappa = \frac{k}{\left(A_N^K\right)^{1/(1-\alpha)}} = \frac{K}{AL}$$

might do the trick, with

$$A = \left(A_N^K\right)^{1/(1-\alpha)}.$$

- Notice that

$$\frac{\dot{\kappa}}{\kappa} = \frac{\dot{k}}{k} - \frac{1}{1-\alpha} g$$

so

$$\frac{\dot{\kappa}}{\kappa} = s\kappa^{\alpha-1} - \delta - \frac{1}{1-\alpha} g$$

or

$$\dot{\kappa} = s\kappa^{\alpha} - \left(\delta - \frac{1}{1-\alpha} g \right) \kappa.$$

- Formally, using this new state variable κ we can proceed just like with k in the Solow model.

Steady State

- We now have a new state variable:

$$\kappa = \frac{k}{\left(A_N^K\right)^{1/(1-\alpha)}}$$

which follows the differential equation

$$\dot{\kappa} = s\kappa^{\alpha} - \left(\delta - \frac{1}{1-\alpha}g\right)\kappa.$$

in the North, with s replaced by $s/(d^C d^K)$ in the South.

- It converges to a steady state value of:

$$\bar{\kappa} = \left(\frac{s}{\delta + g/(1-\alpha)}\right)^{1/(1-\alpha)}.$$

with $s/(d^C d^K)$ in place of s in the South.

Output per Worker

- Real GDP per capita (in units of the consumption good per person) along the steady state is:

$$
\begin{aligned}
y &= k^{\alpha} = \left(\left(A_N^K\right)^{1/(1-\alpha)} \bar{\kappa}\right)^{\alpha} \\
&= \left(A_N^K\right)^{\alpha/(1-\alpha)} \left(\frac{s}{\delta + g/(1-\alpha)}\right)^{\alpha/(1-\alpha)}
\end{aligned}
$$

in the North, with $s/(d^C d^K)$ replacing s in the South.

- Over time, the North's technological change in producing capital goods g drives world economic growth at rate

$$
g_y = \frac{\alpha}{1-\alpha} g.
$$

- Note that the South will grow just as fast whether of not they have any technological change.

- How can this be? The terms of trade continually shift in their favor.

- Furthermore, as in the classic Solow model, overall growth comes to a halt if $g = 0$.

- Looking across countries at a point in time, trade makes A_N^K common to both of them.

- But since $s/(d^C d^K)$ is lower than s we have

$$\frac{y_N}{y_S} = \left(\frac{s}{s/(d^C d^K)}\right)^{\alpha/(1-\alpha)} = \left(d^C d^K\right)^{\alpha/(1-\alpha)}$$

- Hence the model predicts "parallel growth."

- Notice that the relevant relative savings rate variable is

$$\frac{s}{s/(d^C d^K)} = \frac{\iota_N}{\iota_S} = \frac{s_N/(P_N^K/P_N^C)}{s_S/(P_S^K/P_S^C)}$$

- That's how MRW calculate it.

- But, notice that the interpretation is quite different.

Krugman, "A Model of Innovation, Technology Transfer, ..."

What's Been Missing

- Explanations of growth so far all depend on exogenous technological change.
  - In Solow its aggregate technological change.
  - In Trade in Capital, its technological change only in producing capital goods.

- Explanations of differences in levels of GDP per capita across countries all depend on capital accumulation.
  - In Mankiw Romer and Weil, its the actual savings rate.

- In Trade in Capital it has to do with comparative advantage in capital good production and trade costs.

- We'll now model differences in technology across countries and the building blocks for endogenous technological change.

“A Model of Innovation, ...” JPE (1979)

- Simple formalization of ongoing technological change (product innovation) in a model of trade.

- Captures observation of a product cycle: North (developed countries) are the innovators.

- North produces for a time but eventually production of a good moves to South (developing countries).

- Thus captures the role of technology diffusion as well.

- Had all the ingredients of an endogenous growth model, but didn't put it together with monopolistic competition to endogenize research.

Preferences

- Consider n varieties of goods:

$$U = \left\{ \sum_{i=1}^{n} c(i)^{(\sigma-1)/\sigma} \right\}^{\sigma/(\sigma-1)} . \tag{1}$$

- Krugman parameterizes with $\theta = (\sigma - 1)/\sigma$, but I like to see the elasticity of substitution explicitly.

- If it was written today the summation would be an integral.

- Krugman thinks about n increasing over time, hence must assume $\sigma > 1$ so that you can survive without goods yet to be invented.

- Given a fixed expenditure $Y = wL$ and prices $p(i)$ how do you choose consumption?

- Set it up as a big Lagrangian

$$\pounds = \left\{ \sum_{i=1}^{n} c(i)^{(\sigma-1)/\sigma} \right\}^{\sigma/(\sigma-1)} + \lambda \left[Y - \sum_{i=1}^{n} p(i)c(i) \right]$$

- Take the derivative with respect to any two intermediates $i = a, b$, setting each derivative to zero.

- Simplify and take the ratio of the first order conditions to get

$$\frac{c(a)}{c(b)} = \left[\frac{p(a)}{p(b)} \right]^{-\sigma} . \qquad (2)$$

Production

- Two countries (regions): North and South.

- Goods are divided into two sets $n = n_N + n_S$.

- North knows how to produce all of them but South only knows how to produce n_S.

- All goods are produced with efficiency 1, thus the advantage of the North is in innovation not production.

- Labor forces L_N and L_S.

- Wages w_N and w_S, later we can worry about why they are what they are.

- South can thus produce the n_S goods at a cost of w_S and North can produce any good at a cost of w_N.

Market Structure

- Interestingly, Krugman assumed perfect competition.

- Odd, since at the same time, in other papers he was introducing monopolistic competition into trade models.

- ... for which he won the Nobel Prize.

- Ironic since monopolistic competition would have allowed him to endogenize the innovation rate in the North.

- Thus, the price of any good produced in the North is $p_N = w_N$ and in the South is $p_S = w_S$.

- The relative quantities per variety (depending on where they're produced) is

$$\frac{c_N}{c_S} = \left[\frac{p_N}{p_S}\right]^{-\sigma}.$$

- Each of these goods will be purchased by everyone in the World.

Solving Krugman's Model I

- Solution to the static model relates to DFS.

- Order goods by South's comparative advantage, which is trivial in this case.

- Thus, the South produces goods $1, 2, ..., \overline{n}$ for $\overline{n} \leq n_S$ while the North produces $\overline{n} + 1, \overline{n} + 2, ..., n$.

- Remember that $\overline{n}$ is not necessarily n_S, forget that $\overline{n}$ is an integer, and plot the relative productivity curve ($\overline{n}$ on the horizontal axis), with a step at $\overline{n} = n_S$.

- Still have the demand relationship between any good produced in the South vs. any good produced in the North:

$$\frac{c_S}{c_N} = \left[\frac{p_S}{p_N}\right]^{-\sigma} = \left[\frac{w_S}{w_N}\right]^{-\sigma}.$$

- Furthermore, labor supply dictates

$$\frac{c_S}{c_N} = \frac{L_S/\overline{n}}{L_N/(n-\overline{n})}$$

Solving Krugman's Model II

- Put these two expressions together to get:

$$\frac{w_S}{w_N} = \left[\frac{c_S}{c_N}\right]^{-1/\sigma} = \left[\frac{L_S/\overline{n}}{L_N/(n-\overline{n})}\right]^{-1/\sigma}, \tag{3}$$

  which is an increasing function of $\overline{n}$.

- Think of (3) as the labor-market-clearing condition: demand for goods from the South must be sufficient to match the supply of them when everyone's working.

- Plot this curve against the relative productivity curve, which ensures that firms can break even hiring workers at those wages. The intersection is the World equilibrium (as in DFS).

- Two possibilities: (1) crosses at $\overline{n} = n_S$ in which case (3) tells us the relative wage or (2) crosses at $w_S/w_N = 1$ in which case (3) can be solved for $\overline{n}$.

- Idea is that the North has to have a large enough technological lead to maintain a wage advantage over the South, given that its only inherent advantage is in innovation.

Outsourcing I

- Outsourcing means taking a good that only North can produce and transfering the technology to the South (so that n_S increases by 1).

- To see the potential gain to the North, need to think about the cost of living (the cost of a unit of U):

$$P = \left\{ \sum_{i=1}^{n} p(i)^{-(\sigma-1)} \right\}^{-1/(\sigma-1)} .$$

- If $w_S < w_N$ then the advantage to a Northern firm is to get the work done more cheaply (if $w_S = w_N$ there's no point).

- We'll see that if n_S is small enough, then workers in the North benefit from outsourcing, since prices decline.

- But, a rising value of n_S also raises the wage in the South.

- When n_S is large, workers in the North are hurt by outsourcing since it increases prices for products already imported from the South.

Outsourcing I

- Only interesting if $w_S < w_N$ in which case $\overline{n} = n_S$.

- Outsourcing means n_S increases by 1 (hence $n - n_S$ decreases by 1).

- Price index is

$$\begin{aligned} P &= \left[n_S w_S^{-(\sigma-1)} + (n - n_S) w_N^{-(\sigma-1)} \right]^{-1/(\sigma-1)} \\ &= \left[n w_N^{-(\sigma-1)} + n_S \left(w_S^{-(\sigma-1)} - w_N^{-(\sigma-1)} \right) \right]^{-1/(\sigma-1)}. \end{aligned}$$

- Thus, the standard of living in the North is

$$\begin{aligned}\frac{w_N}{P} &= w_N \left[n w_N^{-(\sigma-1)} + n_S \left(w_S^{-(\sigma-1)} - w_N^{-(\sigma-1)} \right) \right]^{1/(\sigma-1)} \\ &= \left[n + n_S \left(\left(\frac{w_N}{w_S} \right)^{(\sigma-1)} - 1 \right) \right]^{1/(\sigma-1)}\end{aligned}$$

- Since $\left(\frac{w_N}{w_S}\right)^{(\sigma-1)} - 1 \geq 0$ we know that $\frac{w_N}{P}$ is at least as large as the value in autarky, $n^{1/(\sigma-1)}$.

Outsourcing III

- Have two equations:

$$\frac{w_N}{P} = \left[n + n_S \left(\left(\frac{w_N}{w_S} \right)^{(\sigma-1)} - 1 \right) \right]^{1/(\sigma-1)}$$

  and

$$\frac{w_N}{w_S} = \left[\frac{L_S/n_S}{L_N/(n-n_S)} \right]^{1/\sigma}$$

- When n_S increases due to outsourcing, there are two effects, going in opposite directions.

- Starting at a low value of n_S:

- Given the relative wage, an increase in n_S increases the real wage in the North as it leads to lower prices.

- But, an increase in n_S also raises the relative wage in the South, thus raising the price of all the goods that the North imports from the South.

- If n_S is near zero then the real wage in the North is near its autarky value. In this case effect I. dominates so outsourcing benefits workers in the North.

- But, eventually increasing n_S stops raising the real wage in the North.

* In fact, with a large enough value of n_S the wage in the South becomes equal to the wage in the North and the real wage in the North is pushed back down to its autarky level.

- Clearly, at some value of n_S less than that, increases in n_S lower the real wage in the North.

Technology Dynamics I

- Section III of Krugman is a nice way to think about world growth.

- The idea is that the North country determines growth via its innovation rate, ι.

- Then the imitation rate τ of the South determines how far it is behind the North.

- But, they both end up growing at the same rate.

- Sometimes called “bathtub” dynamics since innovations flow into the North bathtub while imitated innovations flow out.

- As with the level of water in a real bathtub, we’re looking for something to remain constant over time.

Technology Dynamics II

- Let $n(t)$ be the number of goods at date t, ignoring integer constraints:

$$n(t) = n_S(t) + n_N(t).$$

- Innovation (dot means derivative with respect to time):

$$\dot{n}(t) = \iota n(t).$$

so that $n(t)$ grows exponentially at rate ι.

- Imitation:

$$\dot{n}_S(t) = \tau n_N(t).$$

- All together:

$$\begin{aligned}\dot{n}_N(t) &= \dot{n}(t) - \dot{n}_S(t)\\ &= \iota n(t) - \tau n_N(t)\end{aligned}$$

Technology Dynamics III

- Once again: the key step is to choose the right state variable.

- Conjecture that we can understand what's going on in terms of $x(t) = n_N(t)/n(t)$.

- Take the time derivative:

$$\begin{aligned}
\dot{x}(t) &= \frac{\dot{n}_N(t)}{n(t)} - \frac{n_N(t)}{n(t)}\frac{\dot{n}(t)}{n(t)} \\
&= \frac{\iota n(t) - \tau n_N(t)}{n(t)} - x(t)\iota \\
&= \iota - (\iota + \tau)x(t)
\end{aligned}$$

- Graph it with $\dot{x}$ on the vertical axis and x on the horizontal.

- Note that from any initial value of x the system will converge to $\bar{x}$ satisfying

$$0 = \iota - (\iota + \tau)\bar{x}.$$

- Thus

$$\bar{x} = \frac{\iota}{\iota + \tau}.$$

- Hence in the long run, the ratio of North exclusive innovations to those imitated by the South will converge to:

$$\frac{n_N(t)}{n_S(t)} = \frac{\iota}{\tau}$$

- While both $n_N(t)$ and $n_S(t)$ grow at exponential rate ι, their ratio remains constant.

Romer (1990) Introduction: Premises

- Three premises:

  1. Technological change "trial and error, experimentation, refinement, and scientific investigation, the instructions that we follow for combining raw materials ..." lies at the heart of economic growth. Technological change drives capital accumulation.

  2. Technological change is in part driven by actions of people driven by market incentives.

  3. Technology is a non-rival good, so once invented (a fixed cost) it can be used in any number of places at the same time.

- Note that Krugman (1979) captured 1 and 3, but not 2.

Romer Public Goods

- Distinguishes goods along two dimensions:

  1. Rivalry: does use by one preclude use by another

  2. Excludability: can the owner prevent others from using it

	rival	non-rival
excludable	iPhone understanding of math	design of iPhone recorded music
non-excludable	parking space at Mall	basic science mathematical ideas

- The technologies that have raised our standard of living are mostly in the north-east quadrant.

- Non-rivalry opens up huge potential at a global scale.

Romer Implications of Premises

- Non-rivalry of technology implies output doubles by doubling traditional inputs (holding technology fixed).

- In a competitive market, rival factors exhaust revenue, thus ruling out second premise.

- Could still have technology provided by govt. funded scientists, or unintentionally through learning by doing and spillovers.

- With decreasing returns in rival inputs, technology can be financed. But then you violate the replication argument.

- Romer argues that you're led to imperfect competition.

Production Function

- Country endowed with labor L and human capital H (distinct factors of production as we saw in Mankiw, Romer, and Weil).

- Human capital divided between production and research

$$H = H_Y + H_A.$$

- Production function for final goods (consumed or turned into specialized capital goods):

$$Y(H_Y, L, X) = H_Y^{\alpha} L^{\beta} X^{1-\alpha-\beta}.$$

- The capital stock X is composed of many differentiated varieties (same idea as in Krugman, 1979) of what are here "producer durables":

$$X = \left(\sum_{i=1}^{A} x_i^{1-\alpha-\beta} \right)^{1/(1-\alpha-\beta)}$$

which later becomes

$$X = \left(\int_0^A x(i)^{1-\alpha-\beta} \right)^{1/(1-\alpha-\beta)}.$$

- Put it all together and you have

$$Y = H_Y^{\alpha} L^{\beta} \int_0^A x(i)^{1-\alpha-\beta}.$$

- The convenient restriction Romer introduced was that the elasticity of substitution between varieties is the same as the captial share in production.

- Since it's constant returns to scale in H_Y, L, x, (given a fixed A) he can interpret with many competive price-taking firms producing Y.

Invention Function

- Inventions are the result of inventors H_A interacting with existing knowledge A:

$$\dot{A} = \delta H_A A.$$

- Each individual inventor takes the whole stock of A as given since it is a non-rival input.

- Its the source of a big positive externality from invention, giving a possible rationale for subsidizing it.

- Remember that human capital can be allocated to production or research.

- In production it gets a was w_H while in research the payoff is $P_A \delta A$.
- Here P_A is the value of an idea for a new type of producer durable.
- Although the idea is non-rival, it is excludable through patent protection or other means, so has value.
- It can be sold to a firm that produces a unique capital good.

The Market for Capital Goods

- With an idea, a capital goods firm can produce a distinct $x(i)$ at unit cost η by reshaping η units of final output (whose price is 1).

- It can then rent $x(i)$ to a final goods producer at a rental price $p(i)$.

- Demand for capital goods comes from final goods producers who (given H_Y and L) solve:

$$\max_{x} \left\{ H_Y^{\alpha} L^{\beta} \int_0^A x(i)^{1-\alpha-\beta} - \int_0^A p(i)x(i) \right\}.$$

- The first order condition is:

$$(1-\alpha-\beta)H_Y^{\alpha}L^{\beta}x(i)^{-\alpha-\beta}-p(i)=0.$$

- It becomes the demand curve faced by a capital goods producer:

$$p(x)=(1-\alpha-\beta)H_Y^{\alpha}L^{\beta}x^{-\alpha-\beta}$$

  where now price is a function of the quantity x of this unique producer durable.

- We could also invert it to get

$$x(p)=\left(\frac{p}{1-\alpha-\beta}\right)^{-1/(\alpha+\beta)}H_Y^{\alpha/(\alpha+\beta)}L^{\beta/(\alpha+\beta)}.$$

- Note, the elasticity of demand is $1/(\alpha+\beta)$.

The Capital Goods Producer

- The capital goods producer, who is a monopolist since he has the rights to an idea, solves

$$\max_x \{p(x)x - r\eta x\}.$$

- Implicit in this formulation is the assumption that the firm can always work its production process in reverse (since remember that x never depreciates). That's what "putty putty" means.

- The first order condition implies the famous price markup rule over marginal cost $r\eta$:

$$\bar{p} = \frac{1}{1-\alpha-\beta} r\eta.$$

- In general, a monopolist faced with elasticity of demand σ charges a markup of

$$\frac{\sigma}{\sigma - 1} = \frac{1/(\alpha+\beta)}{1/(\alpha+\beta) - 1} = \frac{1}{1-\alpha-\beta}.$$

- The resulting profits are

$$\pi = \bar{p}x(\bar{p}) - r\eta x(\bar{p}),$$

where

$$x(\bar{p}) = \bar{x} = (1-\alpha-\beta)^{2/(\alpha+\beta)}\,(r\eta)^{-1/(\alpha+\beta)}\,H_Y^{\alpha/(\alpha+\beta)}L^{\beta/(\alpha+\beta)}$$

The Value of an Idea

- The value of an idea, P_A, is equal to the present value of future profits:

$$P_A(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \pi(\tau) d\tau$$

- Differentiating both sides with respect to t:

$$\begin{aligned} \dot{P}_A(t) &= -\pi(t) + r(t) \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \pi(\tau) d\tau \\ &= -\pi(t) + r(t) P_A(t) \end{aligned}$$

- More intuitive as market return on investment equals dividend plus capital gain:

$$r(t)P_A(t) = \pi(t) + \dot{P}_A(t).$$

- If P_A and r are constant, which they will be with steady state growth, then

$$P_A = \frac{\pi}{r}$$

which is the value of a perpetuity.

Consumers

- The representative consumer chooses a consumption path to maximize:

$$U = \int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} dt$$

where $\sigma \geq 0$.

- Note that if $\sigma = 1$ you get log preferences.

- From dynamic optimization, the first order condition is:

$$\begin{aligned} \frac{dU'(C(t))/dt}{U'(C(t))} &= \rho - r(t) \\ &= \frac{d \ln U'(C(t))}{dt} \end{aligned}$$

- Note that in our case:

$$U'(C(t)) = C(t)^{-\sigma}$$

- The consumer's first order condition thus yields

$$\frac{\dot{C}}{C} = \frac{r(t) - \rho}{\sigma}$$

Relationship to Solow

- Define capital K as foregone consumption (as in the Solow model).

- The quantity $\bar{x}$ of each type of capital good is related to K via:

$$K = \eta A \bar{x}, \tag{4}$$

much like the calculation we did in Krugman (1979) for consumption per variety.

- Now we can simplify the production function:

$$
\begin{aligned}
Y &= H_Y^\alpha L^\beta \int_0^A x(i)^{1-\alpha-\beta} \\
&= H_Y^\alpha L^\beta A \bar{x}^{1-\alpha-\beta} \\
&= H_Y^\alpha L^\beta A \left(\frac{K}{\eta A}\right)^{1-\alpha-\beta} \\
&= A^{\alpha+\beta} H_Y^\alpha L^\beta K^{1-\alpha-\beta} \eta^{\alpha+\beta-1} \qquad (5)
\end{aligned}
$$

Equilibrium

- Equilibrium paths for quantities and prices must satisfy:

  1. Consumers make optimal consumption and savings decisions taking the path of interest rates as given

  2. Workers with human capital decide whether to work in research or production taking the path of A, P_A, and w_H as given.

  3. Final goods producers choose employment of labor and human capital as well as rentals of producer durables to minimize costs taking prices as given.

  4. Capital good producers maximize profit taking their demand curve and the interest rate as given.

5. Potential entrants take the price of an idea as given.
6. The supply of each good equals demand.

Balanced Growth

- Focus on an equilibrium for which A, K, and Y all grow at the same rate (show how that is consistent with the production function).

- Requires that H_A remains constant over time.

- If K and A grow at the same rate, then (4) implies $\bar{x}$ will be constant.

- Production function (5) implies marginal product of H_Y and of H_A grow at same rate as A.

- Need P_A to be constant so that marginal revenue products of H_A and H_Y grow at same rate:

$$P_A = \frac{\pi}{r} = \frac{\frac{1}{1-\alpha-\beta} r\eta\bar{x} - r\eta\bar{x}}{r} = \frac{\alpha+\beta}{1-\alpha-\beta}\eta\bar{x}$$

Allocation of Human Capital

- Equality of wages for both uses of human capital

$$\delta A P_A = \alpha H_Y^{\alpha-1} L^{\beta} A \bar{x}^{1-\alpha-\beta}$$

or

$$\begin{aligned}
\delta \frac{\alpha+\beta}{1-\alpha-\beta}\eta &= \alpha H_Y^{\alpha-1} L^{\beta} \bar{x}^{-\alpha-\beta} \\
&= \alpha H_Y^{\alpha-1} L^{\beta} (1-\alpha-\beta)^{-2} r\eta H_Y^{-\alpha} L^{-\beta} \\
&= \alpha H_Y^{-1} (1-\alpha-\beta)^{-2} r\eta.
\end{aligned}$$

- Expression for human capital used in production:

$$H_Y = \frac{\alpha}{(1-\alpha-\beta)(\alpha+\beta)\delta} r.$$

- A higher r means you'd rather produce now rather than make an invention which only pays off in the future.

Endogenous Growth

- The growth rate of the economy stems from technological change:

$$\begin{aligned} g &= \frac{\dot{A}}{A} = \delta H_A = \delta\left(H - H_Y\right) \\ &= \delta H - \Lambda r, \end{aligned}$$

where

$$\Lambda = \frac{\alpha}{\left(1 - \alpha - \beta\right)\left(\alpha + \beta\right)}$$

- If we're not at a corner solution, growth is decreasing in r.

- But, a consumer facing a high r will choose more rapid growth in consumption:

$$g = \frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}$$

- Combining these two relationships:

$$g = \delta H - \Lambda \left(\sigma g + \rho \right)$$

so that

$$g = \frac{\delta H - \Lambda \rho}{1 + \Lambda \sigma}.$$

- For finite discounted utility we need:

$$\left(1 - \sigma \right) g < \rho.$$

1 Eaton and Kortum (1999): International Technology Diffusion ...

Basic Idea

- Retain Krugman's (1979) model of non-rival technology potentially useable anywhere.

- Endogenize the production of new technology.

- Following Romer: technology has value due to patentability (excludability) and imperfect competition.

- You pay to take a draw from the Pareto distribution and if your idea is good enough you can patent it at home and also abroad.

- Bring in any number of countries.

- Difficult to work out if you also have trade.

International Technology Diffusion (1)

- In Eaton and Kortum (IER, 1999: 537-570) we assume techniques discovered in country i become available in n with an exponentially-distributed lag L_{ni}:

$$\Pr[L_{ni} \leq x] = 1 - e^{-\epsilon_{ni} x}.$$

- Mean lag between arrival of an idea in i and implementation in n is $1/\epsilon_{ni}$ (could let $\epsilon_{ii} \to \infty$ to represent imediate availability in the country of invention).

- Thus, the number of ideas in n by date t is Poisson, with parameter:

$$T_n(t) = \sum_{i=1}^{N} \int_0^t [1 - e^{-\epsilon_{ni}(t-s)}] R_{is} ds.$$

- The Poisson arrival rate is thus $R_n(t) = dT_n(t)/dt$.

Another Derivation of the Tech. Frontier (1)

- Let $H_n(z,s)$ be the technology frontier in country n at date s, i.e. the probability that the state of the art is below z.

- Probability that no new idea arrives in $[s, s+ds]$ to bump you above z is:

$$e^{-aR_n(s)\left(z/\underline{q}\right)^{-\theta}ds} = e^{-R_n(s)z^{-\theta}ds}$$

- Hence, you're still below z at date $s+ds$ with probability:

$$H_n(z, s+ds) = H_n(z,s)e^{-R_n(s)z^{-\theta}ds}.$$

Another Derivation of the Frontier (2)

- Take logs:

$$\ln H_n(z, s + ds) = \ln H_n(z, s) - R_n(s) z^{-\theta} ds.$$

- Thus:

$$\frac{\partial \ln H_n(z, s)}{\partial s} = -R_n(s) z^{-\theta}.$$

- Integrate from $s = 0$ to t, given $H_n(z, 0) = 1$:

$$H_n(z, t) = e^{-T_n(t) z^{-\theta}}.$$

International Technology Diffusion (2)

- Thus, in each destination country n the distribution of efficiency is Fréchet with parameter $T_n(t)$, depending on research done in all countries.

- Average productivity is proportional to $(T_n(t))^{1/\theta}$. Typically delivers technological catch-up of backward countries, converging to constant proportional productivity gaps across countries.

- Natural to assume the quality of an idea is universal (due to non-rivalry of technology): A good technology invented in i is eventually used everywhere.

- Thus, technology diffusion generates a correlation in efficiency across countries, something we will assume away in the basic trade model.

Back to Basics

- Eaton and Kortum (1999) in line with all of Romer's premises.

- Lucas (*Economica*, 2009: 1-19), using results from Alvarez, Buera and Lucas (NBER WP # 14135, 2008) drops one of the premises.

- For Lucas there are no blueprints. What matters is the ideas in people's heads.

- Ideas are still non-rival. When we meet and talk, we both leave with the better of our two ideas.

- But, the idea in your head are rival. Lucas doesn't give you a technology by which you can hire a bunch of people and have them produce using the idea in your head.

Human Capital and Technology (1)

- Individual starts with productivity below z, and in meeting people in $(t, t+h)$, gets αh independent draws from distribution H.

- Probability that those interactions leave him below z is $H(z,t)^{\alpha h}$.

- Thus, technological frontier evolves according to:

$$G(z, t+h) = G(z,t)H(z,t)^{\alpha h}$$

- So

$$\frac{\partial \ln G(z,t)}{\partial t} = \alpha \ln H(z,t)$$

Human Capital and Technology (2)

- Natural to assume people you meet are drawn from the technological frontier itself:

$$\frac{\partial \ln G(z,t)}{\partial t} = \alpha \ln G(z,t)$$

- Given initial distribution $G(z, 0)$, the solution is

$$\ln G(z,t) = e^{\alpha t} \ln G(z,0).$$

- Taking initial condition to be Fréchet:

$$G(z,0) = e^{-Tz^{-\theta}} = G(z;T)$$

- We get:

$$G(z,t) = e^{-Te^{\alpha t}z^{-\theta}}.$$

Human Capital and Technology (3)

- Thus, letting $T(t) = Te^{\alpha t}$:

$$G(z, t) = e^{-T(t)z^{-\theta}}.$$

- With $T(t)$ growing at a constant rate α, average efficiency grows at rate α/θ.

- Note that we can undo the effect of time by shifting the distribution by $e^{(\alpha/\theta)t}$:

$$\ln G(e^{(\alpha/\theta)t}z, t) = -Te^{\alpha t}(e^{(\alpha/\theta)t}z)^{-\theta} = -Tz^{-\theta}.$$

- This property of $G(z,t)$ is preserved asymptotically (as $t \longrightarrow \infty$) with much weaker restrictions on the initial distribution $G(z,0)$.

Limiting Results (1)

- Suppose the result so far applies only after the process has gone on for a long time.

- Applying l'hopital's rule:

$$\begin{aligned}
\lim_{t\to\infty} \ln G(e^{(\alpha/\theta)t}z, t) &= \lim_{t\to\infty} \frac{\ln G(e^{(\alpha/\theta)t}z, 0)}{e^{-\alpha t}} \\
&= \lim_{t\to\infty} \frac{(\alpha/\theta)G_1(e^{(\alpha/\theta)t}z, 0)e^{(\alpha/\theta)t}z}{-\alpha e^{-\alpha t}G(e^{(\alpha/\theta)t}z, 0)} \\
&= \lim_{t\to\infty} \frac{-G_1(e^{(\alpha/\theta)t}z, 0)e^{(\alpha/\theta)(1+\theta)t}z}{\theta G(e^{(\alpha/\theta)t}z, 0)} \\
&= \lim_{y\to\infty} \frac{-G_1(y, 0)y^{1+\theta}z^{-\theta}}{\theta G(y, 0)},
\end{aligned}$$

which used the change of variable to $y = e^{(\alpha/\theta)t} z$.

Limiting Results (2)

- We'd like this limit to equal $Tz^{-\theta}$ for any z.

- So, since $\lim_{z \to \infty} G(z, 0) = 1$, need the initial distribution to satisfy:

$$\lim_{z \to \infty} \frac{1}{\theta} G_1(z, 0) z^{1+\theta} = T > 0. \tag{6}$$

- Initial distribution must have a "Pareto tail": if $G(z, 0) = 1 - (z/a)^{-\theta}$ then the limit above is satisfied with $T = a^{\theta}$.

Limiting Results (3)

- Alvarez, Buera, and Lucas (2008) work with $X = Z^{-\theta}$ so that

$$\Pr[X > x] = \Pr[Z \leq x^{-1/\theta}].$$

- Thus, the cdf of X given by $H(x, 0)$ satisfies:

$$1 - H(x, 0) = G(x^{-1/\theta}, 0).$$

- Hence, letting $z = x^{-1/\theta}$ we have

$$H_1(x, 0) = \frac{1}{\theta} z^{1+\theta} G_1(z, 0).$$

- Thus, the asymptotic condition on the initial distribution (6) reduces to $\lim_{x \to 0} H_1(x, 0) = T > 0$ (note that $x \to 0$ when $z \to \infty$).

- Thus, need an initial distribution with positive density even a x approaches 0.

- Not a mass point at zero, i.e. the cdf itself will satisfy $\lim_{x \to 0} H(x, 0) = 0$.

2 Lucas and Moll (2011): Knowledge Growth and the Allocation of Time

Lucas and Moll (1)

- Builds on Lucas (2009).

- A mass one of individuals having different productivities A of producing a homogeneous good.

- Let productivity be $A = Z^{-\theta}$:

$$\text{Pr}[Z \leq z|t] = F(z, t).$$

- Corresponding density $f(z, t)$.

- Call z the individual's cost.

- If he spends $s(z,t)$ portion of his time searching, his production is:

$$y(z,t) = [1 - s(z,t)]z^{-\theta}.$$

- Aggregate (and per capita) output is:

$$Y(t) = \int_0^\infty [1 - s(z,t)]z^{-\theta} f(z,t)dz$$

Lucas and Moll (2)

- Given $s(z,t) = s$, individual gets to learn from someone else with probability $\alpha(s)\Delta$ over the time interval $(t, t+\Delta)$.

- Here $\alpha(s)$ is some increasing function of search effort.

- Evolution of cost. The measure of individuals whose cost remains above z by date $t+\Delta$:

$$
\begin{aligned}
1 - F(z, t+\Delta) &= \int_z^\infty \left[1 - \alpha(s(y,t))\Delta F(z,t)\right] f(y,t)dy \\
&= 1 - F(z,t) - F(z,t)\int_z^\infty \alpha(s(y,t))\Delta f(y,t)dy
\end{aligned}
$$

- Forming the time derivative:

$$\frac{\partial F(z,t)}{\partial t} = F(z,t)\int_z^{\infty} \alpha(s(y,t))f(y,t)dy$$

- Differentiate with respect to z to get:

$$\frac{\partial f(z,t)}{\partial t} = -\alpha(s(z,t))f(z,t)F(z,t) + f(z,t)\int_z^{\infty} \alpha(s(y,t))f(y,t)dy$$

which is called a Boltzmann equation.

Lucas and Moll (3)

- Individuals maximize discounted consumption:

$$V(z,t) = E_t \left\{ \int_t^{\infty} e^{-\rho(\tau - t)} \left[1 - s(Z(\tau), \tau \right] Z^{-\theta} d\tau \, | Z(t) = z \right\}.$$

- Bellman equation:

$$\rho V(z,t) = \max_{s \in [0,1]} \left\{ (1-s) z^{-\theta} + \frac{\partial V(z,t)}{\partial t} + \alpha(s) \int_0^z \left[V(y,t) - V(z,t) \right] f(y,t$$

- Equilibrium, given $f(z,0)$ is a triple (f, s, V) such that (i) given that s, f satisfies the Boltzmann equation and (ii) given f, V satisfies the Bellman equation.