Johannes-Gutenberg University Mainz
Bachelor of Science in Wirtschaftswissenschaften

Macroeconomics II: Behavioural Macro
Summer 2017

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www.macro.economics.uni-mainz.de
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Part II

Behavioural economics

6 Overview

6.1 General idea

• “Behavioral Economics is the combination of psychology and economics that investigates what happens in markets in which some of the agents display human limitations and complications” (Mullainathan and Thaler, 2001)

• Compared to self-interested homo oeconomicus, human behavior deviates in three dimensions: (i) bounded rationality, (ii) bounded willpower and (iii) bounded self-interest

• Bounded rationality: limited cognitive abilities that constrain human problem solving

• Bounded willpower: people sometimes make choices that are not in their long-run interest

• Bounded self-interest: humans are willing to sacrifice their own interests to help others
6.2 Specific papers in behavioural theory

• Bounded willpower
  – Automatic behaviour: Bernheim and Rangel (2004), Laibson (2001)

• Bounded self-interest
  – Altruism: Fehr and Schmidt (1999)
  – Superfairness: Baumol (1986)

• Bounded rationality → see the seminar offered by the chair

• Prospect theory
  – Kahneman and Tversky (1979) - see emotion part of lecture

• Emotions → see previous part of lecture
6.3 What we do here at JGU

How do we allow modern views to enter traditional economic thinking here at the macro chair?

- Makroökonomik I – nothing so far
- Macro II
  - see slides on emotions for emotional economics
  - Bounded willpower
    * Automatic behaviour: Bernheim and Rangel (2004)
    * Time inconsistency: general ideas (as in Strotz, 1955 and Laibson, 1997) and O’Donoghue and Rabin (1999)
- Advanced Macro (Master in International Economics and Public Policy)
  - Bounded willpower
    * Dual selves as in Fudenberg and Levine (2006) including game structure
    * Time inconsistency as in Strotz (1955), full analysis à la Krieger (2011)
  - Bounded rationality
    * to be determined
7 Bounded willpower and automatic behaviour

7.1 The plan

We consider the model of addiction by Bernheim and Rangel (2004)

- When thinking about addiction and drug abuse, three observations can be made (p. 1561)
  - there is a pathological divergence between choice and preference – individuals commit errors
  - using drugs makes individual sensitive to cues about drug use (think of coffee and the Lavazza bar)
  - addicts understand this and try to manage these processes

- Remarkable feature: allow individuals to make mistakes
  - no sense to infer preferences from choice: think about crossing the street in UK (being e.g. a continental European tourist)
  - individuals sometimes are in a “cold” mode where decision making about consumption behaviour is rational. When triggered by some cue (with some probability), individuals are in a “hot” mode and act against their preferences
• Nota bene: In revealed preference approach, “utility maximization and choice are synonymous” (Gul and Pesendorfer, 2008, S. 6)
7.2 The model

- Basic structure (simplified static version still revealing essential insights)
  - There is a normal consumption good, an addictive substance $x$ and a lifestyle $a \in \{E, A, R\}$
    - The consumption good can be consumed at arbitrary positive amounts $c$
    - The addictive good is consumed or not, $x \in \{0, 1\}$
    - Lifestyles are Exposure (to cues at, say, a party), Avoidance (stay at home) and Rehabilitation (in a clinical residential center)
  - The individual has a certain income $y$ which is used for normal consumption (whose price is one) and for the addictive good at price $q$
    \[
    c + qx = y
    \] (12)
  - The individual has a utility function $u(c, x, a)$ that rises in all three arguments (with $a$ being ordered as above)
• The “modes” of the individual (strength of self-control)
  – The individual can be in a “cold mode” or in a “hot mode”, which is triggered by probabilistic cues
  – The choice of the lifestyle determines the probability $p^a$ of entering the hot mode
  – It is assumed that the probability rises, the “worse” the lifestyle
    \[ p^E \geq p^A \geq p^R \]

• Sequence of events
  – Wake up in the morning in cold mode
  – Select a “lifestyle” activity $a \in \{E, A, R\}$
  – Observe the cues (when present, given the probabilities $p^a$)
  – When cue pushes to hot mode, consume addictive substance
  – In the cold mode (in the absence of a cue), allocate resources $y$ rationally between $c$ and $x$
7.3 Individual behaviour

- Behaviour of the individual in cold mode
  - Optimal choice of $c$ and $x$
  - Observe that $x$ can not be chosen in continuous amounts (have a drink or not, smoke a cigar or not ...)
  - Choice between $(c = y$ and $x = 0)$ or $(c = y - q$ and $x = 1)$
  - To make things precise, assume the utility function reads (this is not from Bernheim and Rangel)
    \[
    u = \chi c^\gamma + x + \alpha b \tag{13}
    \]
    where $\chi, \alpha > 0$ and $0 < \gamma < 1$ and
    \[
    b = \begin{cases} 
      3 & \text{for lifestyle } a = \text{Exposure (party)} \\
      2 & \text{Avoidance (stay at home)} \\
      1 & \text{Rehab (clinical center)}
    \end{cases}
    \]
Figure 12 Budget constraint and pseudo-indifference curves for $c$ and $x$ for a “drug-prone” individual (left panel) and a “drug-resistant” individual (right panel) (‘pseudo’ as $x$ is indivisible)
• Drug-resistant and drug-prone individuals

  – We define drug-resistant individuals as those that optimally choose not to consume the addictive good \( x \)
  
  – Drug-prone individuals find it optimal to consume the addictive good even in cold mode
  
  – Drug-resistant individuals have a higher \( \chi \), i.e. they value normal consumption goods more (relative to the addictive good \( x \)) than drug-prone individuals
  
  – Indifference curves turn clock-wise in the above figure
  
  – We study a drug-resistant individual in what follows (drug-prone individuals cannot commit any errors)
• (Not necessarily optimal) Behaviour of the individual

  – In cold mode, individual consumes $c$ only
  – Utility is given by the level as shown in the indifference curve in the right panel above
  – Precise utility level in cold mode is, setting $c = y$ and $x = 0$ in (13)

$$u(y, 0, b) = \chi y^\gamma + \alpha b$$

(14)

  which also depends on the lifestyle $a$ (via $b$)
  – When a cue occurs, the individual consumes the addictive good and utility falls to

$$u(y - q, 1, b) = \chi [y - q]^\gamma + 1 + \alpha b$$

  – To understand that it falls, remember that we look at drug-resistant individual in right panel of above figure (see Exercise 7.5.1)
  – This is an error in behaviour – behaviour does not maximize utility
7.4 Summary

- Many individuals seem to have bounded willpower
- An example of bounded willpower consists in drug use
  - There is a divergence between choice and preference (defined as 'errors')
  - Using drugs makes individuals sensitive to using drugs (not present in our simplified version here)
  - Addicts understand these interactions
- Bounded willpower is captured by exogenous link between cue and consumption of drug $x$
  - Divergence between choice and preference: see figure on drug-resistant individual and observe that utility falls
  - In model here, probability of a cue depends on lifestyle, $p^E \geq p^A \geq p^R$. Probability could also depend on degree of addiction (how often has drug been used in the past → see paper)
  - Addicts understand all of this and choose lifestyle rationally
• This seems to be (personal opinion) a “no risk no fun” paper
  – The individual enjoys a certain lifestyle – which is the fun part – which however bears a higher risk of an “accident” (drug use)
  – This resembles investment under uncertainty: on average, returns are higher (from lifestyle) but sometimes there are bad realizations (drug use)

• What is the difference to the cue-theory of consumption paper by Laibson (2001)?
  – Laibson studied the effects of cues as well
  – In the presence of a cue, marginal utilities are altered
  – Individuals can still make an optimal choice, however
  – Here, a cue directly leads to behaviour, without any intermediate choice

• Do we have an emotion paper here which we missed in our emotion part of this lecture?
  – The paper does talk about ’cravings’ and ’hedonic payoffs’ or ’hedonic implications’
  – In this sense, yes, it is an emotion paper as well, not just a paper about bounded willpower
7.5 Exercises

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7.5.1 Addiction and Automatic Processes (Bernheim and Rangel, 2004)

Consider an individual who maximises the following objective function in a "cold mode", where she can rationally choose her own consumption, whereas a "hot mode", is where she would have to consume $x$, is triggered in response to cues:

$$\max_{\{c,x\}} u(c, x, b) = \chi c^\gamma + x^\gamma + \alpha b$$  \hspace{1cm} (15)

$$s.t. \ c + qx = y$$  \hspace{1cm} (16)

Where $c$ is the standard consumption good whose price is normalised to unity, $x \in \{0, 1\}$ represents consumption of the addictive good at price $q$, and $b$ is the activity (or lifestyle) that the individual chooses, with $b \in \{E, A, R\}$ is the state that she can find herself in, with $E \equiv Exposure$, $A \equiv Avoidance$ and $R \equiv Rehab$. Each state carries a certain probability, $p^\alpha$, of being exposed to a cue and thus triggering the "hot mode", which automatically leads to consumption of $x$, such that $p^E > p^A > p^R$. In other words, $E$ is a more risky state than $A$ which is more risky than $R$. Each time a cue occurs, $x$ is consumed.
1. Given that we have either \( x = 1 \) or \( x = 0 \), and using (16) and (15), what behaviour is optimal for the individual, in the "cold mode"? Do states matter here? In other words, compare the utility from consuming \( c \) and \( x \) and from consuming only \( c \), and determine which is larger.

2. Derive the ratio of marginal utilities for this problem, using the first-order conditions.

3. What happens to this ratio when \( q \) increases? When \( \chi \) increases?

4. Draw the indifference curves for our problem. First for an addiction-resistant individual (high \( \chi \)) and then for an addiction-prone individual (low \( \chi \)). Show graphically what happens to an addiction-resistant individual (high \( \chi \)) when a cue occurs.
8 Time consistent and time inconsistent behaviour

8.1 Time consistency

To understand time inconsistency, we first look at what actually time consistency is.

8.1.1 Two-period utility maximization under certainty

- (Reminder – see ’Makroökonomik I’ for certainty, see section 4.1 for uncertainty)

- Setup

  - Agent maximizes
    \[ U_t = \ln c_t + \delta \ln c_{t+1} \]  \hspace{1cm} (17)

  - subject to period 1,
    \[ w_t = c_t + s_t, \]

  - and period 2 constraint,
    \[ (1 + r_{t+1}) s_t = c_{t+1}. \]

- Optimal behaviour reads
  \[ c_t = \frac{1}{1 + \delta} w_t, \quad s_t = \frac{\delta}{1 + \delta} w_t, \quad c_{t+1} = \frac{\delta}{1 + \delta} (1 + r_{t+1}) w_t. \]  \hspace{1cm} (18)
8.1.2 Three-period utility maximization under uncertainty

- Setup

  - We now start one period earlier, in period \( t - 1 \). It is called period 0 (such that \( t \) remains period 1 and \( t + 1 \) remains period 2)
  
  - Our individual maximizes

\[
U_{t-1} = \ln c_{t-1} + \delta \ln c_t + \delta^2 \ln c_{t+1}
\]

  - There are three constraints, one for period 0,

\[
w_{t-1} = c_{t-1} + s_{t-1}
\]

  one for period 1,

\[
(1 + r_t) s_{t-1} = c_t + s_t
\]  \hspace{1cm} (19)

  and one for period 2

\[
(1 + r_{t+1}) s_t = c_{t+1}
\]
• Timing of the models

<table>
<thead>
<tr>
<th>time index</th>
<th>$t-1$</th>
<th>$t$</th>
<th>$t+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>period</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

- two-period model  
  $1st$ period  
  $2nd$ period

- three-period model  
  $1st$ period  
  $2nd$ period  
  $3rd$ period

• Comparison to the two-period setup

  - Individual saves in $1st$ and in $2nd$ period
  - As before individual works only in $1st$ period
  - This assumptions is made for simplicity and does not affect the result
• Optimal behaviour

  – We can describe optimal behaviour by explicit expressions here as well
  – Labour income in period 0 is split between consumption and saving (see Exercise 8.3.1),

\[
\begin{align*}
    c_{t-1} &= (1 - B)w_{t-1}, \\
    s_{t-1} &= Bw_{t-1}
\end{align*}
\]

(20)

where the share $B$ is determined by the discount factor, $B \equiv \frac{\delta + \delta^2}{1 + \delta + \delta^2}$

  – What is saved in $t - 1$ and therefore available in $t$ is again split in $t$ between consumption and savings,

\[
\begin{align*}
    c_t &= (1 - A)(1 + r_t)s_{t-1}, \\
    s_t &= A[1 + r_t]s_{t-1}
\end{align*}
\]

(21)

where the share $A$ amounts to $A \equiv \frac{\delta}{1 + \delta}$

  – Finally, in $t + 1$, the savings of $s_t$ plus interest are consumed,

\[
    c_{t+1} = (1 + r_{t+1})s_t
\]

(22)
8.1.3 Where can we see that decision making is time consistent?

- General feature
  
  - Individuals behave the same, independently of \textit{when} the decision (i.e. whether it was made in $t$ or in $t - 1$)
  
  - \textit{The rule is always the same} and does not change when time goes by
• Where do we see this in this example?

  – Consumption in $t + 1$ from the perspective of $t$ (see (18) in 2pm)
    
    $c_{t+1} = \frac{\delta}{1 + \delta} (1 + r_{t+1}) w_t$

  – Consumption in $t + 1$ from the perspective of $t - 1$ (see (22), (21) and the definition of $A$ in 3pm)
    
    $c_{t+1} = (1 + r_{t+1}) s_t$
    
    $= (1 + r_{t+1}) \frac{\delta}{1 + \delta} [1 + r_t] s_{t-1} \quad (23)$

  – Wage in $t$ in two-period model (2pm) corresponds to $[1 + r_t] s_{t-1}$ in three-period model (3pm)

  – Individual in 2pm starts in $t$ with wage/ endowment $w_t$

  – Individual in 3pm continues in $t$ with endowment $e_t \equiv (1 + r_t) s_{t-1}$

  – We can write (19) as $e_t = c_t + s_t$ and therefore (23) as
    
    $c_{t+1} = \frac{\delta}{1 + \delta} (1 + r_{t+1}) e_t$

  – *The rule is always the same:* Consume in $t + 1$ a share $\delta/(1 + \delta)$ of endowment in $t$ ($e_t$ or $w_t$) plus interest paid in $t + 1$
8.2 Time inconsistency

- The plan for this section
  - We first look at properties of intertemporal utility functions that imply time consistency and time inconsistency
  - We get to know these specifications for discrete and continuous time setups
  - Then (in section 8.2.4) we look at an example for these general departures

- Teaching background
  - (lecture notes)
  - Caplin Leahy (2006) Solution method to be used is dynamic programming, not subgame perfection
8.2.1 Exponential/ geometric discounting

- Standard (Ramsey-Samuelson type) intertemporal utility function in continuous time
  - Intertemporal utility function $U(t)$
    
    $$U(t) = \int_{t}^{\infty} e^{-\rho[\tau-t]} u(c(\tau)) d\tau$$
  
  - Instantaneous utility function $u(c(\tau))$
  - “Standard” here refers to exponential discounting
  - Discount function is $e^{-\rho[\tau-t]}$ with a constant time-preference rate $\rho$
  - Important for comparison later
    * This time preference rate is the (negative) growth rate of the discount function
    * This growth rate is constant (the growth rate being another name for instantaneous discount factor)
• Standard intertemporal utility function in discrete time

  – Intertemporal utility function $U_t$

    $U_t = \sum_{\tau=t}^{T} \delta^{\tau-t} u(c_{\tau})$

  – Instantaneous utility function $u(c_{\tau})$
  – Geometric/ exponential discounting at discount factor $\delta < 1$
  – General version of special (3-period) case seen above in ch. 8.1.2

• Common features of continuous and discrete time formulations

  – Instantaneous discount factors are constant over time
  – Time consistent behaviour results
8.2.2 Non-exponential discounting

- Non-exponential discounting a la Strotz (1955) in continuous time
  - Intertemporal utility function \( U^{\text{Strotz}}(t) \)
    \[
    U^{\text{Strotz}}(t) = \int_t^\infty \lambda(\tau - t) u(c(\tau)) d\tau
    \]
  - Instantaneous utility function \( u(c(\tau)) \)
  - Discount function \( \lambda(\tau - t) \) where discounting depends on length of time \( \tau - t \)
  - Normalization with \( \lambda(0) = 1 \)
  - Discount function \( \lambda(\tau - t) \) can be exponential
    \[
    \lambda_{\text{example 1}}(\tau - t) = b^{-\rho[\tau-t]} \text{ where } b > 1 \text{ (not necessarily } e = 2.7182...\text{)}
    \]
    but also anything else like e.g. hyperbolic
    \[
    \lambda_{\text{example 2}}(\tau - t) = (1 + \alpha[\tau - t])^{-\gamma/\alpha} \text{ where } \alpha, \gamma > 0
    \]
    \[\text{(generalized hyperbolic discount function, Laibson, 1997)}\]
- Common feature of discount functions
  - Let us define the growth rate of the discount function a.k.a. the instantaneous discount factor as
    \[ g^\lambda \equiv \frac{d\lambda(\tau - t)/d(\tau - t)}{\lambda(\tau - t)} \]
  - For non-exponential discount functions like \( \lambda_{\text{example 2}} \), it falls over time
  - As an example, the instantaneous discount factor of (24) is \( \gamma / (1 + \alpha \tau) \) (see Laibson, 1997, or Exercise 8.3.2)
  - Time inconsistent behaviour results
8.2.3 Quasi-hyperbolic discounting

- Quasi-hyperbolic discounting à la Phelps and Pollak (1968) and Laibson (1997) in discrete time
  - Discount factors for $t, t+1, t+2, \ldots, t+n$ are given by $1, \beta \delta, \beta \delta^2 \ldots \beta \delta^n$
  - Exponential discounting is a special case with $\beta = 1$
  - “Quasi-hyperbolic” (i.e. it mimics (24)) discounting takes place for $\beta < 1$
    * Discounting is exponential for all points in time as of $t+1$ – but not for the time between $t$ and $t+1$
    * Instantaneous discount factor falls in time
quasi-hyperbolic \[ \left\{ \frac{1}{\beta \delta^{\tau - t}} \right\} \] for \( \tau = t \) \( \tau > t \)

exponential \( e^{-\rho (\tau - t)} \)

hyperbolic \( (1 + \alpha [\tau - t])^{-\gamma/\alpha} \)

**Figure 13** *Discount functions and functional forms (figure from Laibson, 1997)*
• Implications of non-exponential discounting
  
  – time inconsistent behaviour
  
  – decision made in $t$ implies a behaviour in $T$ that differs from behaviour in $T$ if decision was made at some point after $t$
  
  – behaviour depends on when an individual makes a decision
  
  – The rule is not always the same
8.2.4 O’Donoghue and Rabin (1999)

- Let us now look at an explicit example for time-inconsistent behaviour due to quasi-hyperbolic discounting

- We will find that people make plans and then do not stick to them – despite the absence of any new information

- The analysis is taken from O’Donoghue and Rabin (1999)
• Preferences

  – Time-consistent preferences (TC)

\[
U_{t}^{TC} = \sum_{\tau=t}^{T} \delta^{\tau-t} u_{\tau}
\]  \hspace{1cm} (25)

where \( t \) is today and \( T > t \) is the end of the planning horizon, \( 0 < \delta \leq 1 \) is the discount factor and \( u_{\tau} \) is instantaneous utility in \( \tau \)

  – We can write these preferences as

\[
U_{t}^{TC} = u_{t} + \sum_{\tau=t+1}^{T} \delta^{\tau-t} u_{\tau}
\]

and see the discounting to amount to \( 1 = \delta^{0}, \delta, \delta^{2} \) and so on

  – Intertemporal preferences that imply time-inconsistencies are “present-biased preferences” (O’Donoghue and Rabin, 1999)

\[
U_{t} = u_{t} + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u_{\tau}, \ \beta < 1
\]  \hspace{1cm} (26)

where the new crucial parameter is \( \beta < 1 \).

  – Discount factors here are \( 1, \beta \delta, \beta \delta^{2}, \beta \delta^{3} \ldots \)
• The decision problem
  
  – We assume that there is only one decision to be made
  – The decision implies an activity which can be performed at any point in time \( \tau \geq t \)
  – Costs of the activity in \( \tau \) are given by \( c_\tau \), benefits \( v \) always accrue at \( T \)
  – We set the usual discount factor equal to one, \( \delta = 1 \)
  – Intertemporal preferences therefore read
    \[
    U_t = u_t + \beta \sum_{\tau=t+1}^{T} u_\tau, \quad \beta < 1
    \]
  
  – Then, the gain (relative to the status quo) from performing this activity is given by (see Exercise 8.3.2)
    \[
    \left\{ \begin{array}{l}
    V_t(t) = \beta v - c_t \\
    V_t(\tau) = \beta v - \beta c_\tau
    \end{array} \right. \quad \text{if the activity is undertaken in } \left\{ \begin{array}{l}
    t \\
    \tau > t
    \end{array} \right. \quad (27)
    \]
• Sophistication of individuals

• We would like to understand optimal behaviour for three types of individuals
  
  – TCs: time consistent individuals with $\beta = 1$
  
  – naifs: time inconsistent with $\beta < 1$
  
  – sophisticated: time inconsistent with $\beta < 1$

• Definition of optimal behaviour as "perception-perfect strategy"
  
  – TCs: choose point in time for action such that present value gain $V_i$ is highest
  
  – naifs: do the same, _ignoring_ that they have a present bias also in the future
  
  – sophisticated: do the same thing, taking into account that they have a present bias also in the future

• Example for benefits and costs
  
  – Individual needs to write a (seminar/ research/ professional) paper
  
  – There are four evenings left
  
  – As an alternative, the individual can enjoy a movie
  
  – The reward is $\nu$, the costs are 1, 2, 3, and 11 on evenings 1 to 4 (e.g. because quality of movie rises)
• Optimal behaviour of TCs

  – By (27), the TCs compute

\[
\begin{align*}
V_1(1) &= v - c_1 \\
V_1(\tau) &= v - c_\tau
\end{align*}
\]

\Rightarrow V_1(1) > V_1(\tau) \iff c_\tau > c_1

  – TCs write paper in period 1
• Optimal behaviour of naifs

  – From (27), naifs compute

\[
\begin{align*}
V_1(1) &= \beta v - c_1 \\
V_1(\tau) &= \beta v - \beta c_\tau
\end{align*}
\]

\Rightarrow V_1(1) > V_1(\tau) \iff \beta c_\tau > c_1

  – If $\beta$ is only small enough, action is postponed to the future

    * Imagine $\beta = .1$
    * then $\beta c_2 = .2 < c_1 = 1$ and they therefore
    * postpone to evening 2

  – When do they plan to write the paper?

    * From the perspective of evening 1, they believe that the paper will be written on evening 2
    * From the perspective of evening 1, $c_3 > c_2$ and therefore
    * It is better to write on evening 2 rather than on evening 3

  – Let us now go “into the future”, i.e. we make this decision again in period 2

    * Naifs compare $\beta c_3 = .3$ to $c_2 = 2$
    * They again postpone (in contrast to what they believed in period 1) and so on
    * They therefore write the paper on the last evening with the highest costs
• Optimal behaviour of sophisticated
  
  – The sophisticated know that they have a present-bias in each period
  – They know that they will postpone (procrastinate) in each period
  – They know that not writing the paper in 1 means writing the paper in period 4
  – The value of doing it in period 4 is

  \[ V_1(4) = \beta v - \beta c_4 \]

  – Now comes the smart move (this is why they are called “sophisticated”): They can compare value in period 4 to the cost of writing the paper today. This implies

  \[ V_1(1) > V_1(4) \iff \beta v - c_1 > \beta v - \beta c_4 \iff \beta c_4 > c_1 \iff 1.1 > 1 \]

  – Hence, they write the paper immediately in period 1 (just as time-consistent individuals)
• Conclusion

  – Why do people make plans and do not stick to them?
  – They have a present-bias and they act naively
  – TCs make plans and each time they revisit the plan, they confirm it
  – Present-biased individuals that are sophisticated might postpone, but they stick to their plan
  – Naive individuals do not stick to their plans
8.3 Exercises

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8.3.1 Three-period savings problem

Consider the following three-period setup, where the individual maximises the following objective function, dependent on consumption in \( t - 1 \), \( t \) and \( t + 1 \):

\[
U (c_{t-1}, c_t, c_{t+1}) = \ln (c_{t-1}) + \delta \ln (c_t) + \delta^2 \ln (c_{t+1})
\]  

(28)

Subject to the following constraints:

\[
w_{t-1} = c_{t-1} + s_{t-1}
\]  

(29)

\[
(1 + r_t) s_{t-1} = c_t + s_t
\]  

(30)

\[
(1 + r_{t+1}) s_t = c_{t+1}
\]  

(31)

where \( w_{t-1} \) is labour income in \( t - 1 \), \( s_{t-1} \) are savings in \( t - 1 \), \( s_t \) are savings in \( t \), and the interest rate is \( r_t \) in \( t \) and \( r_{t+1} \) in \( t + 1 \).
1. Find optimal savings $s_{t-1}$ and $s_t$ as functions of the wage, $w_{t-1}$.

2. Determine optimal consumption $c_{t-1}$, $c_t$ and $c_{t+1}$.

8.3.2 Time (in)consistency (Laibson, 1997; O’Donoghue and Rabin, 1999)

Consider the three main types of discount functions listed below:

- exponential: $e^{-\rho(\tau-t)}$
- hyperbolic: $(1 + \alpha [\tau - t])^{-\frac{2}{\alpha}}$
- quasi-hyperbolic: $\beta \delta^{\tau-t}$

where $\rho, \alpha, \gamma > 0$, and $\beta, \delta \in (0, 1)$, $\tau \in [t, \infty)$ and $e$ is the Euler number.

Also note that a discount function is considered consistent if its growth rate does not depend on the time distance being evaluated.

1. Compute the growth rate of each discount function with respect to the time distance $\tau - t$.

2. Which discount functions are consistent/inconsistent over time?


9 Conclusion

- Behaviour economics studies
  - bounded rationality
  - bounded willpower
  - bounded self-interest

- Bounded rationality
  - see Bachelor seminar at chair
  - or wait for Master studies

- Bounded willpower
  - We studied time-inconsistent behaviour (O’Donoghue and Rabin, 1999)
  - General principles of utility functions (Strotz, 1955, Laibson, 1997)
  - Automatic behaviour (Bernheim and Rangel, 2004)
• Bounded self-interest
  
  – see Sobel (2005) or
  
  – use a utility function for an individual $A$ that reads $u(c^A, c^B)$ and that takes utility of individual $B$ into account

• These departures from neoclassical economics allow us to
  
  – better understand human behaviour per se
  
  – better understand human behaviour in economic situations
  
  – better develop models applied to understand economic issues ...
  
  – ... such as macro – to which we now turn