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ECON 5118 Macroeconomic Theory

Additional Errata in Textbook

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This document provides additional corrections and comments on Michael Wickens (2008) *Macroeconomic Theory*, Princeton University Press. You should first download the original errata on the book's web site.

Chapter 2

- P. 22, second paragraph should read "The implications for consumption can be seen in figures 2.6 and 2.7. In figure 2.6 ... "

- P. 26. The Euler equation (2.19) is a non-linear equation in c_{t+1}, c_t, and k_{t+1}. Let

$$f(x) = \frac{U'(c_{t+1})}{U'(c_t)} \left[F'(k_{t+1}) + 1 - \delta\right],$$

where $x = [c_{t+1}\ c_t\ k_{t+1}]^{\mathrm{T}}$. The first-order Taylor approximation of f about $x^* = [c^*\ c^*\ k^*]^{\mathrm{T}}$ is

$$\begin{aligned} f(x) &\simeq f(x^*) + \nabla f(x^*)^{\mathrm{T}}(x - x^*) \\ &= \frac{U'(c^*)}{U'(c^*)}[F'(k^*) + 1 - \delta] \\ &\quad + \frac{U''(c^*)}{U'(c^*)}[F'(k^*) + 1 - \delta](c_{t+1} - c^*) \\ &\quad - \frac{U'(c^*)}{[U'(c^*)]^2}U''(c^*)[F'(k^*) + 1 - \delta](c_t - c^*) \\ &\quad + \frac{U'(c^*)}{U'(c^*)}F''(k^*)(k_{t+1} - k^*) \\ &= F'(k^*) + 1 - \delta \\ &\quad + \frac{U''(c^*)}{U'(c^*)}[F'(k^*) + 1 - \delta](c_{t+1} - c_t) \\ &\quad + F''(k^*)(k_{t+1} - k^*). \end{aligned}$$

It follows that the first-order Taylor approximation of the Euler equation is

$$\begin{aligned} \beta[F'(k^*) &+ 1 - \delta \\ &+ \frac{U''(c^*)}{U'(c^*)}[F'(k^*) + 1 - \delta]\Delta c_{t+1} \\ &+ F''(k^*)(k_{t+1} - k^*)] \simeq 1. \end{aligned}$$

The coefficient $[F'(k^*) + 1 - \delta]$ is missing for the Δc_{t+1} term in the book. Using $F'(k^*) = \delta + \theta$ (2.21) and rearranging, equation (2.23) should read

$$(c_{t+1} - c^*) \simeq (c_t - c^*) - \frac{F''(k^*)U'(c^*)}{(1+\theta)U''(c^*)}(k_{t+1} - k^*). \tag{2.23}$$

Substitute (2.24) into (2.23), we have

$$\begin{aligned} (c_{t+1} - c^*) &= (c_t - c^*) - \frac{F''(k^*)U'(c^*)}{(1+\theta)U''(c^*)} \\ &\quad [(1+\theta)(k_t - k^*) - (c_t - c^*)] \end{aligned}$$

or

$$\begin{aligned} (c_{t+1} - c^*) = \left[1 + \frac{F''(k^*)U'(c^*)}{(1+\theta)U''(c^*)}\right](c_t - c^*) \\ - \frac{F''(k^*)U'(c^*)}{U''(c^*)}(k_t - k^*) \end{aligned} \tag{2.23a}$$

Equations (2.23a) and (2.24) give the matrix equation

$$\begin{bmatrix} c_{t+1} - c^* \\ k_{t+1} - k^* \end{bmatrix} = \begin{bmatrix} 1 + \frac{F''U'}{(1+\theta)U''} & -\frac{F''U'}{U''} \\ -1 & 1 + \theta \end{bmatrix} \begin{bmatrix} c_t - c^* \\ k_t - k^* \end{bmatrix}.$$

- P. 30, end of third paragraph: The Inada conditions should be $\lim_{k\to\infty} F_k = 0$, $\lim_{k\to 0} F_k = \infty$, $\lim_{n\to\infty} F_n = 0$, $\lim_{n\to 0} F_n = \infty$.

- P. 33, second equation: The last term inside the square bracket of the Lagrangian should be

$$-\frac{\phi i_{t+s}^2}{2k_{t+s}}.$$

The i_t/k_t terms in the last two equations should also be i_{t+s}/k_{t+s}.

Chapter 3

- P. 43, Figure 3.2: The label $F(k, t)$ of the top curve should be y_t. This is because F is a function of three variables, K_t, N_t, and t, not k_t and t.

- P. 43, Figure 3.3: The labels yk_t and yk^* should be γk_t and γk^*
- P. 45, first equation: The last term should be $\alpha\gamma$ instead of $a\gamma$.
- P. 47, 5th equation: the optimization problem should be

$$\max_{c^{\#}_{t+s}, k^{\#}_{t+s+1}} \sum_{s=0}^{\infty} \tilde{\beta}^s \left[\frac{(c_t^{\#})^{1-\sigma} - (1+\eta)^{-(1-\sigma)(t+s)}}{1-\sigma} \right] (1+\eta)^{(1-\sigma)t}.$$

The last term is $(1+\eta)^{(1-\sigma)t}$ instead of $(1+\eta)^{(1-\sigma)t+s}$ because s has been included in $\tilde{\beta}$.

- P. 47, 7th equation: the second term in the denominator inside the square bracket of the Lagrangian should also be $(1+\eta)^{-(1-\sigma)(t+s)}$. The first-order conditions that follow, however, are not affected.
- P. 53: The last equation before section 3.6 should read $A^* = A(\alpha/(1-\alpha))^{-(1-\alpha)}$.

Chapter 4

- P. 56: Equation (4.7) should be

$$W_t = \frac{a_{t+n}}{\prod_{s=1}^{n-1}(1+r_{t+s})} + \sum_{s=1}^{n-1} \frac{c_{t+s}}{\prod_{u=1}^{s}(1+r_{t+u})} + c_t. \tag{4.7}$$

Also, equation (4.8) should be

$$= \sum_{s=1}^{n-1} \frac{x_{t+s}}{\prod_{u=1}^{s}(1+r_{t+u})} + x_t + (1+r_t)a_t. \tag{4.8}$$

- P. 59, second last sentence should read "First we take a linear approximation to the Euler equations, (4.4)."
- P. 60, third line should read "Solving (4.4) and (4.15) we obtain ..."
- P. 60, equation (4.16): The approximation sign should be an equality.
- P. 69: The last term in the second first-order condition should be p^D_{t+s-1} instead of p^D_{t+s}:

$$\frac{\partial \mathcal{L}_t}{\partial D_{t+s}} = \beta^s U_{D,t+s} + \lambda_{t+s} p^D_{t+s}(1-\delta) - \lambda_{t+s-1} p^D_{t+s-1} = 0.$$

Consequently, equation (4.23) should be

$$U_{D,t+1} = U_{c,t+1}\left[(1+r_{t+1})p_t^D - (1-\delta)p^D_{t+1}\right]. \tag{4.23}$$

Also, equation (4.25) becomes

$$\frac{c_{t+1}}{D_{t+1}} = \frac{\alpha}{1-\alpha}\left[(1+r_{t+1})p_t^D - (1-\delta)p^D_{t+1}\right]. \tag{4.25}$$

- P. 70: From the last equation we have

$$\frac{p_t^D d_t}{c_t} \simeq \left[\frac{\Delta c_{t+1}}{c_t} - \frac{1}{1-\alpha}(r_{t+1}-\theta) + \delta\right] \frac{p_t^D D_t}{c_t}.$$

Since $p_t^D d_t / c_t \geq 0$ and $p_t^D D_t / c_t > 0$, the expression inside the square bracket is nonnegative. For example, if interest rate r_{t+1} rises high above the steady-state value of θ, nondurable consumption must increase in period $t+1$ to a level such that

$$\frac{\Delta c_{t+1}}{c_t} \geq \frac{1}{1-\alpha}(r_{t+1}-\theta) - \delta.$$

- P. 71: The fourth line should read "... where $U_c > 0, U_l > 0, U_{cc} \leq 0, U_{ll} \leq 0, U_{n,t} = -U_{l,t}$, and ...".
- P. 71, third equation: The labour income and other income terms in the Lagrangian should be $w_{t+s}n_{t+s}$ and x_{t+s} respectively.
- P. 71: In the sentence before equation (4.28), the reference to equation (2.12) is incorrect. It should be the equation before (4.23).
- P. 74: The first term in the first equation should be $\partial \mathcal{P}_t / \partial n_{t+s}$.
- P. 74, third equation: The first-order condition for b_{t+s} should be

$$\frac{\partial \mathcal{P}_t}{\partial b_{t+s}} = -(1+r)^{-s}(1+r) + (1+r)^{-(s-1)} = 0, \quad s > 0.$$

- P. 76, first equation: The first-order condition should be

$$\frac{\partial \mathcal{P}_t}{\partial n_{t+s}} = (1+r)^{-s}(F_{n,t+s} - W_{t+s} + \lambda \Delta n_{t+s+1}) - (1+r)^{-(s-1)}\lambda \Delta n_{t+s} = 0.$$

(subtraction for the last term instead of addition.)

- P. 76, the sentence before equation (4.34) should be "The first first-order conditions gives ...".
- P. 79: The last equation should be

$$w_t = F_{n,t} = -\frac{U_{n,t}}{U_{c,t}}.$$

- P. 80, third equation: The labour demand function should be

$$n_t^d = \left[\frac{w_t}{(1-\alpha)A}\right]^{-1/\alpha} k_t.$$

The exponential term for the square brackets in the sixth and eighth equations should also be $-1/\alpha$.

- P. 81, third equation: Delete the term $-\ r$ near the end of the equation.

Chapter 5

- P. 91, second equation: In period t, the first term of the GBC should be g_{t-1} instead of g_t. The same applies to the GBC in periods $t+1$ and $t+n-1$.

- P. 91: The sixth equation should be

$$b_{t+n} = b_t + \sum_{s=1}^{n} \Delta b_{t+s} = b_t + \left[\frac{(1+R)^n - 1}{R}\right] \Delta g_t.$$

The next equation is

$$\frac{b_{t+n}}{(1+R)^n} = \frac{b_t}{(1+R)^n} + \left[\frac{1}{R} - \frac{1}{R(1+R)^n}\right] \Delta g_t.$$

- P. 91, the last paragraph before section 5.3.4: I do not quite get how the last equation works so this is my opinion on how the paragraph should read:

It can be shown that wealth is unaffected by this as its values in periods $t-1$ and t are

$$\begin{aligned} W_{t-1} &= \sum_{s=0}^{\infty} \frac{x_{t+s-1} - T_{t+s-1}}{(1+R)^s} + (1+R)b_t, \\ W_t &= \sum_{s=0}^{\infty} \frac{x_{t+s} - T_{t+s}}{(1+R)^s} + (1+R)b_t \\ &= W_{t-1} - \Delta T_t - \frac{\Delta T_{t+1}}{1+R} \\ &= W_{t-1} - \Delta T_t + \frac{(1+R)\Delta T_t}{1+R} \\ &= W_{t-1}. \end{aligned}$$

Therefore the temporary tax cut does not have any wealth effect and so consumption remains unchanged.

- P. 97, equation (5.5): The last term should be $M_t/P_t y_t$.

- P. 97: Equation (5.9) should be

$$P_t d_t = P_t D_t - R_t B_t.$$

- P. 97: The last sentence should read "Since the nominal rate of growth $\pi_{t+1} + \gamma_{t+1}$ is nearly always strictly positive, equation (5.8) is stable and can be solved backward."

- P. 98, section 5.4.1: We can apply the result of first-order difference equation with a constant d_t/y_t to (5.12) to get (5.14). Equation (5.13) does not make sense mathematically. This is because we are taking the limit $n \to \infty$ on the left-hand side, n should not appear on the right-hand side of the equation.

- P. 99, second paragraph, line 3: the term $(y_t + s)$ should be y_{t+s}.

- P. 108, second last line: The optimal solution is $\tau_t = g_t/F(k_t)\ \ldots$.

- P. 114: Starting from the second equation, the last term in the denominator of the middle expression should be $U_{lc,t}n_t$:

$$\frac{V_{l,t}}{V_{c,t}} = \frac{(1+\mu)U_{l,t} + \mu(U_{cl,t}c_t - U_{ll,t}n_t)}{(1+\mu)U_{c,t} + \mu(U_{cc,t}c_t - U_{lc,t}n_t)} = w_t.$$

Comparing with the household optimal condition (5.24), we need

$$\begin{aligned} \frac{V_{l,t}}{V_{c,t}} &= \frac{(1+\mu)U_{l,t} + \mu(U_{cl,t}c_t - U_{ll,t}n_t)}{(1+\mu)U_{c,t} + \mu(U_{cc,t}c_t - U_{lc,t}n_t)} \\ &= \frac{(1+\tau_t^c)U_{l,t}}{(1-\tau_t^w)U_{c,l}}. \end{aligned} \tag{5.35}$$

(l_t should be n_t and τ_t^l in the denominator of the second equality should be τ_t^w. As a result, the rest of the analysis may be as follows:)

Since our model consider a representative household, preferences must be homothetic. Therefore for all $\theta > 0$,

$$\frac{U_c(\theta c, \theta l)}{U_l((\theta c, \theta l)} = \frac{U_c(c,l)}{U_l((c,l)}.$$

Differentiate with respect to θ and then set $\theta = 1$, we have

$$\frac{U_{cc,t}c_t + U_{lc,t}l_t}{U_{c,t}} = \frac{U_{cl,t}c_t + U_{ll,t}l_t}{U_{l,t}},$$

or, using $n_t + l_t = 1$,

$$\frac{(U_{cc,t}c_t - U_{lc,t}n_t) + U_{lc,t}}{U_{c,t}} = \frac{(U_{cl,t}c_t - U_{ll,t}n_t) + U_{ll,t}}{U_{l,t}}.$$

If we assume in the above that $U_{lc,t}/U_{c,t} = U_{ll,t}/U_{l,t}$, then

$$\frac{(U_{cc,t}c_t - U_{lc,t}n_t)}{U_{c,t}} = \frac{(U_{cl,t}c_t - U_{ll,t}n_t)}{U_{l,t}}. \tag{5.37}$$

Substituting (5.37) into (5.35) gives

$$\frac{V_{l,t}}{V_{c,t}} = \frac{U_{l,t}}{U_{c,t}} = \frac{(1+\tau_t^c)U_{l,t}}{(1-\tau_t^w)U_{c,l}}.$$

This implies that $\tau_t^c = \tau_t^w = 0$ or $\tau_t^c = -\tau_t^w$, which means that both taxes should be zero or the government subsidizes consumption at the same rate as it taxes labour.

Two comments:

1. The assumption $U_{lc,t}/U_{c,t} = U_{ll,t}/U_{l,t}$ is very restrictive.
2. Government spending g_t is absent in the utility function. The results of zero labour and consumption taxes do not hold when g_t is a public good.

Chapter 8

- P. 183: The second last equation should be

$$\beta U_{m,t+1} = \lambda_{t+1} R_{t+1}.$$

- P. 187, the last two equations: Since

$$U_{l,t+1} = \frac{\eta}{l_{t+1}}, \quad S_{m,t+1} = -\psi \frac{c_{t+1}}{m_{t+1}^2},$$
$$U_{c,t+1} = \frac{1}{c_{t+1}}, \quad S_{c,t+1} = \frac{\psi}{m_{t+1}},$$

equation (8.13) becomes

$$\frac{\psi \eta c_{t+1}}{l_{t+1} m_{t+1}^2} = \left(\frac{1}{c_{t+1}} - \frac{\psi \eta}{l_{t+1} m_{t+1}} \right) R_{t+1}.$$

Using $m_{t+1} = \psi c_{t+1}/s_{t+1}$ and rearranging give

$$m_{t+1} = \frac{c_{t+1}}{R_{t+1}} \left(\frac{\eta s_{t+1}/l_{t+1}}{1 - \eta s_{t+1}/l_{t+1}} \right). \qquad (8.14)$$

- P. 189, equation (8.18): The last term $(1+\theta)b$ should be θb.

- P. 190, second paragraph: If $T_m + \pi < 0$, then $T_{mc}(T_m + \pi) \geq 0$ since we have assumed that $T_{mc} \leq 0$. It follows that $\Delta = T_{mm}(1+T_c) - T_{mc}(\pi + T_m)$ is not necessary positive. The problem can be resolved by assuming $T_{mc} = 0$.

- P. 195: There is an extra k_{t+1} term in the first line of the first equation. The correct version should be

$$c_t + (1+\pi_{t+1})(k_{t+1} + b_{t+1} + m_{t+1})$$
$$= (1-\tau_t) w_t n_t + (1+R_t^k) k_t + (1+R_t^b) b_t + m_t$$

- P. 195, last three equations: The term π_{t+s+1} should be π_{t+s}.

- P. 196, equations (8.25) and (8.26): The term π_{t+s+1} should be π_{t+s}.

- P. 196, last line: The term $(1 + R_{t+s}^b)$ should be R_{t+s}^b. The term π_{t+s+1} should be π_{t+s}.

- P. 197: In view of the corrections in the last equation on p. 196, the intertemporal household budget constraint should be

$$\lambda_{t-1}(1+\pi_t)(k_t + b_t + m_t)$$
$$= \sum_{s=0}^{\infty} \lambda_{t+s} \left[c_{t+s} - (1-\tau_{t+s}) w_{t+s} n_{t+s} + R_{t+s}^b m_{t+s} \right].$$

Equation (8.29) is

$$\lambda_{t-1}(1+\pi_t)(k_t + b_t + m_t)$$
$$= \sum_{s=0}^{\infty} \beta^s \left(U_{c,t+s} c_{t+s} - U_{l,t+s} n_{t+s} + U_{m,t+s} m_{t+s} \right).$$

- P. 197, 7th equation: The signs in the economy's resource constraint are incorrect. The equation should be

$$F(k_t, n_t) = c_t + k_{t+1} - (1-\delta) k_t + g_t.$$

- P. 197, last line: A minus sign is missing in front of the term $U_{l,t+s} n_{t+s}$.

- P. 200, third and second last equations: In the first-order conditions for capital and money, the last term should be $(1 + \pi_{t+s})$ instead of $(1 + \pi_{t+s+1})$. The same applies to the following definition of $1 + r_{t+s}$.

Chapter 9

- P. 208, equation (9.1): The price should be P instead of p.

- P. 208, last equation: The share on the left-hand side should be

$$\frac{W_i X_i}{PQ}.$$

- Sections 9.3.2 and 9.3.3: The analysis is static in nature and so the time subscripts t for all variables can be dropped. Section 9.3.2 contains a number of typographical and analytical errors. See the Appendix for a revised version of the whole section.

- P. 214, the 8th and 9th equations: The exponential terms for the expressions inside the square brackets should be ϕ instead of $1/\phi$. Therefore the two equations should read

$$\begin{aligned} n_t(i) &= A_i^{\phi-1}\left[\frac{\phi-1}{\phi}\frac{P_t}{W_t}\right]^{\phi} y_t, \\ y_t(i) &= A_i^{\phi}\left[\frac{\phi-1}{\phi}\frac{P_t}{W_t}\right]^{\phi} y_t. \end{aligned}$$

- P. 215: Using

$$P_t = \left[\sum_{i=1}^{N} p_t(i)^{1-\phi}\right]^{1/(1-\phi)}$$

and

$$p_t(i) = \frac{\phi}{A_i(\phi-1)}W_t,$$

it can be shown that

$$v_t = N^{-1/(1-\phi)}$$

for $A_i = 1, i = 1, \ldots, N$. The range of ϕ is $(0, \infty)$. Therefore $v < 1$ for $0 < \phi < 1$ (in the short run) but $v > 1$ for $\phi > 1$. If $0 < \phi < 1$, however, $n_t(i), y_t(i)$, and $P_t(i)$ become negative ...

- P. 215, sixth equation: Using

$$P_t = \left[\sum_{i=1}^{N} P_t(i)^{1-\phi}\right]^{1/(1-\phi)},$$

we have

$$\frac{\partial P_t}{\partial P_t(i)} = \left[\frac{P_t}{P_t(i)}\right]^{\phi}, \quad i = 1, \ldots N.$$

The total differential of P_t is therefore

$$dP = \sum_{i=1}^{N}\frac{\partial P_t}{\partial P_t(i)}dP_t(i) = \sum_{i=1}^{N}\left[\frac{P_t}{P_t(i)}\right]^{\phi} dP_t(i).$$

As a result the last inequality

$$\frac{dP_t}{P_t} < \frac{dP_t(i)}{P_t(i)}$$

is incorrect. An alternative analysis is as follows:

Suppose that in period $t+1$ inflation rates for all the N inputs are the same at $1+\pi$. Then

$$\begin{aligned} P_{t+1} &= \left(\sum_{i=1}^{N}\left[(1+\pi)P_t(i)\right]^{1-\phi}\right)^{1/(1-\phi)} \\ &= (1+\pi)\left(\sum_{i=1}^{N} P_t(i)^{1-\phi}\right)^{1/(1-\phi)} \\ &= (1+\pi)P_t. \end{aligned}$$

Therefore P_t and all the $P_t(i)$ have the same inflation rate. The result is well-known in index number theory. When the production function is the CES function, the input price index is the so-called Llyod-Moulton index.[1] The index satisfies the "proportionality" test, that is, when the next period price vector is a scalar multiple $(1+\pi)$ of the current period price vector, the overall price index is equal to $(1+\pi)$.[2]

- P. 221, second equation: The last term should be p_{t-2} instead of p_{t-1}.

- P. 221: The fourth equation

$$\pi_t = \rho(p_t^{\#} - p_{t-1}) + (1-\rho)\pi_{t-1}$$

follows directly from the first equation

$$p_t = \rho p_t^{\#} + (1-\rho)(\pi_{t-1} + p_{t-1}).$$

There is no need to solve the second equation by the auxiliary equation.

- P. 221: The fifth equation should be

$$\pi_t = \rho(1-\gamma)(p_t^* - p_{t-1}) + \gamma E_t\pi_{t+1} + (1-\gamma)(1-\rho)\pi_{t-1}.$$

- P. 222, last equation: The variable β_s should be β^s.

- P. 223: In equation (9.31) α cannot be 1 since this makes $\alpha/(1-\alpha)$ and $\beta/(1-\alpha)$ undefined. Instead (9.27) should be used with $\alpha = 1$ and $\beta = 0$. In the Calvo model we need $\rho = 1$. The discussion on perfectly flexible prices should be based on comparing $\pi_t = \Delta p_t$ with $p_t^* - p_{t-1}$, not with $p_t^* - p_t$. Therefore equations (9.25) and (9.27) should be used instead of (9.31).

- P. 224–7, equations (9.32) and (9.35): Remove the minus sign of the first term on the right hand side:

$$\begin{aligned} \Delta p_t = \left(1 - \frac{\beta}{1-\alpha}\right)\pi + \frac{\alpha}{1-\alpha}(p_t^* - p_t) \\ + \frac{\beta}{1-\alpha}E_t\Delta p_{t+1}. \quad (9.32) \end{aligned}$$

$$\begin{aligned} \pi_t = \left(1 - \frac{\beta}{1-\alpha}\right)\pi + \frac{\alpha}{1-\alpha}(p_t^* - p_t) \\ + \frac{\beta}{1-\alpha}E_t\pi_{t+1}. \quad (9.35) \end{aligned}$$

Repeat with equations (9.39), (9.41), and (9.42).

- P. 226, equation (9.40): The sign in front of $\tilde{a}_t/\phi$ should be plus instead of minus.

[1] P.J. Lloyd (1975) "Substitution Effects and Biases in Nontrue Price Indices," *American Economic Review*, 65(3), 301–313.

[2] For details see, Bert M. Balk (1995) "Axiomatic Price Index Theory: A Survey," *International Statistical Review*, 63(1), 69–93.

Appendix

Section 9.3.2 contains a numbers of typographical errors and conceptual problems. These include

- The aggregator function c lacks weight parameters that sum to one. As a result the real wage W/P is dependent on N, the number of goods and services in the model.
- The model is static in nature so the time subscript t is a distraction.
- Although the number of households and the number of firms are assumed to be the same, the use of the same index i for both causes some confusion.
- Leisure of the household is mistaken as labour supply in equation (9.10).

In view of the large number of corrections needed a revised version is provided here.

9.3.2 Price Determination in the Macroeconomy with Imperfect Competition

Modern macroeconomic theories of price determination emphasize the fact that in the economy a large number of different goods and services are produced. A widely used model of price setting when these goods are imperfect substitutes is that of Dixit and Stiglitz (1977). We consider a variant of this that is closely related to work by Blanchard and Kiyotaki (1987), Ball and Romer (1991), and Dixon and Rankin (1995) (see also Mankiw and Romer (1991) and the articles cited therein). For simplicity, the model is highly stylized.

We assume that the economy is composed of N firms each producing a different good that is an imperfect substitute for the other goods, and that a single factor of production is used, namely, labour that is supplied by N households. The production function for the ith firm is assumed to be

$$y_i = F_i(n_i), \quad i = 1, \ldots, N,$$

where n_i is the labour input of the ith firm. The production function is indexed by i to denote that each good may be produced with a different production function. Profit of the ith firm is

$$\Pi_i = P_i F_i(n_i) - W_i n_i, \tag{9.4}$$

where P_i is the output price and W_i is the wage rate paid by firm i.

9.3.2.1 Households

We assume that there are also N households and these are classified by their type of employment, with each household working for one type of firm. Households are assumed to have an identical instantaneous utility function:

$$U(c_j, l_j) = u(c_j) + \eta l_j^{\varepsilon}, \quad j = 1, \ldots, N,$$

where c_j is household j's aggregate consumption, l_j is leisure, and $n_j + l_j = 1$. We assume that u is increasing and concave.

We also assume that aggregate consumption c_j of household j is obtained by aggregating over the N different types of goods and services c_{ij} using the constant elasticity of substitution function

$$c_j = \left(\sum_{i=1}^{N} \beta_i c_{ij}^{\rho} \right)^{1/\rho}, \quad 0 \neq \rho < 1, \quad \sum_{i=1}^{N} \beta_i = 1. \tag{9.5}$$

The elasticity of substitution is $\phi = 1/(1-\rho)$; we recall that a higher value of ϕ implies greater substitutability. Thus goods and services are imperfect substitutes if ϕ is finite.

Total consumption of all households is

$$c = \sum_{j=1}^{N} \left(\sum_{i=1}^{N} \beta_i c_{ij}^{\rho} \right)^{1/\rho}.$$

Total consumption of the ith good or service is

$$c_i = \sum_{j=1}^{N} c_{ij}, \quad i = 1, \ldots, N.$$

Total household expenditure is

$$Pc = \sum_{i=1}^{N} P_i c_i.$$

This implies that the general price level is

$$P = \sum_{i=1}^{N} P_i \frac{c_i}{c}. \tag{9.6}$$

The budget constraint of household j is given by

$$Pc_j = \sum_{i=1}^{N} P_i c_{ij} = W_j n_j + \frac{1}{N} \sum_{i=1}^{N} \Pi_i, \quad j = 1, \ldots, N,$$

where each household is assumed to hold an equal share in each firm.

In the absence of capital (and trading in shares) the budget constraint is static. Consequently, optimization can be carried out each period without regard to future periods. Thus, in the absence of assets, the intertemporal aspect of the DGE model of the model is eliminated. We assume, therefore, that household j maximize utility with respect to $\{c_{1j}, \ldots, c_{Nj}, n_j\}$ subject to their budget constraint and to $n_j + l_j = 1$. The household maximization problem is

$$\max_{c_{ij}, n_j} \quad u\left(\left[\sum_{i=1}^{N} \beta_i c_{ij}^{\rho}\right]^{1/\rho}\right) + \eta(1-n_j)^{\varepsilon}$$

$$\text{subject to} \quad \sum_{i=1}^{N} P_i c_{ij} = W_j n_j + \frac{1}{N}\sum_{i=1}^{N} \Pi_i$$

The Lagrangian is

$$\mathcal{L} = u\left(\left[\sum_{i=1}^{N} \beta_i c_{ij}^{\rho}\right]^{1/\rho}\right) + \eta(1-n_j)^{\varepsilon} - \lambda\left[\sum_{i=1}^{N} P_i c_{ij} - W_j n_j - \frac{1}{N}\sum_{i=1}^{N} \Pi_i\right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial c_{ij}} = \beta_i u'(c_j)\left(\frac{c_j}{c_{ij}}\right)^{1-\rho} - \lambda P_i = 0, \quad i = 1, \ldots, N$$

$$\frac{\partial \mathcal{L}}{\partial n_j} = -\eta\varepsilon(1-n_j)^{\varepsilon-1} + \lambda W_j = 0.$$

The first-order condition for c_{ij} implies that

$$\frac{c_{ij}}{c_j} = \left[\frac{\beta_i u'(c_j)}{\lambda P_i}\right]^{\phi}, \quad i = 1, \ldots, N. \tag{9.7}$$

Each household's problem can also be expressed in terms of maximizing utility with respect to aggregate consumption c_j, as the Lagrangian can be rewritten as

$$\mathcal{L} = u(c_j) + \eta(1-n_j)^{\varepsilon} - \lambda\left[P c_j - W_j n_j - \frac{1}{N}\sum_{i=1}^{N} \Pi_i\right].$$

The first-order condition with respect to c_j is

$$u'(c_j) - \lambda P = 0,$$

which implies that $\lambda = u'(c_j)/P$. Equation (9.7) can be written as

$$\frac{c_{ij}}{c_j} = \left(\frac{\beta_i P}{P_i}\right)^{\phi}, \quad i = 1, \ldots, N.$$

The right-hand side of the above equation is independent of j so that the consumption ratio c_{ij}/c_j of the ith good or service is constant across households. The aggregate consumption ratio of the ith good or service is therefore

$$\frac{c_i}{c} = \left(\frac{\beta_i P}{P_i}\right)^{\phi}, \quad i = 1, \ldots, N. \tag{9.8}$$

Substituting (9.8) into (9.6) gives the general price level expressed solely in terms of individual prices:

$$P = \sum_{i=1}^{N} P_i \left(\frac{\beta_i P}{P_i}\right)^{\phi}.$$

Solving for P gives

$$P = \left(\sum_{i=1}^{N} \beta_i P_i^{1-\phi}\right)^{1/(1-\phi)}. \tag{9.9}$$

From the first-order condition with respect to labour, the total supply of labour by household j is

$$n_j = 1 - \left[\frac{u'(c_j) W_j}{\eta\varepsilon P}\right]^{1/(\varepsilon-1)}. \tag{9.10}$$

If $\varepsilon \le 1$, an increase in W_j will raise labour supply n_j. If labour markets are competitive, households have the same utility function (implying complete markets) and work equally hard (implying firms are indifferent about who they hire), in which case W_j will be equal across households. We denote the common wage by W. If households have different utility functions (or do not work equally hard), then the marginal utilities will differ and so will wages.

9.3.2.2 Firms

The problem for the ith firm is to maximize profits subject to its demand function, equation (9.8). In the absence of investment and government expenditures, we have $c_i = y_i = F_i(n_i)$. Equation (9.4) becomes

$$\Pi_i = P_i c_i - W n_i.$$

The first-order condition for profit maximization is

$$\frac{d\Pi_i}{dc_i} = P_i + \frac{\partial P_i}{\partial c_i} c_i - W\frac{dn_i}{dc_i} = 0,$$

where

$$\frac{dc_i}{dn_i} = F_i'(n_i).$$

Assuming that in (9.8) the effects of changes in individual P_i and c_i on the overall P and c are small, we have

$$\frac{\partial c_i}{\partial P_i} = -\phi c \frac{(\beta_i P)^{\phi}}{P_i^{\phi+1}} = -\phi\frac{c_i}{P_i}.$$

The price elasticity of demand for good i is therefore

$$\varepsilon_{Di} = \frac{\partial c_i}{\partial P_i}\frac{P_i}{c_i} = -\phi.$$

The above first-order condition becomes

$$P_i\left(1 - \frac{1}{\phi}\right) = \frac{W}{F_i'(n_i)},$$

or

$$P_i = \frac{\phi}{\phi - 1} \frac{W}{F_i'(n_i)}. \tag{9.11}$$

This is a key result. It indicates that price is a markup over W/F_i', which is the marginal cost of an extra unit of output; the markup or wedge is $\phi/(\phi-1) > 1$. As $\phi \to \infty$, i.e., as the consumption goods become perfect substitutes, the markup tends to unity and price falls to equal marginal cost. This solution is the standard outcome for monopoly pricing. Prices vary across goods due to differences in the marginal product of labour, $F_i'(n_i)$. Equation (9.11) implies that firms have some control over their prices. This entails a source of inefficiency because output, and hence consumption, are lower than in perfect competition. An increase in the economy-wide wage would therefore cause an increase in the price of each good and in the general price level.

The demand for labour can be obtained from equation (9.11). Suppose that the production function is Cobb-Douglas so that

$$y_i = A_i n_i^{\alpha_i}, \quad \alpha_i \leq 1,$$

where A_i can be interpreted as an efficiency term for the ith firm. Labor demand is then given by

$$n_i = \left[\frac{\phi}{\alpha_i A_i(\phi - 1)} \frac{W}{P_i}\right]^{-1/(1-\alpha_i)}. \tag{9.12}$$

The greater ϕ is, and hence the lower the markup, the greater labour demand and output are, reflecting once more the inefficiency of monopolies in terms of lost output and employment.

Since each household works for one firm only, so matching $n_i = n_j$ in equations (9.10) and (9.12) gives

$$\left[\frac{\phi}{\alpha_i A_i(\phi - 1)} \frac{W}{P_i}\right]^{-1/(1-\alpha_i)} = 1 - \left[\frac{u'(c_j)W}{\eta\varepsilon P}\right]^{1/(\varepsilon-1)}.$$

Solving for P_i gives

$$P_i = \frac{\phi W}{\alpha_i A_i(\phi - 1)} \left\{1 - \left[\frac{u'(c_j)W}{\eta\varepsilon P}\right]^{1/(\varepsilon-1)}\right\}^{1-\alpha_i} \tag{9.13}$$

Thus differences between firm prices are due to A_i and α_i . Equation (9.13) implies that, if $\varepsilon < 1$, an increase in the economy-wide real-wage rate would raise the relative price of firm i.

In the special case where the efficiency term A_i and the production elasticities are the same, so that $A_i = A$ and $\alpha_i = \alpha$, firm prices will be identical. By equation (9.9) $P = P_i$ and equation (9.13) becomes[3]

$$P = \frac{\phi W}{\alpha A(\phi - 1)} \left\{1 - \left[\frac{u'(c_j)W}{\eta\varepsilon P}\right]^{1/(\varepsilon-1)}\right\}^{1-\alpha},$$

which can be rewritten as

$$\frac{1}{\alpha A} \frac{\phi}{\phi - 1} \frac{W}{P} \left\{1 - \left[\frac{u'(c_j)}{\eta\varepsilon} \frac{W}{P}\right]^{1/(\varepsilon-1)}\right\}^{1-\alpha} = 1. \tag{9.14}$$

Since all households and all firms are identical, $c_j = c/N = y/N$. If $\varepsilon < 1$, W/P is unambiguously negatively related to $u'(c_j)$ and therefore positively related to c and therefore output y. That is, an increase in the real wage will raise output. Moreover, the lower the markup $\phi/(\phi-1)$ is, the greater the response of output to the real wage will be. Equation (9.14) also shows that the economy is then neutral with respect to nominal values. An example of this is when each production function is linear in labour when $\alpha = 1$. In this case

$$\frac{W}{P} = A\left(\frac{\phi - 1}{\phi}\right)$$

so that employment is determined by the supply side (equation (9.10)).

[3] If $\beta_i = 1$ for all i as specified in the textbook, then $P = N^{1/(1-\phi)} P_i$.