1. **Differential equations**

   Solve the following differential equations and comment on their stability conditions

   (a)
   $$\dot{x}_t = a x_t$$

   (b)
   $$\dot{x}_t = b + a x_t.$$

2. **The Solow growth model in discrete time**

   (a) Show that in the discrete-time Solow model the dynamics of k_t are given by
   $$(1+n)(1+g)k_{t+1} = sf(k_t) + (1-\delta)k_t.$$

   (b) Now assume that the production function is Cobb-Douglas, i.e.
   $$F(K_t, A_t L_t) = K_t^{\alpha}(A_t L_t)^{1-\alpha}.$$
   Derive the steady state of the economy.

   (c) Assume that the model parameters have the following values
   $$g = 0.03, \ , n = 0.01, \ s = 0.2, \ \alpha = \frac{1}{3} \text{ and } \delta = 0.04.$$
   The log-linear approximation of the dynamics of k_{t+1} is given by
   $$\hat{k}_{t+1} = \left[\alpha A \left[k^*\right]^{\alpha-1} + B\right] \hat{k}_t,$$
   with $A = s/[(1+n)(1+g)]$ and $B = (1-\delta)/[(1-n)(1-g)]$. Assume that initially k_t is 50% below its steady state. Simulate the convergence process.

   (d) Now assume that initially k_t is 50% above its steady state. Simulate the convergence process.

   (e) Derive the savings rate s^* corresponding to the "*golden rule*" level of the economy.

3. **CES production function**

Consider the following CES production function

$$Y_t = F(K_t, A_t L_t) = \left[\alpha K_t^p + (1-\alpha)(A_t L_t)^p\right]^{\frac{1}{p}},$$

where we have defined $p \equiv 1 - 1/\sigma$. The parameter σ denotes the elasticity of substitution.

Derive the limit of this function when

(a) $p \to 1$

(b) $p \to 0$

(c) $p \to -\infty$.

(d) Comment on the substitutability of K_t and $A_t L_t$ and plot the isoquants of the function for all cases.