

Advanced Macroeconomics

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Tutorial 8: Towards Behavioural Growth

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1. *A Dual-Self Model of Impulse Control (see Fudenberg & Levine, 2006)*

Consider an infinite-horizon consumer making a savings decision. Wealth y may be divided between consumption and savings according to the savings rate $a \in [0, 1]$. The consumer is represented by the short-run self and the long-run self. Parameter δ is the time preference rate.

Preferences for the short-run self are as follows:

$$u(y_t, 0, a_t) = \log[(1 - a_t)y_t]$$

Savings are then invested in an asset that returns wealth:

$$y_t = Ra_{t-1}y_{t-1}$$

Where $R = 1 + r$. The short-run self wishes to spend all wealth on consumption. The choice of variable a imposes a self-control constraint via a linear cost:

$$C(y_t, a_t) = \gamma \{\log[y_t] - \log[(1 - a_t)y_t]\}$$

Given the reduced form of preferences for the long-run self:

$$U = \sum_{t=1}^{\infty} \delta^{t-1} [(1 + \gamma) \log[(1 - a_t)y_t] - \gamma \log(y_t)]$$

and assuming a constant savings rate a , find the optimal saving behaviour and give an interpretation to this result.

2. Deterministic OLG Model

Consider an agent living for two periods. The constraint in the first period reads:

$$w_t = c_t + s_t$$

Where w_t is wage at time t , c_t is consumption and s_t represents savings. In the second period, i.e. at $t + 1$, the constraint reads:

$$(1 + r_{t+1}) s_t = c_{t+1}$$

Where r_{t+1} is the interest rate, and consumption at $t + 1$ is given by the value of savings at t plus interests.

(a) Solve the maximisation problem:

$$\max_{s_t} u(c_t) + \beta u(c_{t+1})$$

(b) Given the Cobb-Douglas preferences below, find the optimal consumption and saving paths:

$$\max_{s_t} \gamma \ln c_t + (1 - \gamma) \ln c_{t+1}$$

(c) Now, let there be many firms, employing capital K and labour L to produce output Y according to:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}$$

Where A is total factor productivity, held constant.

Capital stock in $t + 1$ is a function of labour size (in each period) and savings:

$$K_{t+1} = Ls_t$$

Households behave exactly as in b above.

Using your results from b , find the expression for the capital stock at $t + 1$ and draw its phase diagram. Give an interpretation of the graph.