Bachelor Business Administration and Economics 6th Semester

Macroeconomics and Behaviour

2014 Summer Term

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Problem Set 2

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Aghion, Howitt (1997): Endogenous Growth Theory

Question 1 (Exogenous Technological Change)
Given an exponential growth rate of technology, g, and the aggregate production function:

$$Y = [A(t)L(t)]^{1-\alpha}[K(t)]^{\alpha}, \tag{1}$$

where $A(t) = A_0 e^{gt}$ is a productivity parameter that reflects the current state of technological knowledge, find

a) the capital and output growth rates, $\dot{K}/K$ and $\dot{Y}/Y$;

b) the capital and output per person growth rates, $\dot{k}/k$ and $\dot{y}/y$.

Question 2 (Optimal Saving Behaviour)
Objective function representing welfare of society as a whole:

$$\max_{\{C(\tau)\}} \int_t^{\infty} e^{-\rho[\tau-t]} u(C(\tau)) d\tau, \tag{2}$$

where ρ is a time preference rate, $C(\tau)$ is consumption at time point τ, and the instantaneous utility function is given by:

$$u(C(\tau)) = \frac{[C(\tau)]^{1-\sigma} - 1}{1-\sigma} \text{ with } \sigma > 0. \tag{3}$$

A resource constraint requires that net capital investment is given by the difference between output $Y(K(t), L)$, depreciation $\delta K(t)$ and consumption $C(t)$,

$$\dot{K}(t) = Y(K(t), L) - \delta K(t) - C(t). \tag{4}$$

a) Derive the optimal consumption behaviour, i.e. the Keynes-Ramsey rule:

$$\frac{\dot{C}(t)}{C(t)} = \frac{Y_K(K(t), L) - \delta - \rho}{\sigma}. \tag{5}$$

What does it tell you?

b) Compute the optimal saving rate $s^* = \frac{Y^* - C^*}{Y^*}$ in the steady state, assuming the Cobb-Douglas production function without the technological growth:

$$Y = F(K, L) = L^{1-\alpha} K^{\alpha}, \; 0 < \alpha < 1. \tag{6}$$