Master in International Economics and Public Policy 1st semester

Advanced Macroeconomics

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Tutorial 10: Volatility & Infinite Horizon Models

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1. *Stochastic Model of Natural Volatility*

   Let technology follow a Cobb-Douglas specification (with Harrod-neutral technical progress):

   $$Y_t = K_t^{\alpha} (A_t L)^{1-\alpha}$$

   Where $A_t$is total factor productivity, K_t the capital stock and L_t represents hours worked. The accumulation of capital is given by the following relationship:

   $$K_{t+1} = (1-\delta) K_t + I_t$$

   With δ, the depreciation rate. While the technological level (or total factor productivity) grows as follows:

   $$A_{t+1} = (1+q_t) A_t$$

   Where:

   $$q_t = \left\{ \begin{array}{c} \bar{q} \\ 0 \end{array} \right\} \text{ with probability } \left\{ \begin{array}{c} p_t \\ 1-p_t \end{array} \right\}$$

   This probability depends on resources R_t invested into R&D.

   The resource constraint in this economy reads:

   $$C_t + I_t + R_t = Y_t$$

   Where we have aggregate consumption, investment and R&D expenditure, respectively C, I and R. And assuming optimal behaviour from the household, aggregate consumption and R&D expenditure are determined by:

   $$C_t = s_C Y_t \quad ; \quad R_t = s_R Y_t$$

   Where s_C and s_R are constant savings rates.

   (a) Show that after a jump in technological progress, the growth rate is higher at first and then progressively decreases over time. Draw a diagram showing this process.

   (b) What is the equilibrium for capital per effective labour ($\frac{K_t}{A_t L_t} = k_t$)? Find the temporary steady-state.

   (c) Explain why the steady-state is called temporary and what happens to it after technological jumps.

   (d) Draw the implication of technological jumps on production over time and give an interpretation.

2. *Oil Shocks in the Economy*

   Consider an economy which utilises oil as an intermediate good, but which affects its entire production process:

$$Y_t = AK_t^{\alpha}O_t^{\beta}L^{1-\alpha-\beta}$$
$$\alpha, \beta \in (0,1) \;\; ; \;\; \alpha+\beta<1$$

   (a) Consider a representative firm in the economy. Specify its profit schedule as a function of the factors of production and find the first-order conditions. What is the price of oil equal to?

   (b) Use your result above to rewrite the overall production function, i.e. rewrite O_t using your result above, and insert this result into Y_t.

   (c) What would the outcome of a rise in oil prices be on productivity?

3. *Infinite Horizon RBC set-up*

   Consider the following infinite horizon problem. The economy follows the usual Cobb-Douglas process:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}$$

   While households maximise:

$$\max_{\{c_t\}} E_t\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right]$$

   Subject to:

$$a_{t+1} = \left(1+r_t\right)a_t + w_t - c_t$$

   (a) What is the optimality rule for households?

   (b) Consider now a central planner, seeking to maximise the following function:

$$\max_{\{C_\tau\}} E_t\left[\sum_{\tau=t}^{\infty}\beta^{(\tau-t)}u\left(C_t\right)\right]$$

   Subject to:

$$K_{t+1} = \left(1-\delta\right)K_t + Y_t - C_t$$

   Using these two expressions and the production function, find the optimality conditions for the central planner.