1. **Consumer maximization probelm (2 periods)**

   Solve the following optimization problem of the representative consumer

$$\max_{c_1,c_2,k_2} V_1 = \max_{c_1,c_2,k_2} U(c_1) + \beta U(c_2) \tag{1}$$

   subject to

$$c_1 + k_2 = F(k_1) + (1-\delta)k_1 \tag{2}$$

$$c_2 = F(k_2) + (1-\delta)k_2 \tag{3}$$

   Period utility is given by

$$U(c_t) = \log c_t \tag{4}$$

   and output is given by

$$y_t = F(k_t) = k_t^\alpha, \tag{5}$$

   where $t = 1, 2$.

   - Derive the Euler equation for the specific utility- and production function by employing the Lagrange approach.
   - What does your result say about the consumption path of the consumer?

2. **Consumer maximization probelm (infinite periods)**

   Solve the following optimization problem of the representative consumer

$$\max_{\{c_{t+s}\}_{s=0}^{\infty}} V_t = \max_{\{c_{t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \tag{6}$$

   subject to

$$c_s + k_{s+1} = F(k_s) + (1-\delta)k_s \quad \text{for all } s > 0. \tag{7}$$

   Assume again that period utility is given by (4) and that output is given by (5).

   - Derive the Euler equation for the specific utility- and production function by employing the Lagrange approach.
   - What does your result say about the consumption path of the consumer?

3. **Empirical relevance**

   - Download the dataset from the course website.
   - Open the dataset. It provides quarterly data for output, consumption, and investment.
   - Detrend the data using the Hodrick-Prescott filter with $\lambda = 1600$.
   - Compute both the trend and the cyclical component of consumption and output.
   - Compute both variances and correlations of the detrended data.
   - Do your empirical results coincide with the stylized facts of business cycles?
   - For your empirical analysis use either the statistical program package *gretl*[1], *Excel* for which you can download[2] a Hodrick-Prescott filter Add-In, or any other software package that contains the filter.

4. **The Hodrick-Prescott (HP) filter**

   Recall from the lecture that the minimization problem of the HP-filter is given by

$$\min_{\{y_t^{lr}\}} \sum_{t=1}^{T} \left(y_t - y_t^{lr}\right)^2 + \lambda \sum_{t=2}^{T-1} \left[\left(y_{t+1}^{lr} - y_t^{lr}\right) - \left(y_t^{lr} - y_{t-1}^{lr}\right)\right]^2 . \tag{8}$$

   Recall also that $y_t = y_t^{lr} + y_t^{sr}$.

   Explain what happens to the short-run (cylical) and the long-run (trend) component if

   - $\lambda = 0$
   - $\lambda \to \infty$.

   In order to assess the two extreme cases first think about it in terms of equation (8) and then use the data from problem 3. For $\lambda \to \infty$ you can try out large values for λ such as 64.000 or larger.

[1] The program is free of charge, you can download it from `http://gretl.sourceforge.net/`.

[2] The Add-In is also free of charge, you can download it from `http://www.web-reg.de/hp_addin.html`.