Bachelor Business Administration and Economics 6th Semester

Macroeconomics and Behaviour

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Problem Set 3

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Question 1 (Fudenberg and Levine (2006))
Consider an infinite-lived consumer making a savings decision. Wealth y may be divided between consumption and savings according to the savings rate $a \in [0,1]$. The consumer is represented by the short-run self and the long-run self. Parameter δ is the time-preference rate. preference of the short-run self has utility

$$u(y_t, 0, a_t) = \log[(1-a_t)y_t], \tag{1}$$

Savings are invested in an asset that returns wealth:

$$y_t = Ra_{t-1}y_{t-1}. \tag{2}$$

The short-run self wishes to spend all wealth on consumption. The choice of variable a imposes a self-control constraint via a linear cost,

$$C(y_t, a_t) = \gamma\{\log(y_t) - \log[(1-a_t)y_t]\}. \tag{3}$$

Given the reduced form of preferences for the long-run self:

$$U = \sum_{t=1}^{\infty} \delta^{t-1} \left[(1+\gamma)\log[(1-a_t)y_t] - \gamma\log(y_t)\right] \tag{4}$$

and assuming constant savings rate a, find the optimal saving behaviour and give an interpretation to this result.

Question 2 (Foellmi et al. (2011))
With respect to the model discussed in Foellmi et al (2011), consider a simplification of it with $\alpha = 0$. It means that the agent aims to maximize:

$$\max_{\{c_t, k_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{5}$$

with the instantaneous utility function

$$U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{6}$$

subject to the binding budget constraint

$$c_t + k_{t+1} = f(k_t) \tag{7}$$

where the production function $f(\cdot)$ is increasing and continuous with $f(0) = 0$, $c_t \geq 0$ and $k_t \geq 0$ for all $t = 0, 1 \dots$

Obtain the Euler equation in the standard formulation of a Ramsey model,

$$\left(\frac{c_t}{c_{t+1}}\right)^{-\sigma} = \beta f'(k_{t+1}), \tag{8}$$

and give an interpretation to it.

Question 3 (Households and intertemporal optimization)

Consider an agent that lives for two periods. The constraint in the first period (at t):

$$w_t = c_t + s_t \tag{9}$$

where w_t is wage income, c_t – instantaneous consumption, s_t – savings. In the second period, $t + 1$, when the individual is retired, the constraint reads

$$(1 + r_{t+1})s_t = c_{t+1}, \tag{10}$$

where r_{t+1} is the interest rate.

a) Solve the maximization problem

$$\max_{s_t} E_t\{u(c_t) + \beta u(c_{t+1})\} \tag{11}$$

given that the interest rate r_{t+1} is a discrete random variable with probabilities π_i for each realization $i = 1, 2, \dots, n$.

b) Given the Cobb-Douglas preferences

$$E_t\{\gamma \ln c_t + (1 - \gamma) \ln c_{t+1}\}, \tag{12}$$

find the optimal consumption and saving levels.