Johannes-Gutenberg University Mainz
Bachelor of Science in Wirtschaftswissenschaften

# Macroeconomics II: Behavioural Macro

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www.macro.economics.uni-mainz.de
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# Part III
# How behavioural macro could look like

## 10 The plan

- We take three typical macroeconomic fields
  - business cycle analysis
  - unemployment and
  - growth
- We get to know standard models that allow us to understand why there are
  - business cycles
  - unemployment and
  - growth

- We then replace our well-known but far-off-track homo oeconomicus by more emotional counterparts
- We see how predictions in emotional (or behavioural) macro models differ from standard predictions: what can we now understand that we did not understand earlier?
- Is this prediction in any sense meaningful i.e. can we empirically distinguish between the extended version and the original one? (though one)

# 11 Unemployment and time inconsistency

## 11.1 Models of unemployment

Macro I told us that we can distinguish between

- Models of labour supply ("voluntary unemployment")
- Traditional views of unemployment based on static models
- Modern models of unemployment looking at the dynamics of a labour market (search and matching models)

### 11.1.1 A reminder of voluntary unemployment

... understood as a labour supply decision

- The setup
  - Consider an individual that values consumption $c$ and leisure $l$ and is described by

$$u(c,l) = \left[\gamma c^{\theta} + (1-\gamma) l^{\theta}\right]^{1/\theta}, \quad \theta < 1, 0 < \gamma < 1$$

  - Real budget constraint (wage expressed in units of consumption good)

$$c = \left(\bar{l} - l\right) w$$

    where $\bar{l}$ is time endowment of the individual and $w$ is the real wage

- Optimal leisure/ labour supply decision
  - The amount of leisure

$$l = \frac{1}{1 + \left(\frac{\gamma}{1-\gamma}\right)^{\frac{\theta}{1-\theta}} w^{\frac{\theta}{1-\theta}}} \bar{l}$$

– Does leisure increase in labour income $w$?

$$\frac{dl}{dw} \lesseqgtr 0 \Leftrightarrow \theta \gtreqless 0$$

– Leisure increases if the income effect dominates the substitution effect

### 11.1.2 A reminder of involuntary unemployment

- Real wage lies above the market clearing wage
- workers are off their labour supply curve
- more workers would want to work at the going wage $\bar{w}$ which exceeds the market clearing wage
- Examples for involuntary unemployment: why is the real wage too high?
  - minimum wage (but keep the monopsony setup in mind)
  - wage bargaining between trade unions and employers' federations
  - efficiency wages set by firms (Solow, 1979): firms pay a wage that is higher than the market clearing wage as this allows firms
    - to have a larger pool of applicants and
    - to motivate workers to provided more effort (identify more with the firm, be more careful ...)

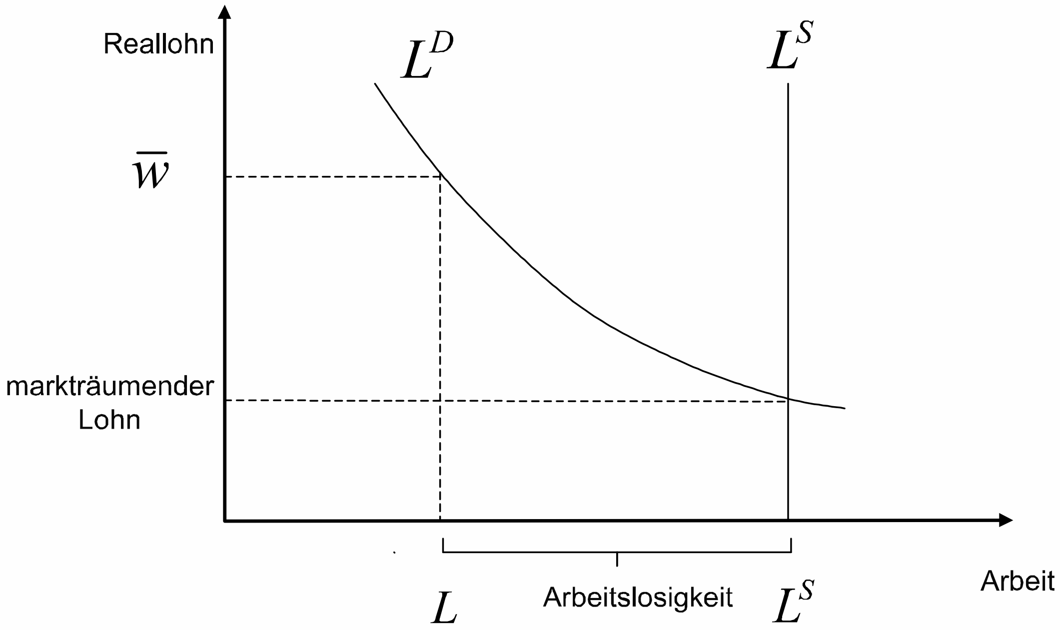


**Figure 14** *Real wage rigidity and unemployment*

### 11.1.3 A reminder of matching models of unemployment

- Pissarides (2000) Equilibrium unemployment theory
- The central assumptions and insights
  - Finding a job and finding a worker takes time due to incomplete information
  - Search processes play an important role
  - Adjustment of the unemployment rate takes time
  - One can compute how much time this adjustment process takes
  - Vacancies (job opening by firm) play an important role

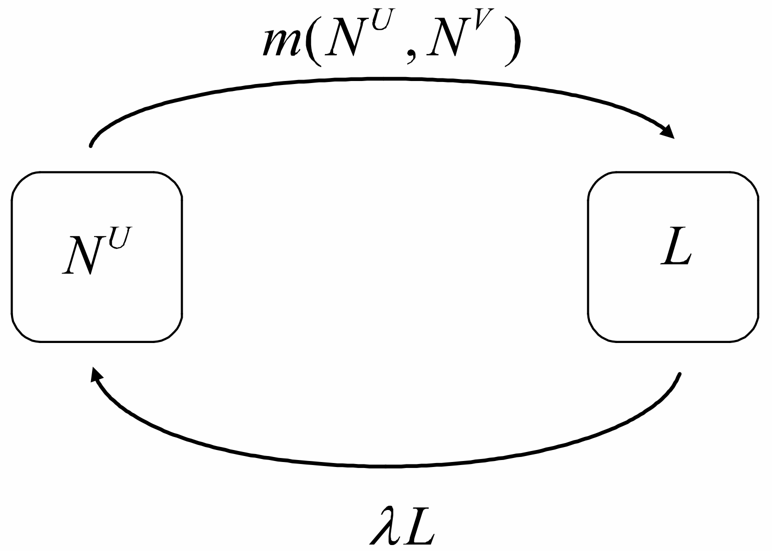


**Figure 15** *Inflows $\lambda L$ into the pool of $N^U$ unemployed workers and outflows which are determined by the matching function $m\left(N^U, N^V\right)$ – notation: separation rate $\lambda$, number $L$ of employed workers and number $N^V$ of vacancies*

- Dynamics of the unemployment rate
  - We fix the number of vacancies per unemployed worker for simplicity (see Masters programme for details)
  - Denote the
    - individual job finding rate by $\mu$
    - the initial unemployment rate at some $t = 0$ by $u_0$
  - The unemployment rate is then given by

$$u(t) = \frac{\lambda}{\lambda + \mu} + \left( u_0 - \frac{\lambda}{\lambda + \mu} \right) e^{-(\lambda+\mu)t}$$

- In words
  - The unemployment rate at $t = 0$ is given by $u_0$
  - The unemployment rate for $t \to \infty$ is given by $\frac{\lambda}{\lambda+\mu}$
  - We can therefore define $u^* \equiv \frac{\lambda}{\lambda+\mu}$ as the long-run unemployment rate
  - When $u_0 \left\{ \begin{matrix} > \\ < \end{matrix} \right\} u^*$, the unemployment rate $u(t) \left\{ \begin{matrix} \text{falls} \\ \text{rises} \end{matrix} \right\}$ over time $t$

## 11.2 A pure search model of unemployment

- Understanding the dynamics of unemployment should proceed in steps
- The starting point is the analysis of *one* unemployed worker
- We ask
  - how this unemployed worker can behave optimally and
  - how this behaviour affects his or her expected duration in unemployment
- What do we learn from a methodological perspective?
  - Beyond the analysis of unemployment, we get to know 'Bellman equations'
  - They are a (the?) standard tool in economics to solve maximization problems
- Reading
  - Cahuc and Zylberberg (2004, ch. 3) for the economics
  - Wälde (2012) for the methods
- Once this is understood, one would proceed to an equilibrium analysis of unemployment (that would explain the number $N^V$ of vacancies which were assumed to be constant)

### 11.2.1 The basic idea

- Reason for search: lack of information about job availability and the wage paid per job
- Setup
  - look at *one* unemployed worker
  - S/he receives unemployment benefits
  - Intensity of search is *not* chosen
  - Can *not* look for another job once employed
  - Stationary environment
- Question we can ask: which wage is accepted once an offer is made?

### 11.2.2 Expected utility once employed

- Unemployed does
  - not know which wage will be offered once a job is found
  - know that wages are drawn from a (continuous cumulative) distribution $H(w)$ with density $h(w)$
  - see next figure ...

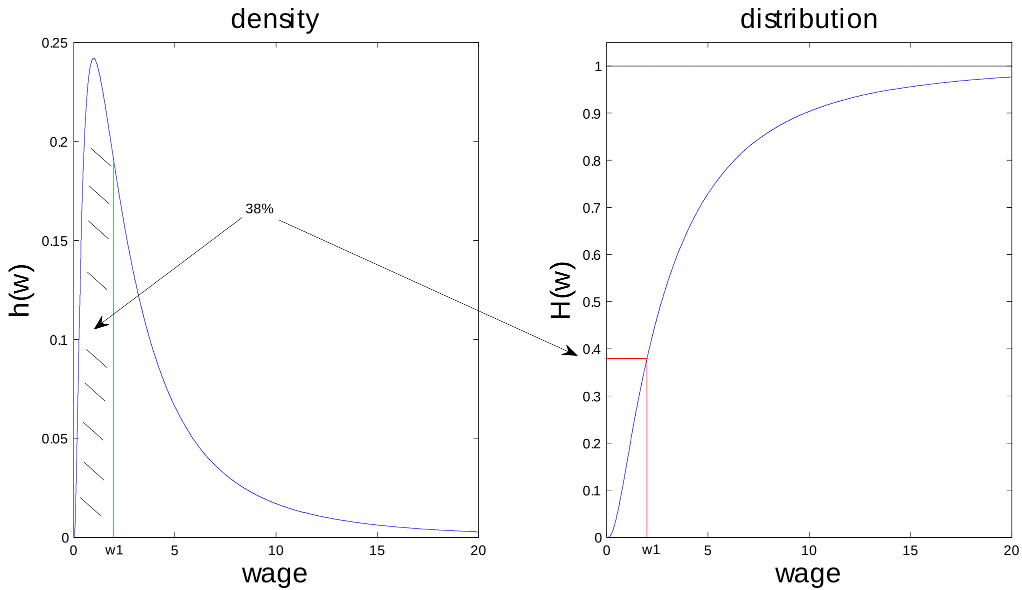


**Figure 16** *Illustration of wage offer density and distribution from which workers draw the wage*

- Unemployed does
  - not know which wage will be offered once a job is found
  - know that wages are drawn from a (continuous cumulative) distribution $H(w)$ with density $h(w)$
- Worker is risk neutral
  - utility function is linear in income
  - here: utility function is given by real labour market income (wage or benefit)
- When employed the worker loses the job
  - at (separation) *rate* $s > 0$, meaning that
  - the *probability* to lose the job over period of time of length $dt$ is given by $sdt$
  - (Poisson process in continuous time)
- Real instantaneous interest rate $r$: invest a Euro in $t$ and receive $1 + rdt$ Euro in $t + dt$
- Discount factor of $\frac{1}{1+rdt}$ useful for computing present values

- This gives us value of being employed between $t$ and $t + dt$

$$V_e = \frac{1}{1 + rdt} \left[ wdt + (1 - sdt) V_e + sdtV_u \right]$$

where $w$ is the instantaneous wage rate and $V_u$ is the value of being unemployed and $(1 - sdt)$ is the probability to keep the current job

- (technically: this is heading towards a Bellman equation)

- rearrange this to make it simpler $\rightarrow$ Exercise 11.5.1

$$rV_e = w + s \left[ V_u - V_e \right]$$

(this is presentation in terms of classic Bellman equation)

- rewrite this for later purposes as

$$V_e (w) - V_u = \frac{w - rV_u}{r + s} \tag{32}$$

### 11.2.3 The optimal search strategy

- we assume job searcher only meets one employer at a time
- an offer consists of a fixed wage $w$
- choice between 'accept' or 'reject'
- optimality criterion: is $V_e$ or $V_u$ higher?
- accept $\Leftrightarrow V_e(w) > V_u$, which from (32) is the case if and only if $w > rV_u \equiv x$
- we have thereby defined the reservation wage $x$
- intuition why ever reject
  - Disadvantage from accepting a job consists in the inability to further look for jobs (as there is *no* on-the-job search)
  - Employee is stuck with wage $w$ for a potentially long time
  - It might be better to reject and hope for better offer (with higher wage $w$)

### 11.2.4 The discounted expected utility (value function) of a job seeker

- Arrival rate of job: $\lambda$
- $\lambda$ reflects labour market conditions, personal characteristics (age, educational background), effort (time and carefulness put into writing applications, not modeled here)
- Unemployment benefits $b$ and opportunity costs of search $c$ give instantaneous utility when unemployed, $z \equiv b - c$
- Value of receiving an offer

$$V_\lambda = \int_0^x V_u h(w)\, dw + \int_x^\infty V_e(w)\, h(w)\, dw$$

- Value of being unemployed over a period of length $dt$

$$V_u = \frac{1}{1 + rdt} \left( zdt + \lambda dt V_\lambda + (1 - \lambda dt) V_u \right)$$

- Rearranging (see Exercise 11.5.1), we get the Bellman equation for unemployed worker

$$rV_u = z + \lambda \int_x^\infty \left[ V_e(w) - V_u \right] h(w)\, dw$$

### 11.2.5 Reservation wage

- Last equation has intuitive interpretation, but hard to use for comparative statics
- But note that it is also an expression for the reservation wage $x = rV_u$
- After further steps (see Exercise 11.5.1), we get final expression for the reservation wage $x$

$$x = z + \frac{\lambda}{r+s} \int_x^\infty (w-x)\, h(w)\, dw$$

- Interpretation as above for $rV_u$, apart from $r+s$ in denominator
  - $\frac{\pi}{r}$ is the present value (when discounting with $r$) of receiving income (profits) $\pi$ forever
  - $\frac{\pi}{r+s}$ is the present value of receiving $\pi$ as long as it randomly stops at rate $s$
  - hence $\frac{\int_x^\infty (w-x)h(w)dw}{r+s}$ is the present value of receiving a wage above $x$ until exit rate $s$ hits
  - $z$ is received instantaneously as a flow and $\lambda$ is the arrival of a job offer

### 11.2.6 Hazard rates and average duration in unemployment

- What is hazard rate (exit rate with which an individual leaves unemployment)?

$$\text{exit rate} = \lambda \left[1 - H(x)\right]$$

where $\lambda$ is the job offer rate and $1 - H(x)$ is the probability of accepting a job

- What is the average duration $T_u$ in unemployment?

$$T_u = \frac{1}{\lambda \left[1 - H(x)\right]}$$

(using a standard property of Poisson processes, duration is exponentially distributed)

- This forms basis of some simple policy analyses
  - What are determinants of duration in unemployment (unemployment rate)?
  - How does a change in reservation wage $x$ affect duration in unemployment?
  - How can reservation wage $x$ be influenced?

## 11.3 Search unemployment and time inconsistency

- Remember time inconsistent behaviour as in O'Donoghue and Rabin (1999) above (ch. 8.2.4)
- Individuals can be time consistent, naif and sophisticated
- This framework was applied to pure search by Paserman (2008) building on DellaVigna and Paserman (2005)
- We first look at the setup with time-consistent (TC) individuals ($\beta = 1$)
- Then we allow for time inconsistency ($\beta < 1$)

### 11.3.1 Pure search in discrete time with time consistent behaviour

Describing the individual

- We extend objective function (25), which reads $U_t^{TC} = \Sigma_{\tau=t}^{T} \delta^{\tau-t} u_\tau$, by
  - letting the planning horizon start at 0
  - specifying utility from consumption explicitly
  - adding search effort $e_t$ and cost $c(e_t)$ from search and
  - taking uncertainty into account

$$U_0^{TC} = E_0 \Sigma_{t=0}^{\infty} \delta^t \left[ u(c_t) - c(e_t) \right] \tag{33}$$

- As before, discounting takes place at the discount factor $0 < \delta < 1$
- Uncertainty arises as consumption depends on
  - the employment status of the worker and
  - on the uncertain wage level as workers draw from a wage distribution
- We capture the effect of uncertainty by using an expectations operator $E_0$ (compare the two-period setup in (17))

- Consumption is given by
  - the (uncertain) wage $w$ when employed
  - unemployment benefits $b$ when unemployed
- The probability of being employed in $t+1$ depends on search effort in $t$

$$\text{Prob}\left(c_{t+1} = w\right) = p\left(e_t\right)$$

  with effort increasing the employment probability, $p'\left(e_t\right) > 0$
- With a probability $q$, an employed worker loses a job

Optimal behaviour

- Consider an unemployed worker in 0
- The objective function (33) can be written as

$$U_0^{TC} = u(b) - c(e_0) + \delta\left[E_0 u(c_1) - E_0 c(e_1)\right] + E_0 \Sigma_{t=2}^{\infty} \delta^{\tau - t}\left[u(c_t) - c(e_t)\right] \quad (34)$$

  where

$$E_0 u(c_1) = p(e_0) E_w u(w) + (1 - p(e_0)) u(b)$$

- Expected utility
  - depends on the probability of being employed
  - on utility $u(b)$ when unemployed and
  - on expected utility $E_w u(w)$ when having a job
- Now let the worker choose effort $e_0$ and the reservation wage $R$ optimally
- How does the optimality condition look like?

- How does the optimality condition look like?

$$\begin{aligned} c'(e_0) &= \delta\left[p'(e_0)\,E_w u(w) - p'(e_0)\,u(b)\right] \\ &= \delta p'(e_0)\left[E_w u(w) - u(b)\right] \end{aligned} \tag{35}$$

- It tells us that (as always) marginal costs must equal marginal benefits
  - Marginal costs are given by instantaneous marginal costs $c'(e_0)$ from effort
  - Benefits occur (i) in the future (next period discounted by $\delta$) with (ii) only a certain probability
  - Marginal benefits are the increase in the gain $E_w u(w) - u(b)$ from getting a job
- We see from this equation that optimal effort is independent of time
  - the individual lives in a stationary environment
  - the general condition for any point $t$ in time reads

$$c'(e_t) = \delta p'(e_t)\left[E_w u(w) - u(b)\right] \tag{36}$$

- See Exercise 11.5.2 for an example where one can explicitly compute $e_0$ (and thereby $e_t$)
- (We do not look at the reservation wage $R$ here explicitly)

### 11.3.2 Pure search and time inconsistency

- Instead of (34), the objective function now contains the present-bias parameter $\beta$ from (26) and reads

$$U_0^{TC} = u(b) - c(e_0) + \beta \left\{ \delta \left[ E_0 u(c_1) - E_0 c(e_1) \right] + E_0 \Sigma_{t=2}^{\infty} \delta^{\tau - t} \left[ u(c_t) - c(e_t) \right] \right\}$$

  where $0 < \delta, \beta < 1$ and $e_t$ again is effort put into finding a job

- The first-order condition for $e_0$ now reads

$$c'(e_0) = \delta \beta p'(e_0) \left[ E_w u(w) - u(b) \right]$$

  and displays the $\beta$

- The first-order condition from the perspective of zero for $t = 1$ or higher has the *same* structure as the time-consistent case (36)

$$c'(e_t) = \delta p'(e_t) \left[ E_w u(w) - u(b) \right] \text{ for } t \geq 1$$

  as there is no present-bias for $t \geq 1$ from the perspective of 0

- This is the same basis of time inconsistent behaviour as in Strotz/ Laibson/ O'Donoghue and Rabin and others

- Individual is assumed to be sophisticated

- Question of Paserman (2008)
  - How large are $\delta$ and $\beta$?
  - Is there really time-inconsistency in real-world data?
  - Is time-inconsistency important (is $\beta$ much smaller than 1)?
- Estimates (see table 2 in Paserman, 2008)
  - $\delta$ (discount factor) is around 0.999 (per week)
  - $\beta$ (measure of time inconsistency) is between .4 and .89
- What does this tell us?
  - Discounting by $\delta$ hardly plays a role
  - Estimates of present bias $\beta$ are significantly below 1 in an economically large sense
  - Present bias is an important feature that should be taken into account in analysis of (economic) behaviour

## 11.4 Conclusion

- What have we learned about unemployment?
  - We looked at the pure-search model
  - There is no instantaneous labour market clearing as it takes time to find a job
  - Finding a job is split into
    - receiving a job offer
    - accepting the job offer
  - Unemployment arises due to necessity to search (involuntary unemployment) and because of rejection of wage offers (voluntary unemployment)
  - Unemployment can be reduced via all channels that reduce the reservation wage
  - Model is "very one-sided" as demand side by firms is not modelled
  - Policy conclusions are (at least) incomplete

- How important is time inconsistent behaviour for unemployed workers?
  - We looked at pure-search model extended for present-bias
  - We found the usual tension in first-order conditions where discounting between one period and the subsequent on depends on when the decision is made
  - Empirically, Paserman shows that measure $\beta$ of present bias can be considerably below 1
  - He finds estimates between .4 and .89
  - Present bias is an important feature of search effort of the unemployed
- Are there policy implications?
  - Policy tools for time-inconsistent behaviour can be applied here as well
  - Any commitment device is desirable
  - Taxation in the form of "sin taxes" might be advisable for public employment agencies as well

## 11.5 Exercises



### 11.5.1 Pure search model of unemployment

Consider the following value function for an employed worker:

$$V_e = \frac{1}{1+rdt}\left[wdt + (1-sdt)\,V_e + sdtV_u\right]$$

Where $\frac{1}{1+rdt}$ is a discount factor, computing the present value of being employed, $1+rdt$ is the value at $t+dt$ in Euros of investing 1 Euro in $t$ and earning interest $r$. And $1-sdt$ is the probability of keeping a job between $t$ and $t+dt$.

Also, consider the following value function for an unemployed worker:

$$V_u = \frac{1}{1+rdt}\left[zdt + \lambda dtV_\lambda + (1-\lambda dt)\,V_u\right],$$

with

$$V_\lambda = \int_0^x V_u h\,(w)\,dw + \int_x^\infty V_e\,(w)\,h\,(w)\,dw.$$

Where $1 - \lambda dt$ is the probability of staying unemployed, and $\lambda dt$ is the probability of receiving an offer. And $z = b - c$, is instantaneous utility when unemployed, which is equal to the difference between unemployment benefits, $b$, and the opportunity cost of searching for a job, $c$.

1. Derive the employed worker's discounted expected utility, $V_e$, as a function of the wage rate, $w$, the job destruction rate, $s$, the interest rate, $r$, and the unemployed worker's discounted lifetime utility, $V_u$, such that time increments no longer appear.

2. Derive the unemployed worker's discounted expected utility, $V_u$, as a function of the net income, $z$, the job-offer arrival rate, $\lambda$, the interest rate, $r$, and the employed worker's discounted lifetime utility, $V_e$, such that time increments no longer appear.

3. Given your answer above, derive the expression for the reservation wage, $x \equiv rV_u$, that solely depends on the model's parameters.

4. What are the effects of the model's parameters on the reservation wage $x$? Take the partial derivatives, using the implicit function theorem.

### 11.5.2 Pure search in discrete time and time consistent behaviour

Consider the discrete-time model of unemployment with search effort. The lifetime utility of the individual is given by

$$U = E_0 \left[ \sum_{t=0}^{\infty} \delta^t \left[ u\left(c_t\right) - c\left(e_t\right) \right] \right],$$

where $0 < \delta < 1$ is the discount factor, $e_t$ is effort at time $t$, and $c\left(e_t\right)$ is the cost of searching for a job. Consumption is given by

$$c_t = \left\{ \begin{array}{c} w \\ b \end{array} \right\} \text{ if the individual is } \left\{ \begin{array}{c} \textit{employed} \\ \textit{unemployed} \end{array} \right. .$$

Unemployed individuals can increase their probability of becoming employed in $t+1$ by increasing their search effort in $t$, that is we have

$$\text{Prob}\left(c_{t+1} = w\right) = p\left(e_t\right),$$

with $p'\left(e_t\right) > 0$, and $0 \leq p\left(e_t\right) \leq 1$.

1. Determine the optimal effort in period 0, i.e. $e_0$, for an unemployed worker. Use the

setup above and the following functional forms

$$
\begin{aligned}
u(c_t) &= \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \\
c(e_t) &= \exp(e_t), \\
p(e_t) &= 1 - \exp(-e_t), \\
E_w u(w) &= u(w),
\end{aligned}
$$

where $\sigma \geq 0$ is the inverse of the intertemporal elasticity of substitution.

2. What happens to optimal effort as $\sigma$ increases? Give an interpretation.

# 12 Growth, cues and automatic behaviour

- One of the most discussed property of any real world economy is its growth rate
  - Usually laymen and policy makers want higher growth rates
  - But then think about global warming – there are arguments that growth rates are too high
- Independently of political or social objective function, we want to understand
  - what drives short-run and especially long-run growth
  - whether drivers of growth can be influenced by policy
- This chapter looks at
  - models of growth and at
  - extensions of those models that allow for behavioural features of decision maker
  - The growth part covers exogenous and endogenous growth models
  - The behavioural growth part allows for cues and automatic behaviour (in the sense of Laibson, 2001)

## 12.1 Models of growth

### 12.1.1 The Solow model with technological progress and population growth

- Technologies and saving behaviour
  - (see e.g. Aghion and Howitt, 1998, ch 1.1)
  - Production technology
$$Y(t) = K(t)^{\alpha} [A(t) L(t)]^{1-\alpha} \tag{37}$$
  - Capital accumulation
$$\dot{K} = sK(t)^{\alpha} [A(t) L(t)]^{1-\alpha} - \delta K(t),$$
    population growth and factor productivity $A$ growth
$$L(t) = L_0 e^{nt}, \quad A(t) = A_0 e^{gt}.$$

- The dynamics of the economy
  - Define an auxiliary variable $\tilde{k}$ to simplify the analysis as

$$\tilde{k}(t) = \frac{K(t)}{A(t)L(t)} \tag{38}$$

  - The growth rate of $\tilde{k}(t)$ is (see Exercise 12.3.1)

$$\frac{d\tilde{k}(t)/dt}{\tilde{k}} = s\tilde{k}(t)^{-(1-\alpha)} - \delta - g - n. \tag{39}$$

  - Its dynamic properties can easily be understood via a graphic analysis

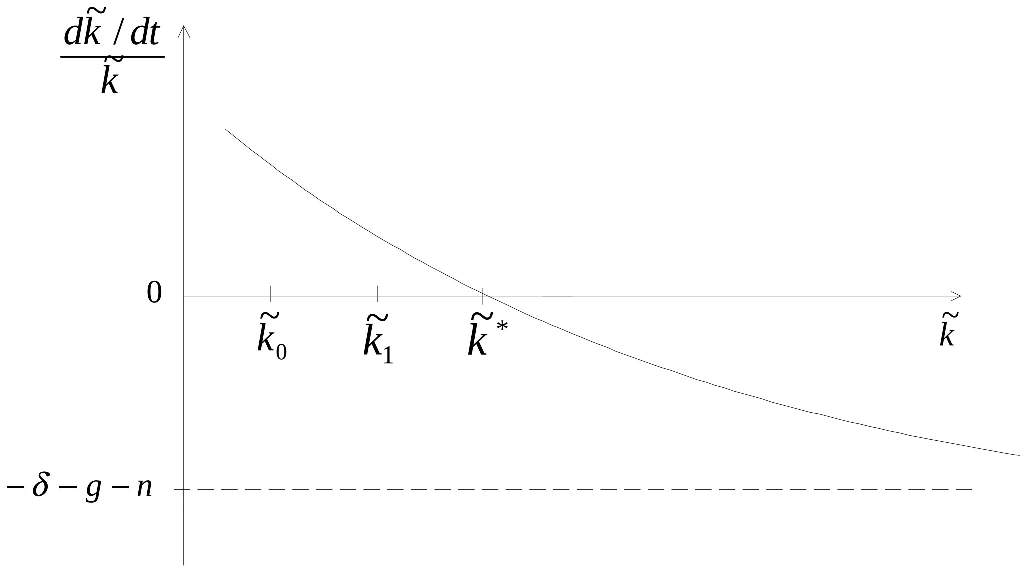


**Figure 17** *A phase diagram analysis for the Solow growth model with technological growth, population growth and exogenous saving rate*

- Growth rates in the long-run equilibrium
  - Steady state value of capital per effective labor $\tilde{k}$ is constant (see Exercise 12.3.1)

$$\tilde{k}^* = \frac{K(t)}{A(t) L(t)} = \left( \frac{s}{\delta + g + n} \right)^{1/(1-\alpha)}. \tag{40}$$

  - Computing the time derivative yields the long run growth rate (note that right hand side of (40) is constant)

$$\frac{\dot{K}(t)}{K(t)} = \frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)} = g + n.$$

- Growth rate of GDP is given by

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha \frac{\dot{K}(t)}{K(t)} + (1 - \alpha) \left[ \frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)} \right] = \frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)} = g + n$$

- In words
  - growth rate of GDP is driven only by growth rate of TFP and population growth
  - capital growth per se does not play a role in the long-run
  - capital is not a "driver" of growth, drivers are TFP and population growth

- Growth rate of GDP per capita $y(t) \equiv Y(t)/L(t)$

$$\frac{\dot{y}(t)}{y(t)} = \frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{L}(t)}{L(t)} = g + n - n = g$$

- In words,
  - inhabitants of a country become richer only by an increase in total factor productivity $A(t)$
  - population growth can "kill" GDP growth, i.e. measures of GDP growth are uninformative about whether a nation is better off over time

- To see the huge importance of the population growth rate $n$, consider the following figures

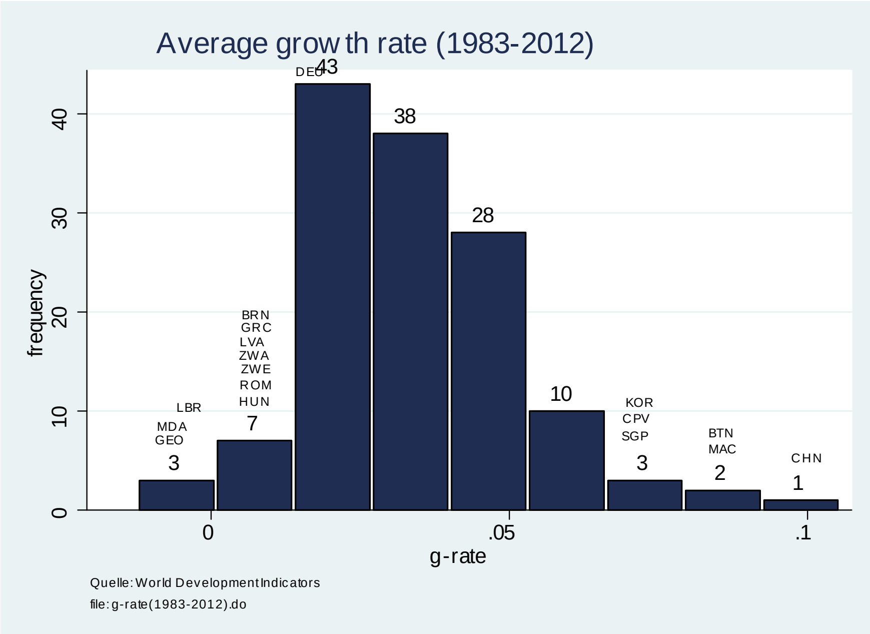


**Figure 18** *Frequencies of growth rates of GDP (averages from 1983 to 2012)*

- For country codes, see wits.worldbank.org/wits/WITS/WITSHELP/Content/Codes/Country_Codes.htm

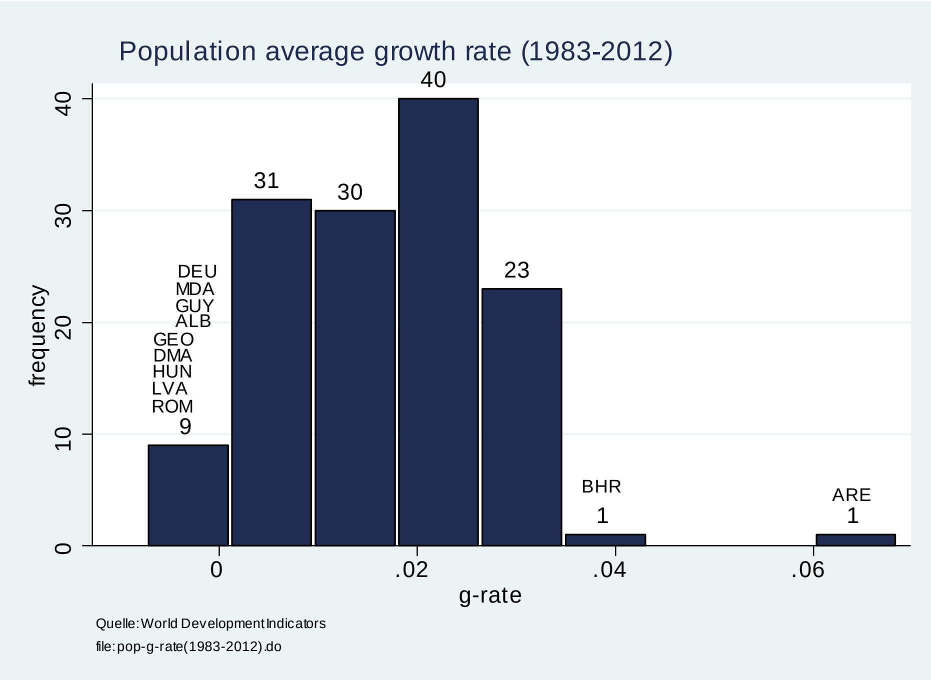


**Figure 19** *Frequencies of growth rates of population (averages from 1983 to 2012)*

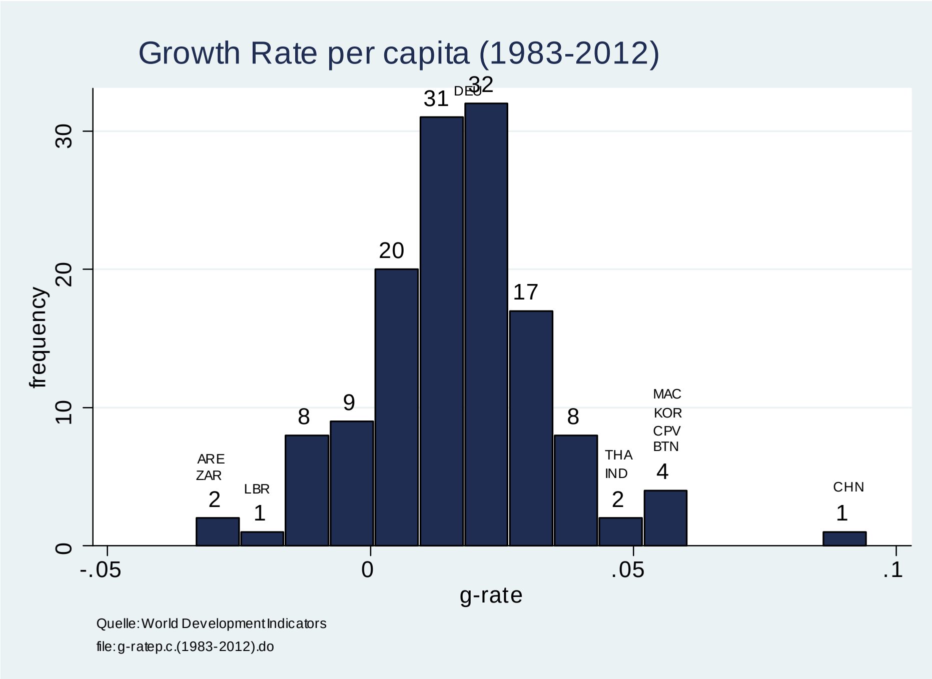


**Figure 20** *Frequencies of growth rates of GDP per capita (averages from 1983 to 2012)*

- What these figures tell us
  - "Only" three countries have negative average growth rates of GDP: Liberia (LBR), Moldava (MDA) and Georgia (GEO)
  - By contrast, 20 countries have negative average growth rates of GDP per capita
  - Stresses again that only per capita values are informative when describing a country
- Policy implications of Solow growth model
  - Growth of total factor productivity seems the only truly convincing policy option
  - Unfortunately, TFP growth is exogenous and the Solow model does not allow us to understand how TFP growth can be influenced
  - Only saving rate $s$ can be influenced by policy – allowing to study only short-run effects
  - We need models with endogenous long-run growth rates

### 12.1.2 A simple model of endogenous growth: The AK model

- Background on the "new" endogenous growth theory
  - Some general discontent with the prediction of the neoclassical growth model
    - Why do countries grow with very unequal growth rates over long periods of time?
    - Why do not all countries catch up?
    - Why is the long-run growth rate unaffected by any economic incentive?
  - As a response, a series of theoretical papers were written that developed new growth models providing an endogenous explanation of long-run growth rates

- Various channels have been identified in the literature
  - Constant returns to scale for all factors of production that can be accumulated (Romer, 1986, Lucas, 1988, Rebelo, 1991)
  - Mechanisms include positive externalities from capital accumulation (Romer, 1986) or accumulation of both physical capital and human capital (Lucas, 1988, Rebelo, 1991)
  - Endogenous technological change (Romer, 1990, Grossman and Helpman, 1991, Aghion and Howitt, 1992)
  - This is achieved by knowledge spillovers from R&D (also a positive externality)
- Second wave of new new growth models followed afterwards ...

- The AK model (Rebelo, 1991)
  - We look here at the simplest model of endogenous growth
  - It allows us to understand the central insight of this literature very quickly
  - We extend the model to allow for a tax rate and government expenditure
  - This illustrates the central insights even better :-)
- Preferences
  - Consider an individual (or a central planner) having the following intertemporal objective function

$$U(t) = \int_t^\infty e^{-\rho[\tau - t]} u(C(\tau))\, d\tau \tag{41}$$

  - Overall utility is denoted by $U(t)$
  - The planning period starts in $t$, time is continuous (whence we have the integral) and the planning horizon goes to infinity
  - The time preference rate is $\rho$
  - Instantaneous utility is characterised by constant relative risk aversion (CRRA)

$$u(C(\tau)) = \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma \geq 0,\ \sigma \neq 1$$

where $C(\tau)$ is consumption at $\tau$

– Time $\tau$ runs from $t$ to infinity

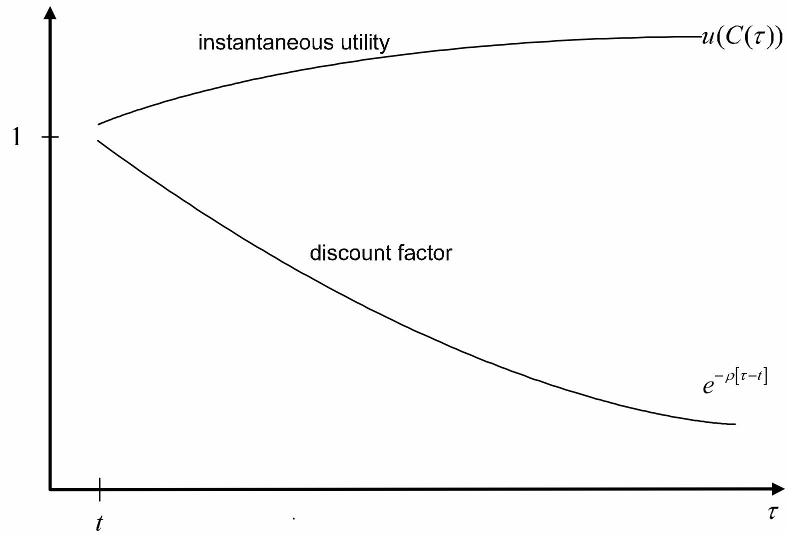


**Figure 21** *Illustration of the objective function (recall Macro I)*

- Technology
  - We follow the approach of constant returns to factors of production that can be accumulated
  - Why should this be the case?
  - Romer (1986): Total factor productivity changes as a function of the knowledge associated with more and more capital, $Y = A(K) K^{\alpha} L^{1-\alpha}$
  - When $A(K)$ happens to equal $K^{1-\alpha}$, we get $Y = KL^{1-\alpha}$
  - Lucas (1988): Accumulate both physical capital $K$ and human capital $H$, $Y = AK^{\alpha} H^{1-\alpha}$
  - Rebelo (1991): Accumulate $K$ and $H$ or simply assume

$$Y(t) = AK(t) \tag{42}$$

  - Why can we have long-run growth? Marginal productivity of capital does *not* fall when $K$ increases (as in the Solow growth model)

- Resource constraint
  - We study the economy as a whole
  - An alternative would consist in solving individual maximization problems (with budget constraints) and then aggregating individual behaviour
  - At the aggregate level, capital rises if output (net of taxes) minus depreciation exceeds consumption,

$$\dot{K}(t) = (1-\tau)\,Y(t) - \delta K(t) - C(t) \tag{43}$$

  - Tax income $\tau Y(t)$ is used for government purposes (not modelled)
- Solving the maximization problem
  - Maximize the social welfare function $U(t)$ given the technology $Y(t)$ and the constraint
  - Optimality condition is provided by the Keynes-Ramsey rule (see Exercise 12.3.2)

$$\frac{\dot{C}(t)}{C(t)} = \frac{(1-\tau)\,A - \rho}{\sigma} \tag{44}$$

  - If the return to capital is $A$ exceeds the time preference rate, there is consumption growth

- The growth rate of the economy – procedure
  - After having obtained our dynamic system consisting of
    * the resource constraint (43)
    * the technology (42) and
    * the Keynes-Ramsey rule (44)
  - we have obtained two differential equations for two variables, $K$ and $C$
  - We proceed similar to models without economic growth (cmp. Makroökonomik I): we look for
    * a long-run equilibrium and (maybe) afterwards for
    * behaviour outside of the long-run equilibrium (transitional dynamics)

- The growth rate of the economy – the question/ the guess
  - If the marginal productivity $A$ of capital is sufficiently large, the growth rate of consumption is positive,

$$\frac{\dot{C}(t)}{C(t)} = \frac{(1-\tau)A-\rho}{\sigma} \equiv g > 0$$

  - Using the resource constraint and the technology, we can express the growth rate of capital as

$$\frac{\dot{K}(t)}{K(t)} = (1-\tau)A - \delta - \frac{C(t)}{K(t)}$$

  - Question: is there a solution to these equations where $\frac{\dot{C}(t)}{C(t)} = \frac{\dot{K}(t)}{K(t)} = g$?

- The growth rate of the economy – verification
    - We now guess that there is a solution for which $\frac{\dot{C}(t)}{C(t)} = \frac{\dot{K}(t)}{K(t)} = g$. Then we would have found a "growth equilibrium" or a "balanced growth path"
    - We verify by writing

$$
\begin{aligned}
g &= (1-\tau) A - \delta - \frac{C(t)}{K(t)} \Leftrightarrow \\
\frac{C(t)}{K(t)} &= (1-\tau) A - \delta - g = (1-\tau) A - \delta - \frac{(1-\tau) A - \rho}{\sigma}
\end{aligned}
$$

    - In words, there is a balanced growth path, if the ratio of consumption to capital is given by

$$
\frac{C(t)}{K(t)} = (1-\tau) A - \delta - \frac{(1-\tau) A - \rho}{\sigma}
$$

    where the right-hand side needs to be positive (see Exercise 12.3.2 for a calibration example)
    - Then, both consumption and capital grow at the rate $g$

- The growth rate of the economy (cont'd)

  – Given that $K$ grows at the rate $g$, we can then compute the growth rate of output (42)

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\dot{K}}{K} = 0 + g = g$$

  – The growth rate of GDP per capita is (assuming constant population) then also given by $g$

- Is there transitional dynamics?

  – Not in this setup

  – $AK$ structure implies constant return $A$ for investments

- What are the determinants of the growth rate?
  - The long-run growth rate is not given by a constant parameter but it is a function of parameters of the model
  - If agents are more impatient ($\rho$ rises), the growth rate would fall (not so in Solow growth model)
  - If the government decreases the tax $\tau$, the growth rate would rise (as the return to capital would rise – not so in the Solow model)
  - When the intertemporal elasticity of substitution $1/\sigma$ rises, the growth rate rises
- This endogeneity is a huge progress compared to exogenous growth rates
  - One can understand why countries grow at different rates
  - One can understand how policy affects economic growth (tax policy, trade policy, labour market policy ...)
  - One can understand whether growth rates are too high (global warming) or too low (poverty)

### 12.1.3 Conclusion

Neoclassical growth theory

- The complete Solow growth model was introduced
- Compared to the version with capital accumulation only ("macro I version"), we allowed for
  - exogenous population growth
  - exogenous technological progress
- In an economy with population growth
  - there is long-run growth of GDP
  - there is *no* long-run growth of GDP per capita
- In an economy with technological progress
  - there is long-run growth of GDP
  - there is also long-run growth of GDP per capita

- Shortcomings of the model
  - One can not understand why countries empirically seem to grow at different long-run growth rates
  - It is also hard to understand why some countries do not seem to catch up at all
  - Any policy measure has no impact on the long-run growth rate

Endogenous growth literature

- A new wave of growth models emerged
  - as of the end of the 1980s
  - that stressed the importance of economic mechanisms that influence the growth rate of an economy
- Various mechanisms why there is endogenous long run growth
  - Positives spillovers from R&D
  - Joint accumulation of human capital and physical capital (technically: constant returns to scale in accumulable factors of production)

- Determinants of long-run growth
  - Size of the economy (shortcoming - see semi-endogenous and non-scale growth models)
  - Preferences of households
  - Technological parameters (e.g. productivities or elasticities of substitution)
  - Policy parameters (e.g. tax or subsidy rates)
- New insights from growth theory
  - Observations on growth rates can better be understood
  - Catching-up, falling-behind or constant (relative) distance can be understood
  - Much more flexibility for growth analyses than with "old" growth theory

## 12.2 Cues and automatic behaviour

- Do humans behave in such a forward looking behaviour as just seen in neoclassical and endogenous growth models?
- Are they not rather influenced by advertisements?
- Are preferences not biased by all types of cues?
- Given the strong role of preferences in predicting the long-run growth rate of an economy, should richer models of human behaviour be taken into account?

### 12.2.1 Reminding of cues

- Laibson's (2001) cue theory of consumption
  - Preferences
$$u\left(c_t^{\text{sweets}}, c_t^{\text{fruit}}\right) = \left(c_t^{\text{sweets}} - x_t\right)^{\alpha}\left(c_t^{\text{fruit}}\right)^{1-\alpha} \tag{45}$$
  - Budget constraint
$$p^s c_t^{\text{sweets}} + p^f c_t^{\text{fruit}} = E$$

• Optimal behaviour

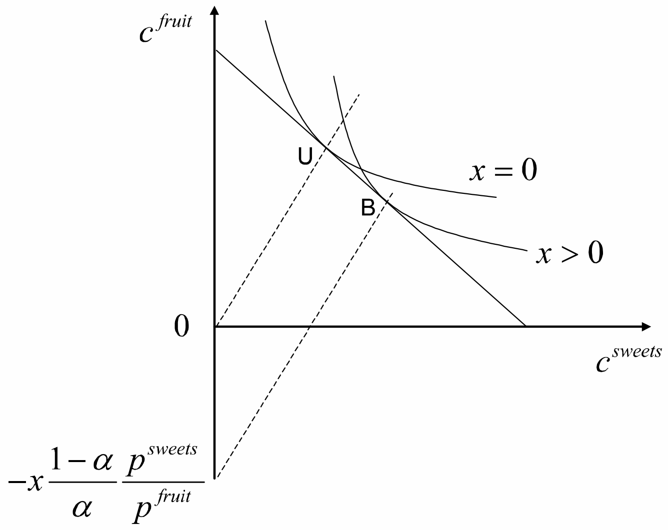


**Figure 22** *Optimal behaviour in the presence of cues*

### 12.2.2 Reminding of automatic behaviour

Bernheim-Rangel (2004) model of addiction

- Preferences

$$u = \chi c^{\gamma} + x + \alpha a \tag{46}$$

- Budget constraint

$$c + qx = y$$

- Optimal behaviour

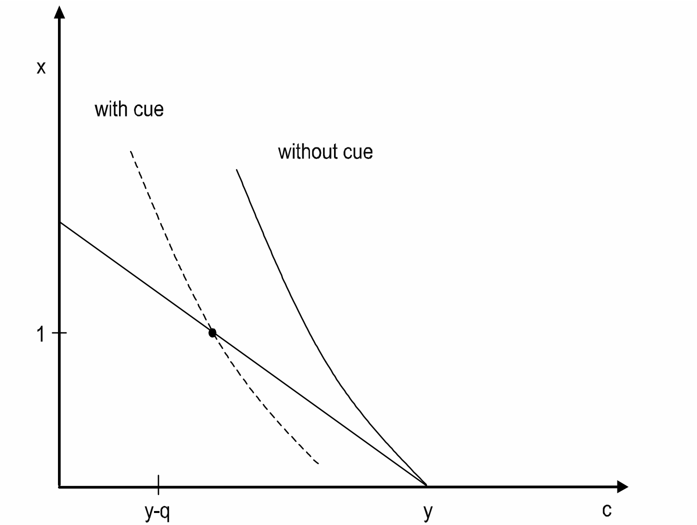


**Figure 23** *The "drug-resistant" individual and indifference curves with and without cue*

### 12.2.3 Linking the two

What would be an interesting question?

- What is the effect of advertising (in the sense of Becker and Murphy, 1993) or the effect of cues (in the sense of Laibson, 2001) on the saving behaviour of an individual?
- What is the effect of addictive goods on consumption and saving behaviour of an individual?
- How do these modified saving behaviours affect the long-run growth rate of an economy?

How could this be modeled?

- One would embed the static utility functions (45) and (46) in an intertemporal utility function like (41)
- One would embed the static budget constraints into dynamic budget or resource constraints like (43)
- One would compute optimality conditions
- One would try to understand them and derive the corresponding growth rates
- A lot remains to be done ...

## 12.3 Exercises



### 12.3.1 Growth paths

Consider the following production sector:

$$Y(t) = K(t)^{\alpha} \left[ A(t) L(t) \right]^{1-\alpha}, \quad \alpha \in (0,1)$$

where production,$Y$, depends on capital, $K$, labour inputs, $L$, and productivity, $A$. Capital accumulates over time, by saving a fixed share of output, $sY$, and depreciates at rate $\delta$:

$$\dot{K} = sY(t) - \delta K(t).$$

Finally, growth rates of the population and total factor productivity (TFP) are constant:

$$\frac{\dot{L}}{L} \equiv n$$
$$\frac{\dot{A}}{A} \equiv g$$

1. Derive the growth rate of capital per effective labour, $\frac{d\tilde{k}/dt}{\tilde{k}}$, where $\tilde{k} \equiv \frac{K}{AL}$.

2. What is the steady-state value of $\tilde{k}$, i.e. where $\frac{d\tilde{k}/dt}{\tilde{k}} = 0$, denoted $\tilde{k}^*$?.

3. Draw the phase diagram of $\frac{d\tilde{k}}{dt}$ with respect to $\tilde{k}$ and provide an intuitive explanation. (Note here we are talking about $\frac{d\tilde{k}}{dt}$, the change in capital per effective labour over time, **not** its growth rate).

4. Now draw the phase diagram of the growth rate of capital per effective labour, $\frac{d\tilde{k}/dt}{\tilde{k}}$, as a function of $\tilde{k}$.

### 12.3.2 Optimal consumption paths

Consider the following problem, where the central planner maximises lifetime utility in aggregate consumption

$$U(t) = \int_t^{\infty} e^{-\rho(\tau-t)} u(C(\tau))\, d\tau$$

where utility is CRRA:

$$u(C(\tau)) = \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma}$$

and aggregate output is produced according to the following technology, with a constant TFP:

$$Y(\tau) = AK(\tau)$$

and aggregate capital accumulates according to

$$\dot{K}(\tau) = (1-\theta) Y(\tau) - \delta K(\tau) - C(\tau),$$

where a share $1-\theta$ of output is reinvested into capital, depreciation is $0 < \delta < 1$, and consumption reduces the accumulation of capital.

1. Using the Hamiltonian method, derive the Keynes-Ramsey rule for consumption, which describes the growth rate of consumption over time as a function of parameters only.

2. Given your answer above, what is the backward solution to the resulting differential equation in $C(\tau)$?

3. Plot the growth rate of consumption, $\frac{\dot{C}}{C}$, as a function of $\theta$, and determine TFP, $A$, using the following initial calibrations,

$$\begin{aligned}
&\theta = 19\% && \text{– VAT in Germany} \\
&\rho = 2\% && \text{– estimated to represent a discount rate of ca. 0.98} \\
&\sigma = \tfrac{3}{4} && \text{– low degree of risk aversion (high IES)} \\
&g \equiv \tfrac{\dot{C}}{C} = 2 && \text{– annual growth rate of Germany's GDP 1971-2015}
\end{aligned}$$

# 13 Business Cycles and Anxiety

## 13.1 Business cycles in an OLG model

- As before for growth, we are developing a model – this time on business cycles and anxiety
- This is all very research-oriented teaching
  - this model does not yet exist but
  - there is more progress than with the growth model
- We start with standard macro and look at the simplest possible DSGE (dynamic stochastic general equilibrium) model
- This builds on business analysis in macro I
  - we had a dynamic structure ...
  - we had no uncertainty ......
- We can use insights from emotion-part of lecture where we studied uncertainty ...

### 13.1.1 The structure of a simple RBC model

- individuals live for 2 periods (e.g. young working and old retired)
- young and old individuals overlap (see figure below)
- rational expectations, all uncertainty is taken into account
- firms act under perfect competition
- closed economy in general equilibrium
- time is discrete

### 13.1.2 Some references

- Kydland and Prescott (1980) "A Competitive Theory of Fluctuations and the Feasibility and Desirability of Stabilization Policy"
- Kydland and Prescott (1982) "Time to Build and Aggregate Fluctuations"
- Wälde (2012) "Applied Intertemporal Optimization" (ch. 8.1)

### 13.1.3 Technology

- Aggregate technology

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

where $K_t$ is capital stock in $t$ and $L_t$ is employment and $0 < \alpha < 1$

- Crucial new aspect

$$A_t \tilde{} \mathrm{LN}\left(A, \sigma^2\right)$$

  - Total factor productivity $A_t$ is lognormally distributed with mean $A$ and variance $\sigma^2$
  - drawing takes place from identical distribution for each $t$
  - TFP $A_t$ is i.i.d. (identically and independently distributed)
  - implication: there is _no_ growth in this model
  - economic importance: TFP is random, there are technology shocks

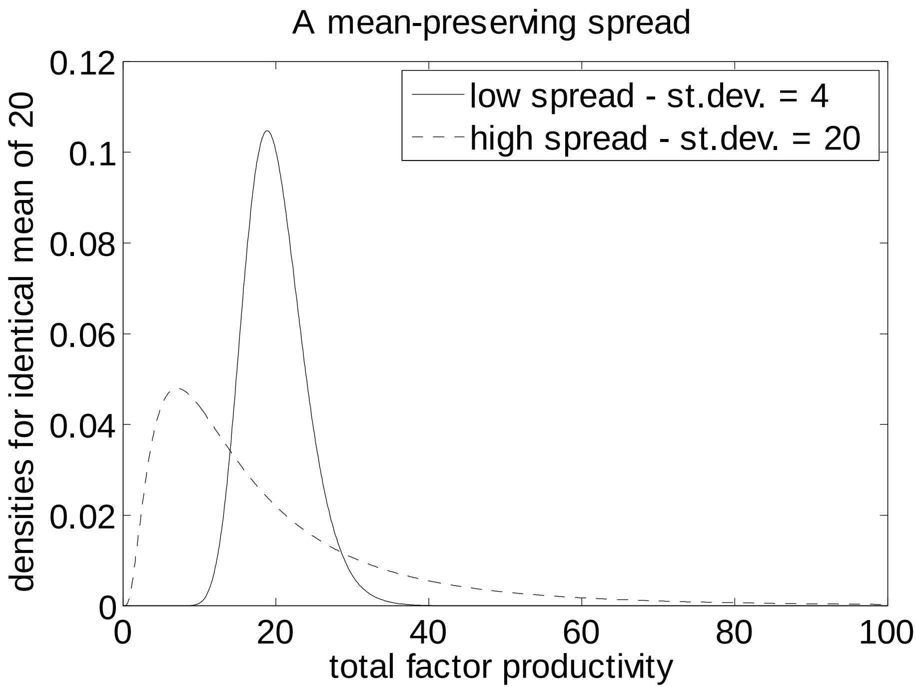


**Figure 24** *Illustrating the distributional assumption for TFP for low and high spread at unchanged mean*

### 13.1.4 Timing

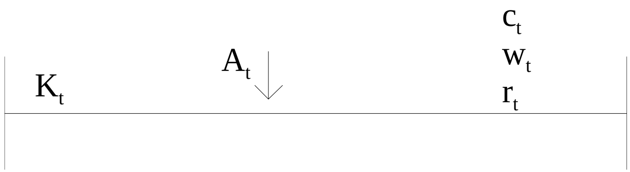


**Figure 25** *The timing of events within a period t*

- As time is discrete and as there is uncertainty, we need to know when various "things" happen
- $K_t$ is inherited from last period
- $A_t$ is realized afterwards (realization of random variable TFP is known – like throwing a realization 4 (or 1 or 2 or 3 or 5 or 6) with a dice
- afterwards firms pay wage and interest rate and households choose consumption

### 13.1.5 Firms

- Is life of firms more complicated? Do their decisions need to take uncertainty into account?
- No, firms maximize profits in a deterministic fashion as
  - they rent factors of production ($K$ and $L$) in each period on spot markets
  - they know realization of TFP before making this decision

$$w_t = p_t \frac{\partial Y_t}{\partial L_t}$$
$$r_t = p_t \frac{\partial Y_t}{\partial K_t}$$

  - firms equate value ($p_t$) marginal productivities to factor rewards ($w_t$ and $r_t$)
  - firms do not bear any risk

### 13.1.6 Households and intertemporal optimization

- (This is familiar from emotion part of the lecture)
- Objective function
  - households/individuals live for 2 periods (only)
  - individual consumes both periods

$$\max E_t \{u(c_t) + \beta u(c_{t+1})\}$$

  and needs to form expectations as consumption (via wage, via TFP $A_t$) is uncertain

  - individual works only in period $t$
  - $E_t$ is the expectations operator saying that individual forms expectations in $t$ and takes all knowledge in $t$ into account

- Constraints
  - constraint in the first period (period $t$)

    $$w_t = c_t + s_t$$

    where $s_t$ is savings in $t$
  - constraint in the second period (individual is retired)

    $$(1 + r_{t+1})\, s_t = c_{t+1}$$

    where left-hand side is income in period $t+1$ (savings plus interest on savings) and right-hand side is consumption expenditure

- An example
  - A Cobb-Douglas utility function

$$E_t \{\gamma \ln c_t + (1-\gamma) \ln c_{t+1}\}$$

  - Optimal behaviour

$$\begin{aligned} c_t &= \gamma w_t \\ s_t &= (1-\gamma) w_t \\ c_{t+1} &= (1+r_{t+1})(1-\gamma) w_t \end{aligned} \tag{47}$$

- Is there any uncertainty left?
  - yes, $r_{t+1}$ is unknown in $t$
  - actual, realized consumption in $t+1$ differs from expected consumption

#### 13.1.7 Aggregation over individuals and firms

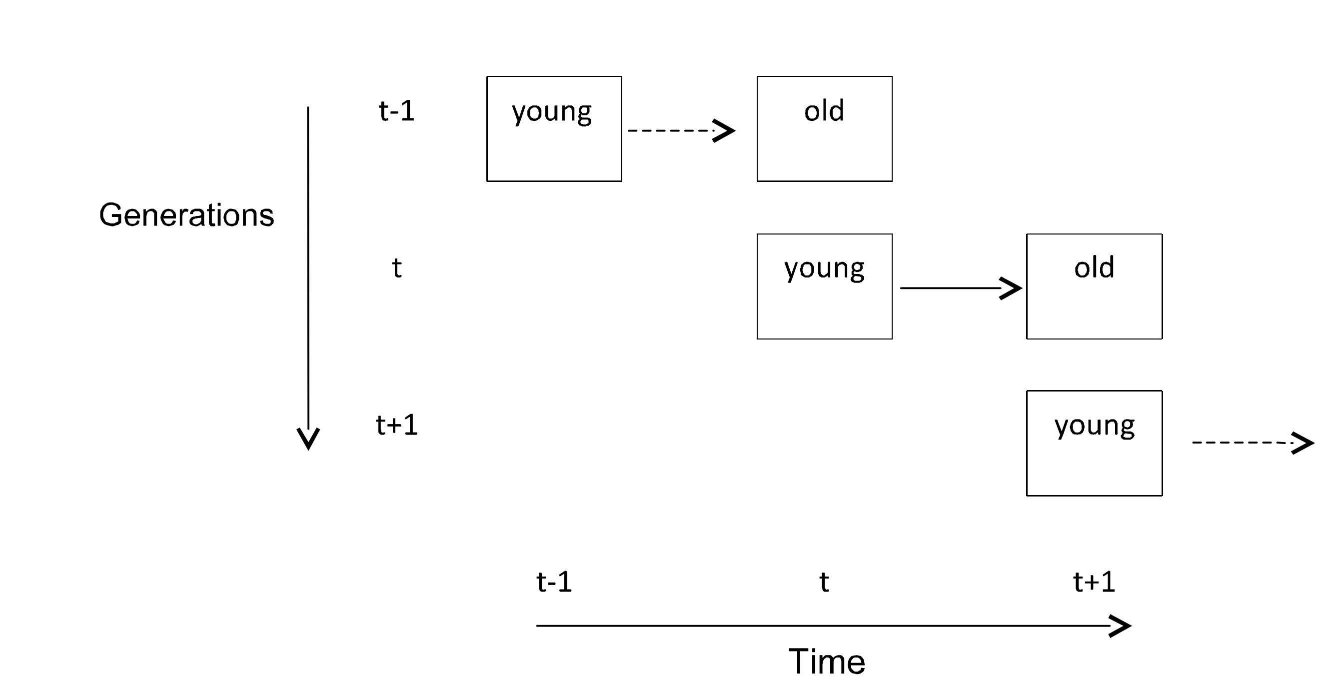


**Figure 26** *Overlapping generations*

- What is the capital stock in $t+1$?
  - Let population size be given by $2L$
  - As individuals work only while young, size of the labour force (in each period) is $L$
  - When the young save in $t$, the capital stock in $t+1$ originates from these savings

$$K_{t+1} = Ls_t$$

- Now construct a difference equation for capital
  - use savings expression from above
  - replace wage by marginal productivity of labour
  - rearrange and find (see Exercise 13.3.1)

$$K_{t+1} = Ls_t = L\left[1-\gamma\right]w_t = \left[1-\gamma\right]\left(1-\alpha\right)A_t K_t^{\alpha} L^{1-\alpha}$$

- This equation describes the intertemporal evolution of the economy by linking period $t$ to period $t+1$ (by looking at the capital stock)
- It allows to understand the role of uncertainty as TFP $A_t$ is on the right-hand side

### 13.1.8 The dynamics of TFP, the capital stock and output

- The phase diagram

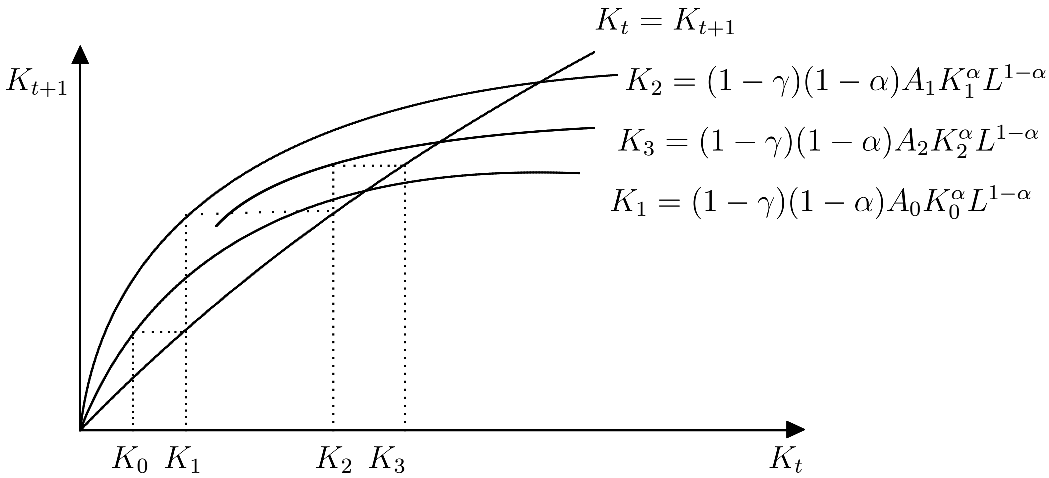


**Figure 27** *Convergence towards a "stochastic steady state"*

- Feeling for dynamics of $K_t$ comes from figure
  - in 0 we can predict $K_1$ but not $K_2$ as TFP $A_1$ is unknown
  - but we know distribution of $A_t$
  - now assume $A_0 < A_t < A_1$ (which is stronger assumption than lognormal distribution from above)
  - in case of $A_t = A_0$ (always recession), we end up at low steady state
  - in case of $A_t = A_1$ (always boom), we end up at high steady state
  - in most cases TFP lies between these extremes
  - in the long-run the capital stock is distributed between $K^{low}$ and $K^{high}$
  - we do not get statements about capital stock in the long run but only about its distribution ("where will it probably be")

### 13.1.9 What have we learned?

- The origins of business cycles
  - Business cycles are the results of shocks to technology
  - (One can just as easily imagine shocks to preferences, international prices, endowment)
  - These shocks are random and their realization can not be predicted
  - Agents know however that there are shocks and they know the distribution of these shocks (rational expectations)
  - Shocks in the real business cycle approach are as exogenous as technological growth in the Solow growth model
- The implications for economic model building
  - One can no longer talk about time paths or a steady state of an economy
  - One needs to talk about distributions and stationary distributions
  - In certain cases (in fact, in this model), one can compute the average capital stock and its variance (advanced bachelor thesis)

- Which of the observed business cycles can plausibly be explained by technology shocks?
  - Oil price shocks of the 1970s
  - Reunification of Germany (negative technology shock)
  - What about the financial crisis starting 2007? $\rightarrow$ A TFP shock was not the cause, but maybe a consequence ...

## 13.2 Business cycles and anxiety: towards a complete analysis

### 13.2.1 The effect of anxiety

- We replace the standard expression $s_t = (1-\gamma)\, w_t$ from (47)
- We now use $s_t = \frac{1-\gamma-(3\zeta-1)\gamma\phi}{1-(3\zeta-1)\gamma\phi} w_t$ from (4) in the anxiety analysis in the emotion-part of the lecture
- For notational and pedagogical simplicity, we write it as

$$s_t = (1-\Gamma)\, w_t \text{ where } \Gamma \equiv \frac{\gamma}{1-(3\zeta-1)\,\gamma\phi} \tag{48}$$

### 13.2.2 Aggregation over individuals and firms

- The capital stock in $t+1$
  - Population size still at $2L$, labour force at $L$
  - Capital stock still given by
$$K_{t+1} = Ls_t$$
- Now construct a difference equation for capital
  - We follow the same steps as before
  - we find (do not see tutorial but compare $\gamma$ with $\Gamma$)
$$K_{t+1} = [1 - \Gamma] (1 - \alpha) A_t K_t^{\alpha} L^{1-\alpha} \tag{49}$$

- Phase diagram analysis
  - qualitatively, there is no difference to fig. 27
  - only the $\gamma$ is replaced by the $\Gamma$
- Economic relevance is huge, however
  - Understand the effect of anticipatory emotions on expected equilibrium capital stock
  - Understand the interaction between precautionary saving ("excess saving" compared to saving in the absence of uncertainty) and emotional saving
  - Can the two be separated, i.e. can we quantify how strong each of these channels is? (also advanced bachelor thesis)

### 13.2.3 How anxiety affects the distribution of wealth

- We now compute the effect of anticipatory emotions on the capital stock in an economy
- To do this, we need to be able to compute the capital stock – of which we know that it evolves in a stochastic way
- We therefore do not know what the *realized* capital stock will be in the future
- We can compute the *expected* capital stock in the future, however!
- We do this in the following way ...

- We do this in the following way ...
  - Rewrite the equation for the capital stock (49) by applying logs

$$\ln K_{t+1} = \ln\left((1-\Gamma)(1-\alpha)L^{1-\alpha}\right) + \alpha\ln(K_t) + \ln(A_t)$$

  - Define $k_t \equiv \ln K_t$, $c_0 \equiv \ln((1-\Gamma)(1-\alpha)L^{1-\alpha})$ and $\varepsilon_t \equiv \ln(A_t)$. Then we obtain

$$k_{t+1} = c_0 + \alpha k_t + \varepsilon_t$$

  - Define further expected wealth as $\mu_t = E_0 k_t$, in words, $\mu_t$ is the expected (logarithmic) capital stock for some future point in time $t$ when we form expectations at 0
  - Applying expectations to this difference equation, we get

$$\mu_{t+1} = c_0 + \alpha\mu_t + \mu_A \tag{50}$$

    where $\mu_A \equiv E_0\varepsilon_t$, in words, $\mu_A$ is the mean of the log of the capital stock
  - This is a simple difference equation for the expected capital stock!

- Analysis of this equation as in all previous examples for dynamic systems
  - Is there a steady state?
  - Is there transitional dynamics?
- Steady state analysis
  - If there is a steady state, it must satisfy $\mu_{t+1} = \mu_t \equiv \mu$
  - Using (50), the expected capital stock in the steady state is given by

$$\mu = c_0 + \alpha\mu + \mu_A \Leftrightarrow \mu = \frac{c_0 + \mu_A}{1 - \alpha}$$

  - (We do not look at transitional dynamics at this point.)
- Back to economics: How does expected wealth depend on emotions?
  - How does expected wealth depend on emotion parameters in (2) and (3), i.e. on $\phi$ (importance of anxiety) and $\zeta$ (personality parameter for variance of return)?

– Answers come from computing the derivative of $\mu$ with respect to emotion parameters – see tutorial or exam :-)

$$\frac{d\mu}{d\phi} = \frac{dc_0/d\phi}{1-\alpha},$$
$$\frac{d\mu}{d\zeta} = \frac{dc_0/d\zeta}{1-\alpha},$$

where one should take into account that $c_0 \equiv \ln\left(\left(1-\Gamma\right)\left(1-\alpha\right)L^{1-\alpha}\right)$ and that $\Gamma = \frac{\gamma}{1-(3\zeta-1)\gamma\phi}$ from (48) collects all the emotion parameters

## 13.3 Exercises

### 13.3.1 Business cycles

Consider a representative household living for two periods only, maximising expected lifetime utility:

$$\max_{\{c_t, c_{t+1}\}} U_t = E_t \left[ u \left( c_t \right) + \beta u \left( c_{t+1} \right) \right], \quad \beta \in (0, 1).$$

The constraint in the first period reads

$$w_t = c_t + s_t,$$

where $w_t$ is wage at time $t$, $c_t$ is consumption and $s_t$ represents savings. In the second period, i.e. at $t+1$, the constraint reads

$$(1 + r_{t+1}) s_t = c_{t+1},$$

where $r_{t+1} \sim N\left(r, \sigma_r^2\right)$ is the stochastic interest rate, and consumption at $t+1$ is given by the value of savings at $t$ plus interests.The representative firm maximises profits

$$\max_{\{K_t, L_t\}} \pi_t = Y_t - (1 + r_t) K_t - w_t L_t$$
$$\text{s.t. } Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},$$

where $K_t$ is aggregate capital at $t$ and $L_t$ represents aggregate labour at $t$. $A_t \sim LN\left(A, \sigma_A^2\right)$ is the log-normally distributed total factor productivity (TFP). Capital next period is determined entirely by aggregate savings

$$K_{t+1} = L_t s_t.$$

1. Solve the maximisation problem of the household by substitution, and determine the consumption Euler equation.

2. Using your answer from above, and the Cobb-Douglas preferences below, find optimal consumption and savings as functions of the wage $w_t$,

$$\max_{\{c_t, c_{t+1}\}} E_t \left[\gamma \ln c_t + (1-\gamma) \ln c_{t+1}\right], \quad \gamma \in (0,1).$$

3. Solve the firm's maximisation problem, and determine the optimal demand functions for $K_t$ and $L_t$ as functions of the wage.

4. Using the results from (*2*) and (*3*) above, derive the law of motion for capital, that characterises this economy and draw its phase diagram (drawing $K_{t+1}$ as a function of $K_t$). Show what happens under various realisations of TFP. Give an interpretation to the graph.