Bachelor Business Administration and Economics 6th Semester

Macroeconomics and Behaviour

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Klaus Wälde (lecture) and Alexey Cherepnev (tutorial)
www.macro.economics.uni-mainz.de

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Problem Set 5

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Question 1 (Phase diagram)
Objective function representing welfare of society as a whole:

$$\max_{\{C(\tau)\}} \int_t^\infty e^{-\rho[\tau-t]} u(C(\tau))d\tau, \tag{1}$$

where ρ is a time preference rate, $C(\tau)$ is consumption at time point τ, and the instantaneous utility function is given by:

$$u(C(\tau)) = \frac{[C(\tau)]^{1-\sigma} - 1}{1-\sigma} \text{ with } \sigma > 0. \tag{2}$$

A resource constraint requires that net capital investment is given by the difference between output $Y(K(t), L)$, depreciation $\delta K(t)$ and consumption $C(t)$,

$$\dot{K}(t) = Y(K(t), L) - \delta K(t) - C(t). \tag{3}$$

The optimal consumption behaviour, i.e. the Keynes-Ramsey rule, is given by:

$$\frac{\dot{C}(t)}{C(t)} = \frac{Y_K(K(t), L) - \delta - \rho}{\sigma}. \tag{4}$$

Suppose output is given by the Cobb-Douglas production function:

$$Y = F(K, L) = L^{1-\alpha} K^\alpha, \quad 0 < \alpha < 1. \tag{5}$$

Analyze equations (3) and (4) jointly in a phase diagram. In doing so, determine

a) zero-motion lines and equilibrium,

b) directions of motion of capital K and consumption C outside of zero-motion lines,

c) trajectories.

Question 2 (2-period saving decision problem with anxiety)
Consider an agent that lives for two periods. The constraint in the first period (at t):

$$w_t = c_t + s_t \tag{6}$$

where w_t is wage income, c_t – instantaneous consumption, s_t – savings. In the second period, $t+1$, when the individual is retired, the constraint reads

$$(1 + r_{t+1})s_t = c_{t+1}, \tag{7}$$

where r_{t+1} is the interest rate.

Anticipatory feelings a_t are modeled as rising in the mean and falling in variance of consumption c_{t+1}:

$$a_t = E_t[c_{t+1}] - \zeta var[c_{t+1}]. \tag{8}$$

They also extend the Cobb-Douglas utility function:

$$U_t = E_t\big\{\gamma[\phi \ln a_t + (1-\phi)\ln c_t] + (1-\gamma)\ln c_{t+1}\big\}. \tag{9}$$

Assuming that $E_t[1 + r_{t+1}] = 0$,

a) compute the first order condition $\frac{dU_t}{ds_t}$;

b) find the optimal savings s_t^*;

c) obtain the comparative statics properties of the model, i.e. find how s_t^* changes with respect to γ and ϕ.