Match efficiency and the cyclical behavior of job finding rates

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Abstract

This paper relaxes the assumption of a constant matching function and shows that fluctuations in the efficiency of matching are an important determinant of job finding rate variation. First, I empirically document time variation in match efficiency for the U.S. economy. Estimates of the matching function are severely complicated by poor data on vacancies. To this end I estimate a model where not only match efficiency but also vacancies are unobserved. The results show that match efficiency is procyclical and can explain 26-35% of job finding rate variation. Second, I show that a search and matching model with endogenous separations and firing costs can account for around 60% of the observed match efficiency fluctuations. Recessions are times when relatively more unemployed workers are not productive enough to form profitable employment relationships which presents itself as a fall in aggregate match efficiency. Firing costs then make firms require a higher minimum productivity level from unemployed workers as a compensation for expected future dismissals. Therefore, the effects of aggregate fluctuations on match efficiency are stronger, as long as the mass of unemployed workers increases with their skill level in the neighborhood of the hiring decision.

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1 Introduction

The rate at which unemployed workers find jobs is a crucial variable in understanding labor market dynamics. To a large extent the job finding rate drives the development of unemployment and therefore its pronounced business cycle swings make it a variable of interest both for research and policy analysis.

A typical way to model the job finding rate and labor market frictions in general is to employ a matching function with constant parameters. Such a function postulates that the number of hired unemployed workers, matches (M_t), is determined by the number of unemployed (U_t) and the number of vacancies (V_t):

$$M_t = Am(U_t, V_t)$$

where A is a scaling parameter (match efficiency) and $m_U, m_V > 0$ and $m(0, V) = m(U, 0) = 0$. The probability that an unemployed worker finds a job then follows directly as $F_t = M_t/U_t$. Hence, the use of a constant matching function implies that changes in the job finding rate are fully driven by fluctuations in vacancies and unemployment.

The matching function is convenient because it enables modeling a whole spectrum of heterogeneities in a parsimonious way. However, if the degree of heterogeneity changes over the business cycle, a matching function with constant parameters cannot capture all the dynamics. In this paper the assumption of constant parameters is relaxed, namely match efficiency is allowed to vary. It turns out that fluctuations in the efficiency of the matching process are an important determinant of variation in the job finding rate.

Estimating the matching function and investigating the possibility that match efficiency varies over the business cycle is severely complicated by the lack of good data on vacancies[1]. To tackle this problem I specify and estimate an unobserved component and time varying coefficient model. Treating both match efficiency as well as the underlying vacancy series as unobserved makes it possible to learn simultaneously about their time series properties. Assumptions on the underlying processes together with some additional information on vacancies at the very end of the sample from the Job Openings and Labor Turnover Survey (JOLTS) database[2] facilitate identification of the two unobserved components.

The obtained results show that match efficiency is procyclical with respect to the business cycle and varies substantially. This means that workers have a harder time finding jobs during a recession not only because there is less vacancies and more unemployed to compete for them, but also because the matching process is less efficient. A decomposition exercise suggests that match efficiency explains $26 - 35\%$ of variation in the job finding

[1] The typically used proxy for vacancies, dating back to 1951, is the Help Wanted Index. This index uses help wanted ads in 51 newspapers across the US and is therefore only a crude measure of vacancies.

[2] The JOLTS database provides high quality data on vacancies, but it dates back only to December of 2000, while the sample used in the empirical part starts in 1948.

probability at business cycle frequencies.

The matching function is a reduced form relationship and therefore one needs a structural model to explain variation in its parameters. I show that a standard endogenous separations model features fluctuations in match efficiency once one explicitly distinguishes between newly hired workers and workers in existing employment relationships. In this model workers are ex-ante identical but each period they draw skills from a (constant) productivity distribution prior to production. Therefore, for an unemployed worker to find a job it is not enough to get matched with a vacancy, but she must also be sufficiently productive to make the employment relationship profitable. Although the individual productivity distribution is constant, aggregate fluctuations affect the threshold level of worker productivity below which employment relationships are not profitable anymore. In recessions it takes workers with relatively higher productivity draws to make an employment relationship viable, while the opposite holds for booms. In other words, recessions are times when a larger fraction of the unemployment pool is not suitable for forming profitable matches. When estimating the matching function using data only on vacancies and unemployment, fluctuations in the individual productivity threshold will present themselves as variation in match efficiency.

Extending the model to include firing costs for workers in existing employment relationships makes the mechanism quantitatively important. Firing costs effectively make employed workers favored by firms. Newly matched workers are required to have a higher minimum productivity level as a compensation for expected future firing costs. At the same time, existing workers are protected, because the firm must pay firing costs upon their dismissal. Thus, if the idiosyncratic productivity distribution in the neighborhood of the hiring decision is such that the mass of workers increases for higher skill levels (which is argued not to be a unreasonable assumption), then aggregate fluctuations will have a stronger effect on match efficiency. The reason is that in such an environment changes in the individual productivity threshold affect a larger number of unemployed workers. The calibrated model can then explain about 60% of match efficiency variation observed in the data. Futhermore, match efficiency turns out to drive about 38% of the job finding rate fluctuations in the model.

The paper is organized as follows. Section (2) describes the estimation of match efficiency in the US economy and section (3) shows the estimation results. Then, section (4) builds a search and matching model with endogenous separations and firing costs to explain the cyclical movements in match efficiency. The model results are presented in section (6). Finally, the last section concludes.

2 Estimating time variation in match efficiency

The starting point of the empirical analysis is the job finding probability: $F_t = M_t/U_t$, where M_t is the number of matches (unemployed workers that find a job) and U_t is the number of unemployed[3] workers in period t. The theoretical number of matches is then modeled by a matching function $M_t = Am(U_t, V_t)$ using data on the number of unemployed and the number of vacancies (V_t).

2.1 State-space representation

The estimation uses quarterly data on the job finding probability taken from Shimer (2007) and the number of unemployed published by the BLS in the period 1948Q1 - 2007Q1. The main idea is to treat both the underlying vacancies and the match efficiency parameter as unobserved states. To help pin down the process for vacancies the job openings series from the JOLTS database (available from December 2000)[4] is used as an unbiased signal[5] of the underlying vacancies for the last six years of the sample.

In the general state space form a $m \times 1$ vector of observables, y_t, is related to a $q \times 1$ vector of unobserved states s_t via the *measurement equation.*

$$y_t = \Theta_{0,t} + \Theta_{1,t} s_t + \epsilon_t \tag{1}$$

where $\Theta_{0,t}$ is a $m \times 1$ vector, $\Theta_{1,t}$ is an $m \times q$ matrix and ϵ_t is an $m \times 1$ vector of serially uncorrelated disturbances with mean zero a covariance matrix R. The unobserved states are assumed to evolve according to a first-order Markov process (the *transition equation*)

$$s_t = \Phi_{0,t} + \Phi_{1,t} s_{t-1} + \eta_t \tag{2}$$

where $\Phi_{0,t}$ is an $q \times 1$ vector, $\Phi_{1,t}$ is an $q \times q$ matrix and η_t is an $q \times 1$ vector of serially uncorrelated disturbances with mean zero and covariance matrix Q.

In the model at hand there are two unobserved states ($q = 2$): match efficiency (A_t) and vacancies (V_t).[7] In the benchmark model vacancies are assumed to be a random walk, while match efficiency is assumed to follow a stationary AR(1) process. The choice of the random walk on vacancies is motivated by its fundamentally close relationship to unemployment, which is an integrated process of order 1 in the given sample.[8] Similarly,

[3]In what follows I will use the number of unemployed and unemployment interchangeably.

[4]The time periods prior to this data are taken as missing observations.

[5]In the JOLTS specification a job opening requires that "1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. The position can be full-time or part-time, and it can be permanent, short-term, or seasonal. Furthermore, active recruitment means include advertising in newspapers, on television, or on radio; posting Internet notices; posting "help wanted" signs; networking or making "word of mouth" announcements; accepting applications; interviewing candidates; contacting employment agencies; or soliciting employees at job fairs, state or local employment offices, or similar sources"[6]. This comprehensive definition suggests that assuming the JOLTS vacancy series to be an unbiased signal of the underlying vacancies is not unreasonable.

[7]Note, that the model works with the *number* of unemployed and vacancies, not with the rates.

[8]The ADF with 4 lags and an intercept (intercept and trend) can reject a unit root at the 11.9% (12%) level. For first differenced unemployment the unit root is rejected (in all specifications: with(out) intercept and intercept with trend) at the 0% level.

the typical proxy for vacancies, the Help Wanted Index (HWI) is also an integrated process of order 1.[9] Moreover, the appendix shows that allowing for a richer non-stationary structure does not change the results much. Assuming match efficiency to be an AR(1) process helps the identification by distinguishing it from the vacancy process. However, the appendix shows that an alternative specification where both states are random walks delivers similar results.

The two states are related to observed variables via two measurement equations ($m = 2$): one for the job finding probability (F_t) and one postulating that the JOLTS job openings series (V_t^J) is a noisy observation of the vacancy state. The former is the main equation facilitating the identification of the two processes. The latter helps pin down further the properties of the vacancy state and especially their level. Remember, however, that the job openings data is available only from 2001Q1. The periods prior to that date can be conveniently handled by the Kalman filter as missing observations. Finally, the matching function is assumed to be of the Cobb-Douglas form with constant returns to scale

$$M_t = A_t U_t^{1-\mu} V_t^{\mu} \tag{3}$$

This functional form is a typical assumption in the literature. Petrongolo and Pissarides (2001) provide an excellent survey of the literature and conclude that the mentioned specification has large empirical support.

Denoting with small letters the logs of variables one can then write the state space representation of the model as

$$\begin{bmatrix} f_t \\ v_t^J \end{bmatrix} = \begin{bmatrix} -\mu u_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & \mu \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_t \\ v_t \end{bmatrix} + \epsilon_t \tag{4}$$

$$\begin{bmatrix} a_t \\ v_t \end{bmatrix} = \begin{bmatrix} (1-\rho_a)\overline{a} \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_a & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{t-1} \\ v_{t-1} \end{bmatrix} + \eta_t \tag{5}$$

where ρ_a is the autoregressive coefficient of log match efficiency and $\overline{a}$ is its unconditional mean. Furthermore, the innovations of the state and measurement equations are assumed to be jointly normally distributed with mean zero and variance covariance matrix

$$E_t \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix} [\eta_t \quad \epsilon_t] = \begin{pmatrix} Q & C' \\ C & R \end{pmatrix} \tag{6}$$

where C is the 2×2 cross-covariance matrix.

[9] The ADF test with 4 lags and an intercept (intercept with trend) rejects the unit root at the 11.4% (40.5%) level. For first differences it rejects at the 0% level.

2.2 Estimation

Maximum likelihood (ML) is used to estimate the elasticity on vacancies in the matching function (μ), the autoregressive coefficient and unconditional mean of log match efficiency (ρ_a and $\overline{a}$) and all the elements of the variance covariance matrix of the innovations (R, C and Q).[10] The Kalman filter is then employed to obtain smoothed states[11] at the ML estimates. Furthermore, to overcome potential endogeneity problems I use the first lag of the regressor as an instrument. The appendix provides explicit exogeneity tests supporting this procedure.

To start the minimization routine one must pick initial values. The starting values for ρ_a, $\overline{a}$ and μ are set to 0.9, -0.6 and 0.3, respectively. The initial values for the covariance matrices are based on error variances from a trial regression of the job finding probability on observed labor market tightness (using the HWI as an indicator of vacancies) in the period 1948Q1-1954Q4. Denote the error variance from the trial regression by W_f. Furthermore, denote by W_v the variance of the (log) job openings series from the JOLTS database. The initial values for the covariance matrices are then $R^{init} = \begin{pmatrix} \omega_{R,f}W_f & 0 \\ 0 & \omega_{Q,v}W_v \end{pmatrix}$, $Q^{init} = \begin{pmatrix} \omega_{Q,f}W_f & 0 \\ 0 & \omega_{Q,v}W_v \end{pmatrix}$. The scaling parameters $\omega_{i,j}$, where $i = Q, R$ and $j = f, v$, are found by a grid search that maximize the log-likelihood of the model. The initial value for the cross-covariance matrix C is a 2×2 zero matrix. Robustness checks show that changing the initial values does little to the results.

Furthermore, to start the Kalman filter routine one must set the initial state vector s_0 and its covariance matrix P_0. Following Durbin and Koopman (2001) the former is set to the unconditional mean of the state vector, while the latter is set to a large number (10^5). This essentially means that there is large uncertainty about the initial state and the data is allowed to speak freely.

3 Estimation results

Table (1) provides the estimated parameter values as well as p-values of diagnostic tests related to the model residuals. After dealing with 4 outliers[12] using a single dummy variable the diagnostic tests show that the standardized model prediction errors satisfy the assumptions of independence, homoscedasticity and normality (more details in the appendix).

The Cobb-Douglas elasticity on vacancies is estimated to be 0.35, which falls within

[10] The minimization itself is done using Chris Sims' csminwel algorithm.

[11] The term "smoothed" might be confusing later on when evaluating the volatility of the states. Note that it refers to running the Kalman filter "backwards". The estimates in period t are then based on not only past information, but also on information from observations t onwards.

[12] The outliers are in quarters 1957Q4, 1958Q1, 1974Q4 and 1975Q1.

the range reported in Petrongolo and Pissarides (2001) and it is close to the estimates in Shimer (2005) and Barnichon (2009). Figure (1) shows the estimated smoothed vacancies and match efficiency which are discussed in detail below.

3.1 Match efficiency

Match efficiency varies substantially with a standard deviation of almost 5.9%[13]. Furthermore, match efficiency is procyclical with respect to the business cycle. Using the HP filter (with smoothing coefficient 1600) to extract cyclical components, the correlation coefficients of match efficiency with (log) unemployment and output are -0.50 and 0.55, respectively. This means that recessions are periods when unemployed workers on average have a harder time finding a job not only because the number of vacancies drops and there are more unemployed workers competing for a given vacancy, but also because the efficiency of the matching process declines.

Not all recessions, however, are characterized by the same behavior of match efficiency. During the recessions in the late 50's and the mid 70's match efficiency experienced the sharpest declines in the range of 15-20%. Smaller, but still sizeable falls in match efficiency happened during the recessions in the early 80's and 90's with falls of 8 and 10%, respectively.

The 1990 recession is peculiar in that match efficiency kept on falling for a few quarters while the economy was already recovering. A similar pattern is apparent for the 2001 recession, where match efficiency picked up after the small decline during the recession, but started falling thereafter. These developments are reflecting the jobless recoveries experienced after these two downturns. Even though output started to rise in the recovery phase, match efficiency remained low keeping down the job finding rate and thus dampening employment growth.

To get an idea of how important the fluctuations in match efficiency are, I decompose the variation of the job finding rate into contributions of match efficiency and labor market tightness. Such a decomposition is not trivial, since the two components are correlated. For a decomposition that appropriately disentangles the covariance term I follow Fujita and Ramey (2009). The starting point is a log deviation of the job finding rate from its trend value (denoted by bars)

$$ln\frac{F_t}{\overline{F}_t} = ln\frac{A_t}{\overline{A}_t} + \mu ln\frac{\theta_t}{\overline{\theta}_t} + \omega_t \tag{7}$$

where ω_t is an error coming from the detrending procedure. In general it will not be the case that the trend components of match efficiency and labor market tightness exactly add up to that of the job finding rate. The above can be expressed generically as

[13]This is the theoretical standard deviation based on the estimated parameters, hence $\sigma_\alpha = \sqrt{\frac{Q(1,1)}{1-\rho_\alpha^2}}$.

$$df_t = df_t^A + df_t^\theta + df_t^\omega \tag{8}$$

One can then show that

$$var(df_t) = cov(df_t, df_t^A) + cov(df_t, df_t^\theta) + cov(df_t, df_t^\omega) \tag{9}$$

where the term $cov(df_t, df_t^A)$ gives the amount of variation in the job finding probability due to match efficiency appropriately taking into account its covariance with labor market tightness. Expressing this variation relative to total volatility in the job finding probability gives:

$$\beta^A = \frac{cov(df_t, df_t^A)}{var(df_t)}$$

From (9) it is clear that $\beta^A + \beta^\theta + \beta^\omega = 1$. Table (2) shows the respective decompositions for HP-filtered and first-differenced data. For HP-filtered data, the contribution of match efficiency to job finding probability fluctuations is somewhere between 26 and 35% depending on the smoothing coefficient. In the case of first-differenced data the contribution is higher at 50%. The difference between the values comes from the different frequencies the filters focus on. First differencing emphasizes high frequencies as does a lower smoothing coefficient in the HP filter. Match efficiency thus seems to gain explanatory power as one focuses on higher frequencies. Zooming in on business cycle frequencies match efficiency accounts for $26-35\%$ of variation in the job finding probability, which is a nontrivial amount.

Figure (2) shows the job finding rate and its counterfactual generated under the assumption that match efficiency is fixed at its average value. Looking at troughs of the two recessions with the largest fall in match efficiency (in 1958 and 1974), the counterfactual job finding rate is 3.5 and 4 percentage points higher, respectively. In other words, during these recessions the fall in match efficiency pushed down further the probability of finding a job by up to 4 percentage points. Given the low cyclical level in the trough, 4 percentage points amount to around 10% of the job finding probability. A similar effect of comparable magnitude occured also in the early 90's, but this time a few quarters *after* the recession ended. Note, however, that even the counterfactual job finding rate picks up after the recession. Hence, although match efficiency contributed to a greater drop in the job finding probability after the recession, it was not the only reason for its delayed bounce-back and hence a jobless recovery.

One can have a closer look at the effect of match efficiency during recessions by decomposing the cumulative falls of the job finding rate again into contributions of match efficiency and labor market tightness. Log-differencing the definition of the job finding probability one can write

$$dF_t \approx F_t(dlog(A_t) + \mu\, dlog(\theta_t)) \tag{10}$$

Figure (3) shows these log contributions to the cummulative drop of the job finding rate during the recessions (the quarters prior to the starting dates of the recessions are indicated on the horizontal axis). It seems that match efficiency contributes to the job finding rate falls more in the onset of recessions. Getting closer to the recovery phase match efficiency contributions slow down and in a few cases they even reverse before the end of the recession. Moreover, the figure points out that the variance decomposition was hiding a lot of heterogeneity. The contribution of match efficiency to the job finding rate fall during the recessions in 1960, 1973 and 1981 was comparable in size to that of labor market tightness. On the other hand during the recessions in 1990 and 2001 match efficiency contributed only very slightly. This is related to the fact that match efficiency fell mostly *after* the recessions in 1990 and 2001. Furthermore, the downturns with the highest contributions of match efficiency are also on average longer and deeper.[14] Therefore, it seems that it is large recessions that are associated with sharp falls in match efficiency. Since contributions of match efficiency to job finding rate falls typically die out 1-2 quarters before the end of the recession, shorter and milder recessions do not permit match efficiency to gather enough momentum.

All the above points to the fact that match efficiency is an important determinant of job finding rate fluctuations. Therefore, specifications of the matching function should not be such that they rule out this channel by assumption.

3.2 Underlying vacancies

Although vacancies are not the main focus of this paper, the estimated vacancies are of separate interest because they provide a methodologically consistent vacancy series that dates back over several business cycles. The alternative typically used is the HWI that dates back to 1951, but is increasingly inaccurate as internet vacancy posting increases in the later part of the sample. The HWI actually stopped being published in May 2008 and was replaced by the Online HWI.[15] The more recent job openings data from the JOLTS database provide a much better indicator of vacancies, but they date back only to 2001 missing all the previous business cycles. On the contrary, the vacancy estimate in this paper enables a methodologically consistent comparison of labor market dynamics over several business cycles, including the more recent ones. The kind of analysis that this deserves is, however, outside the scope of this paper and left for future work.

Figure (1) displays the estimated vacancy state and the HWI. At first sight, the dynamics of the two series is similar (correlation coefficient of 0.81). At the same time, the vacancy estimate is much smoother. Note, however, that the estimated vacancy series is the Kalman smoothed estimate (a conditional expectation) and not a realization. For a fair comparison one needs to compare the estimated theoretical standard deviation of the

[14] On average they are one quarter longer with real GDP growth falling by 1.5 percentage points more.

[15] Barnichon (2009) attempts to link the two indices into a composite HWI. Apart from specific assumptions on the dispersion process of internet use, Barnichon assumes that prior to 1995 the HWI was the ideal characterization of vacancies.

vacancy innovations to a suitable empirical counterpart. To this end one can assume that the HWI is also a random walk[16] and use the standard deviation of its first difference. Such a comparison shows that the estimated vacancies fluctuate less by approximately 10%. A similar result is obtained under the assumption of a richer non-stationary structure for vacancies, as is done in the appendix.

Furthermore, it seems that the HWI index exaggerates vacancy falls in recessions, but is relatively closer to the estimated vacancies in upturns. After 1990, however, there is a clear departure of the two series. This is arguably due to the spur in internet posting of vacancies as was also pointed out by Shimer (2005) and Barnichon (2009).

The Beveridge curve is somewhat weaker for the estimated vacancy series. The correlation coefficient between the unemployment and vacancy rate (HP filtered with smoothing coefficient 1600) is -0.9 when using the HWI and -0.72 when using the estimated vacancies. However, the relationship gets stronger over time and after 1984 it is almost identical (correlation coefficient around -0.85 for both variables). Once again, one needs to keep in mind that the correlation can be affected by the fact that the vacancy series is a conditional expectation and thus smoother.

4 A model to explain match efficiency fluctuations

This section tries to answer the question that has so far been left aside, namely *why* match efficiency fluctuates over the cycle. The starting point is a search and matching model with endogenous separations and with a clear distinction between newly matched workers and workers in existing employment relationships. To keep the model tractable only heterogeneity within the period is modeled. Introducing ex-ante heterogenous agents would only strengthen the results as is discussed shortly in the next subsection.

Imagine an economy with only idiosyncratic productivity risk within each period, but no aggregate uncertainty. Each period all workers draw a value of the individual productivity shock, a, from a *constant* distribution. Employment relationships continue only if the value of production exceeds the joint value of outside options of the firm and worker. Separations are then characterized by a threshold value, $\widetilde{a}$, such that relationships with values of a below the threshold are terminated. Introducing aggregate uncertainty makes the threshold countercyclical. In times of high aggregate productivity even relatively less productive workers can form (sustain) viable employment relationships, while the opposite holds for recessions.

In the standard model newly matched workers are counted as employed even though they can still separate prior to production. The benchmark model in this paper, however,

[16] Although unit root tests do not imply that the HWI is a random walk, they show that the series is non-stationary in the given sample. ADF test with 4 lags and an intercept (intercept with trend) rejects the unit root at the 11.4% (40.5%) level. For first differences it rejects at the 0% level.

explicitly distinguishes between these two types of workers and does not consider newly matched workers as employed unless they actually produce. To make the exposition easier I refer to separations of newly matched workers as *rejections* and to the respective rate as the *rejection rate*.

How does any of this translate into variation in match efficiency? Remember, that a fraction of newly matched workers will get rejected before even starting production and thus will not be counted as employed. Therefore, the job finding probability is no longer determined only by the number of unemployed and vacancies, but also by the rejection rate. Increases in the rejection rate are then reflected as drops in match efficiency, since they are not observed by the econometrician that takes into account only data on vacancies and unemployment.

The question is how well does the benchmark model do quantitatively. The next section builds up the benchmark model and evaluates its performance on the basis of explaining match efficiency fluctuations. It turns out that its performance is very poor. In section (5) I extend the benchmark model by introducing firing costs. This model does a much better job at explaining match efficiency variation and also does better in other dimensions.

4.1 Endogenous separations model

Before I turn to describe the benchmark model it is worth while commenting on the distributional assumption of idiosyncratic productivity shocks. Although identically and independently distributed shocks are not the most realistic they are very convenient. In this environment, the individual history of productivity shocks does not matter. On the other hand, the absence of persistence in individual productivity levels dampens the mechanism through which match efficiency varies. Consider a worker that is separated in period t, enters the unemployment pool and gets immediately rematched and is ready to produce the next period. In the benchmark model in period $t+1$ he has a chance to draw a productivity level that is above the threshold independent of his previous level. However, if idiosyncratic productivity levels were persistent, separated workers would have a greater probability of drawing insufficiently high values of productivity in upcoming periods. Thus, compared to the benchmark model, rejection rates would be higher and more persistent. However, introducing persistence into individual productivity levels deserves attention of its own and his left for future research.

4.1.1 Household behavior

The household consists of a continuum of workers of unit mass. Members of the household pool their incomes from employment and non-employment activities and spend all of it on consumption. Hence, the model abstracts from any investment or labor force participation decisions and the household becomes passive in this sense.

Formally the household maximizes expected life-time utility by choosing aggregate consumption

$$E_t\left[\sum_{j=0}^{\infty}\beta^j\frac{c_{t+j}^{1-\gamma}-1}{1-\gamma}\right]$$

subject to the aggregate budget constraint

$$c_t=\int_{\tilde{a}_t}^{\overline{a}}(w_t(a)n_t)dF(a_t)+bu_t+\Pi_t$$

where total wage income, non-employment income and aggregate profits are spent on consumption. Costs of posting vacancies are assumed to be redistributed to the household.

4.1.2 Matching process

The economy consists of a continuum of workers of unit mass. Let u_t be the mass of unemployed workers available in the matching pool at the beginning of the period. The unemployed meet in the matching market with a continuum of potential firms (infinite mass) to form employment relationships. Firms choose whether or not to post vacancies at the beginning of the period at a flow cost of κ. Let v_t be the mass of firms posting vacancies determined by free entry. The number of matches in period t is then determined by a matching function.

$$m_t=Au_t^{\mu}v_t^{1-\mu} \tag{11}$$

The choice of the Cobb-Douglas functional form with constant returns to scale is consistent with the empirical part of the paper. In the standard model the number of matches is also the number of newly employed workers even though they can still separate prior to production. Here, however, the fraction of workers that is not productive enough to form a profitable employment relationship is not counted as employed. Hence, workers that did not get matched with a vacancy, or those that did but were not productive enough, remain in the unemployment pool.

The matching process runs simultaneously with production and thus workers from severed relationships in period t enter the unemployment pool and are ready to be rematched in the same period and productive in the next. The probability that a worker gets matched with a vacancy in period t is defined as $f_t=m_t/u_t$, while the probability that a firm with an open vacancy gets matched with a worker in period t is $q_t=m_t/v_t$. Remember, that these are *not* equal to the probabilities of finding a job and filling a vacancy, which are defined later in the text.

4.1.3 Employment relationships

An employment relationship consists of a worker and firm pair. Production is given by z_ta_i, where z_t is the aggregate productivity shock and a_i is the worker specific productivity shock. The worker specific shock is assumed to be an identically and independently distributed draw from a log-normal distribution with a cumulative distribution function F. The relationship can be severed exogenously before the shocks materialize and this happens with probability ρ_x. After observing the aggregate and worker specific shocks the employment relationship decides whether to continue and produce or whether to separate. In the event of (exogenous or endogenous) separation there is no production and the worker joins the unemployment pool.

4.1.4 Endogenous separations

Next I provide the value functions describing the problem of firms and workers in the matching market. Denote with $W_{i,t}$ the value at time t of being in a productive employment relationship for a worker with job specific productivity a_i (measured in current consumption units). This is given by

$$W_{i,t} = w_{i,t} + E_t\beta_t \left[(1-\rho_x) \int_{\widetilde{a}_{t+1}}^{\overline{a}} (W_{t+1} - U_{t+1}) dF(a_{t+1}) + U_{t+1} \right] \tag{12}$$

where $w_{i,t}$ is the wage rate, $\beta_t = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$ is the stochastic discount factor, $\widetilde{a}_{t+1}$ is the threshold value of the worker specific shock such that employment relationships with values of a_i below this threshold endogenously separate and $\overline{a}$ is the upper bound of the skill distribution. Hence, the worker gets a wage rate dependent on her idiosyncratic productivity level plus the continuation value of exiting period t in an employment relationship.

The value of being in the matching pool for the worker (U_t) at time t is defined as

$$U_t = b + E_t\beta_t \left[f_t(1-\rho_x) \int_{\widetilde{a}_{t+1}}^{\overline{a}} (W_{t+1} - U_{t+1}) dF(a_{t+1}) + U_{t+1} \right] \tag{13}$$

where the worker enjoys leisure and outcomes of home production worth b units of consumption, the value of being in an employment relationship tomorrow if she is successful in the matching process, or otherwise the future value of remaining unemployed.

Denote with $J_{i,t}$ the value of a productive employment relationship for the firm employing a worker with idiosyncratic productivity a_i. This value is given by

$$J_{i,t} = z_t a_i - w_{i,t} + E_t\beta_t \left[(1-\rho_x) \int_{\widetilde{a}_{t+1}}^{\overline{a}} (J_{t+1} - V_{t+1}) dF(a_{t+1}) + V_{t+1} \right] \tag{14}$$

where the firm gets profits from production plus the continuation value of leaving the period in an employment relationship.

The value of an unfilled vacancy (V_t) is driven down to zero due to the assumption of free entry of firms. This gives then the vacancy posting condition

$$\frac{\kappa}{q_t} = E_t\beta_t(1-\rho_x)\int_{\widetilde{a}_{t+1}}^{\overline{a}} J_{t+1}dF(a_{t+1}) \tag{15}$$

where vacancies are being posted until the expected future payoffs exactly equal the effective costs (κ/q_t).

When deciding whether or not to separate the match weighs the payoffs of staying in the relationship against the outside options. Hence, the employment relationship continues with production when $W_{i,t}+J_{i,t} > U_t$. In other words, the threshold value $\widetilde{a}_t$ is such that it makes the employment relationship exactly indifferent between continuing and separating

$$z_t\widetilde{a}_t - b + E_t\beta_t(1-\rho_x)(1-f_t)\int_{\widetilde{a}_{t+1}}^{\overline{a}} (W_{t+1} - U_{t+1} + J_{t+1})dF(a_{t+1}) = 0 \tag{16}$$

Given $\widetilde{a}_t$ the endogenous separation rate is $F(\widetilde{a}_t)$ and total separations are defined as

$$\rho_t = \rho_x + (1-\rho_x)F(\widetilde{a}_t) \tag{17}$$

4.1.5 Wage bargaining

Wages are assumed to be set according to Nash bargaining and therefore they are the solution to $W_{i,t} - U_t = \eta/(1-\eta)J_{i,t}$, where η is the bargaining power of workers. Using equations (12) to (15) one can obtain the following expression for the wage

$$w_{i,t} = \eta(z_t a_i + \kappa\theta_t) + (1-\eta)b \tag{18}$$

where $\theta_t = v_t/u_t$ is labor market tightness. The wage rate is a weighted average of the firms revenues and savings on hiring costs and the foregone outside option, where the weights are determined by the relative bargaining strengths.

4.1.6 Closing the model

Let n_t be the number of employed workers in period t. Then the law of motion for unemployment is given by

$$u_{t+1} = (1 - f_t(1-\rho_{t+1}))u_t + \rho_{t+1}n_t \tag{19}$$

Tomorrows unemployment pool consists of agents that were unsuccessful in finding a job (either because they did not match with a vacancy, or they did, but were not productive enough), plus newly separated workers that were employed in the previous period. Note that agents that met a vacancy, but in the end did not start a production relationship ($f_t\rho_{t+1}u_t$) are not counted as separated. Rather, I denote these workers as rejected and hence also ρ_t is the rejection rate. In this model the rejection rate is identical to the separation rate. Normalizing the labor force to 1 then gives $1 = u_t + n_t$.

Finally, output is determined by

$$y_t = z_t n_t G(\widetilde{a}_t) \tag{20}$$

where $G(x) = E_t[a|a \geq x] = \int_x^{\overline{a}} a \frac{dF(a)}{1-F(x)}$ is the average productivity of workers with an idiosyncratic draw above x.

4.2 Match efficiency fluctuations

The probability that an unemployed worker finds a job (produces in the next period) is given by $f_t^* = f_t(1 - \rho_{t+1})$ (similarly, the probability that a firm fills a vacancy is $q_t^* = q_t(1-\rho_{t+1})$). Making the matching function explicit one can write $f_t^* = A(1-\rho_{t+1})u_t^{1-\mu}v_t^{\mu}$. One can then define measured match efficiency as

$$A_t = A(1 - \rho_{t+1}) \tag{21}$$

Therefore, unless the rejection rate is constant measured match efficiency varies over time. In other words, in the model agents that get matched with a vacancy, but are not productive enough to start working never actually leave unemployment. Hence, they are not counted as separated, but rather they contribute to a lower job finding rate. Equation (21) provides a direct model counterpart to the match efficiency estimates from section (3).

4.3 Calibration and simulation

In this section the model is calibrated, solved with first-order perturbation techniques and simulated. In the next section the model is then evaluated on the basis of comparing the second moment of measured match efficiency and its empirical counterpart.

To facilitate the exposition of the calibration I divide the parameters of the model into three groups - first, relatively standard parameters in the literature, second parameters that are calibrated to values estimated in the empirical part or found elsewhere in the literature and third, parameters which are calibrated to match statistics in the data. All the parameter values are summarized in table (3).

Standard choices are made for the first group of parameters consisting of the discount factor β, the coefficient of relative risk aversion γ, the standard deviation and persistence of the aggregate productivity shock σ_z and ρ_z, and finally the mean of the idiosyncratic productivity distribution, μ_F.

The second group consists of the exogenous separation rate ρ_x, the elasticity of unemployment in the matching function μ and the bargaining power of workers η. The exogenous separation rate is set to 68% of total separations as in den Haan, Ramey, and Watson (2000). μ is set to 0.65, the value found in the empirical part. Finally, the bargaining power of workers η is set such that the Hoisos condition is satisfied.

The third group contains the value of leisure and home production b, match efficiency A, the flow cost of vacancies κ and the standard deviation of the worker specific productivity distribution σ_F. These parameters are selected to match the mean job finding probability from Shimer (2007) being used in the empirical part (45.4%), an unemployment rate of 12% comonly used in the literature and following den Haan et al. (2000) the mean vacancy filling probability of 71% and the relative standard deviation of separations to output $\sigma(\rho)/\sigma(y) = 3.59$.

I simulate the economy $1,000$ times. Each time $1,237$ quarters are simulated and the first $1,000$ dropped to obtain 237 quarters as in the empirical part. The simulated data are detrended with an HP filter with smoothing coefficient $1,600$ and then the standard deviations and the correlation matrix are calculated for each of the 1000 simulations. The reported statistics are averages over the $1,000$ simulations.

4.4 Model performance

Figure (4) shows the impulse responses to a positive one standard deviation shock to aggregate productivity. All workers become more productive and therefore the minimum requirement on idiosyncratic productivity (the threshold) $\widetilde{a}_t$ falls. This is directly reflected in a fall of the separation rate ρ_t, which leads to a fall in unemployment (also on impact), and a rise in employment and output. At the same time labor market tightness θ_t rises, which together with a fall in the rejection rate makes the job finding probability rise which reinforces the fall in unemployment. Notice, that within a few periods vacancy posting turns negative. This is because the indirect effect of lower unemployment on vacancies dominates. The result is a counterfactually positive relationship between vacancies and unemployment (the Beveridge curve).[17] The correlation coefficient in the model is 0.11, while in the data one finds a value close to -0.9.

Table (4) compares the second moments of labor market variables relative to the volatility of output from the simulated model and the U.S. economy. The model does poorly in explaining the volatilities of labor market variables (as pointed out by Shimer (2005)). Furthermore, the model is able to explain disappointingly little of the observed match efficiency variation (only about 7%). The reason why it fails so blatently is that the rejection rate is related one-for-one with the separation rate. Hence, calibrating the separation rate completely pins down the rejection rate properties as well. Since the average level of separations (ρ) is low and the volatility moderate, the volatility of $1-\rho$ (determining match efficiency) is bound to be small.

[17] Note, that the upward sloping Beveridge curve is a property of the current calibration. One can find parameter values for which the unemployment-vacancy relationship is negative. Specifically the parameters need to be such that when aggregate productivity increases the benefits from future employment relationships dominate the effect of a smaller unemployment pool.

5 Endogenous separations model with firing costs

To break the one-to-one relation between the separation rate and the rejection rate I introduce firing costs for workers in existing employment relationships. This drives a wedge between the two rates making the rejection rate higher. With positive firing costs firms require a higher minimum productivity level from unemployed workers as a compensation for expected future firing costs. At the same time, workers in existing relationships are protected, because the firm suffers firing costs in case of their dismissal.

Although the improved performance of this model rests on the distributional assumption of idiosyncratic shocks, I argue that it is not implausible. The crucial mechanism is that the firms' hiring decision is shifted into an area where the mass of workers is more dense and therefore any fluctuation of the productivity threshold affects more unemployed workers. In other words, all that is needed is that the mass of workers be increasing with the level of idiosyncratic productivity in the neighborhood of the thresholds. Such a distribution seems more plausible than for example productivity levels being uniformly distributed among the unemployed where the number of unemployed with very low skills would be the same as for unemployed with average skills.

The question of quantitative relevance will of course depend on the steepness of this part of the distribution. The model assumes that the skill distributions are identical for unemployed and employed agents. Although such an assumption is questionable, by matching properties associated with the skill distribution of employed workers (such as the separation rate volatility), one disciplines the skill distribution of the unemployed that is crucial for the result. Introducing an additional degree of freedom by modeling a different skill distribution for the unemployed could easily strengthen the results. However, finding empirical counterparts that such a distribution could match is challenging and therefore this route is not taken up here.

In what follows I describe the endogenous separations model adjusted for firing costs. The household problem, matching process and the nature of the employment relationships stay the same as in the benchmark model. However, the hiring and firing decisions now depend on the extra costs that need to be incurred when an existing employment relationship is terminated.

5.1 Endogenous separations

The value of being in the unemployment pool (U_t) at period t is given by

$$U_t = b + E_t \beta_t \left[f_t(1-\rho_x) \int_{\widetilde{a}_{t+1}^N}^{\overline{a}} (W_{t+1}^N - U_{t+1}) dF(a_{t+1}) + U_{t+1} \right] \tag{22}$$

where $\widetilde{a}_{t+1}^N$ is the productivity threshold for newly matched workers. The threshold is the only difference with equation (13) in the standard model.

The value of a job in period t for newly matched and existing workers with idiosyncratic productivity level a_i are

$$W_{i,t}^N = w_{i,t}^N + E_t\beta_t\left[(1-\rho_x)\int_{\widetilde{a}_{t+1}^E}^{\overline{a}}(W_{t+1}^E - U_{t+1})dF(a_{t+1}) + U_{t+1}\right] \tag{23}$$

$$W_{i,t}^E = w_{i,t}^E + E_t\beta_t\left[(1-\rho_x)\int_{\widetilde{a}_{t+1}^E}^{\overline{a}}(W_{t+1}^E - U_{t+1})dF(a_{t+1}) + U_{t+1}\right] \tag{24}$$

where $\widetilde{a}_{t+1}^E$ is the productivity threshold for existing relationships. The only difference between equations (23) and (24) is in the wage rate, which is discussed in the next subsection.

Similarly, the value for the firm of being in a productive employment relationship with a newly hired and existing worker with individual productivity level a_i is, respectively

$$J_{i,t}^N = z_t a_i - w_{i,t}^N + E_t\beta_t\left[(1-\rho_x)\int_{\widetilde{a}_{t+1}^E}^{\overline{a}} J_{t+1}^E dF(a_{t+1}) - F(\widetilde{a}_{t+1}^E)\phi\right] \tag{25}$$

$$J_{i,t}^E = z_t a_i - w_{i,t}^E + E_t\beta_t\left[(1-\rho_x)\int_{\widetilde{a}_{t+1}^E}^{\overline{a}} J_{t+1}^E dF(a_{t+1}) - F(\widetilde{a}_{t+1}^E)\phi\right] \tag{26}$$

where ϕ is the firing cost. The firing cost is assumed to be fully paid by the firm and wasteful. It is thus not a transfer payment to the worker, but rather a tax on the match in the event of separation. Such a specification is justified by the fact that firing costs are (at least partly) of administrative and legal nature, they include for instance loss of efficiency due to disruption of regular work flow etc.

Finally, the value of an open vacancy (imposing the free entry condition) is

$$\frac{\kappa}{q_t} = E_t\beta_t(1-\rho_x)\int_{\widetilde{a}_{t+1}^N}^{\overline{a}} J_{t+1}^N dF(a_{t+1}) \tag{27}$$

The threshold for newly matched workers that determines the lowest profitable idiosyncratic productivity level is such that the surplus of the new match is equal to zero

$$W^N(\widetilde{a}_t^N) + J^N(\widetilde{a}_t^N) - U_t = 0 \tag{28}$$

An analogous reasoning holds for existing employment relationships. However, in this case, one must take into account the firing cost in the outside option of the firm. Essentially, the surplus can be negative up to the value of the firing cost, since the firm saves this cost by holding onto the worker

$$W^E(\widetilde{a}_t^E) + J^E(\widetilde{a}_t^E) - U_t = -\phi \tag{29}$$

5.2 Wage bargaining

Workers coming from the unemployment pool do not posses any contract with the firm from the previous period. Therefore, if they do not come to an agreement with the firm over the wage, no firing costs need to be paid. Assuming Nash bargaining, the wage of newly matched workers is then a solution to $W_{i,t}^N - U_t = \eta/(1-\eta)J_{i,t}^N$. On the other hand, when the firm decides to fire a worker that has been in an employment relationship in the previous period it must pay firing costs. The wage of a worker in an existing employment relationship is then a solution to $W_{i,t}^E - U_t = \eta/(1-\eta)(J_{i,t}^E + \phi)$. Using equations (22) to (27) one can show that the wages of newly hired workers and workers in existing relationships are, respectively

$$w_{i,t}^N = \eta(z_t a_i - \beta(1-\rho_x)\phi + \kappa\theta_t) + (1-\eta)b \tag{30}$$

$$w_{i,t}^E = \eta(z_t a_i + (1-\beta(1-\rho_x)\phi + \kappa\theta_t) + (1-\eta)b \tag{31}$$

where the structure is the same as in the benchmark model. Newly hired workers, however, are penalized because of the threat of having to pay firing costs in the future. On the other hand, workers in existing employment relationships now have a higher wage compared to the benchmark case, because their effective bargaining power increased, since firing them entails a cost for the firm.

5.3 Closing the model

Let the separation rate of existing employment relationships (ρ_t^E) and the rejection rate (ρ_t^N) be defined, respectively

$$\rho_t^E = \rho_x + (1-\rho_x)F(\widetilde{a}_t^E) \tag{32}$$

$$\rho_t^N = \rho_x + (1-\rho_x)F(\widetilde{a}_t^N) \tag{33}$$

Then, the law of motion for unemployment is given by

$$u_{t+1} = (1 - f_t(1-\rho_{t+1}^N))u_t + \rho_{t+1}^E n_t \tag{34}$$

where the only difference compared to equation (19) is that now one needs to distinguish between the separation and rejection rates. Finally, output is determined by

$$y_t = z_t n_t(\omega_t G(\widetilde{a}_t^N) + (1-\omega)G(\widetilde{a}_t^E)) \tag{35}$$

where $\omega_t = \frac{f_{t-1}u_{t-1}(1-\rho_t^N)}{n_t}$ is the fraction of newly employed workers in total employment.

5.4 The effects of firing costs on the productivity thresholds

In the case of zero firing costs the separation rate exactly equals the rejection rate. However, introducing positive firing costs drives a wedge between the two, making the rejection rate larger than the separation rate. This section shows analytically how firing costs increase the idiosyncratic productivity threshold for newly matched workers, while reducing the threshold for workers in existing employment relationships.

The two equations defining the threshold values are equation (29) and (28). First note that one can write the following

$$J^E(a_{t+1}) = J^E(a_{t+1}) - (J^E(\widetilde{a}^E_{t+1}) + \phi) = z_{t+1}(1-\eta)(a - \widetilde{a}^E_{t+1}) - \phi \tag{36}$$

where the first equality follows from the threshold condition (29) and the fact that with Nash bargaining the job value (J^E_t) is proportional to total surplus. The second equality comes from observing that both $J^E(a_{t+1})$ and $J^E(\widetilde{a}_{t+1})$ have all terms common apart from the value of idiosyncratic productivity. Substituting equation (36) into equations (28) and (29) one can obtain analytical expressions for the thresholds.

$$\widetilde{a}^N_t = \frac{1}{z_t}\begin{bmatrix} b + \frac{\eta}{1-\eta}\kappa\theta_t - \beta_t(1-\rho_x)(G(\widetilde{a}^E_{t+1}) - \widetilde{a}^E_{t+1}) + \\ \\ \phi\frac{\beta_t(1-\rho_x)(1+F(\widetilde{a}^E)-\eta)}{1-\eta} \end{bmatrix} \tag{37}$$

$$\widetilde{a}^E_t = \frac{1}{z_t}\begin{bmatrix} b + \frac{\eta}{1-\eta}\kappa\theta_t - \beta_t(1-\rho_x)(G(\widetilde{a}^E_{t+1}) - \widetilde{a}^E_{t+1}) - \\ \\ \phi\left(1-\beta_t(1-\rho_x)\left(1+\frac{F(\widetilde{a}^E_{t+1})}{1-\eta}\right)\right) \end{bmatrix} \tag{38}$$

where $1 + F(\widetilde{a}^E_{t+1}) - \eta > 0$ for any non-negative value of endogenous separations and $1 - \beta(1-\rho_x)\left(1 + \frac{F(\widetilde{a}^E_{t+1})}{1-\eta}\right) > 0$ for low enough values of $F(\widetilde{a}^E_{t+1})$. The steady state effect of firing costs on the threshold for new matches is directly evident from equation (37). Firing costs make the firm demand higher productivity of new matches as a compensation for expected future separations. The opposite reasoning holds for existing matches, where the firm settles for lower productivity levels, because separations now entail a cost. Obtaining an analytical expression for the steady state threshold for existing employment relationships is, however, impeded by the assumption of the log-normal distribution. The appendix shows this steady state effect analytically under the assumption of a uniform distribution for idiosyncratic productivity. Nevertheless, in all the analysis it was always the case that the threshold for existing employment relationships fell with higher values of firing costs.

5.5 The effects of firing costs on second moments of endogenous variables

In this section I keep all parameters at their calibrated values from the benchmark model and gradually increase firing costs. This implies that steady state values of all the endogenous variables need to be recalculated for each value of firing costs. I then solve and simulate the model around the new steady state and calculate the respective second moments.

Figure (5) shows the standard deviations of (logs of) endogenous variables. Notice that, perhaps contrary to ones intuition, introducing firing costs makes both thresholds *more* volatile. With no firing costs the threshold moves according to aggregate productivity changes. In a boom all matches become more productive (and are expected to be more productive in the future due to the persistence of the aggregate shock) and thus firms can keep workers with lower values of a and the threshold falls. This reasoning holds also in the model with positive firing costs. However, a second channel is now operating as well. In a boom, the firm not only expects the employment relationship to be more productive, but it also expects the separation rate to remain low in the future. Therefore, the expected firing costs are lower and every given match is thus more profitable making the threshold fall even further. This additional effect is apparent from the last term in equations (37) and (38) which multiply the firing cost (the appendix shows the increase in volatility explicitly by log-linearizing the threshold expressions around their steady states).

Even though the separation threshold fluctuates more the separation rate variation actually drops with higher firing costs. The reason is that with rising firing costs, the average separation rate falls and thus pushes it into a region with less mass. Then, even though the threshold fluctuates more, the separation rate volatility drops. The opposite is true for the rejection rate which is higher on average with higher firing costs. In this part of the distribution the fluctuations of the threshold are exacerbated.

For example, assuming a uniform distribution of idiosyncratic productivity would mean that a lower average separation rate is not associated with less mass. The volatility of the separation rate would thus increase due to the higher variation of the threshold. At the same time a higher average rejection rate would imply its relatively smaller percentage deviations. This is exactly the opposite to what is needed to explain match efficiency variation while matching the volatility of the separation rate.

Note, however, that the main argument is not that separations are pushed into an area with less mass (in the end the model will be calibrated to fit the separation rate volatility observed in the data). The crucial aspect is that the hiring decision is pushed into an area with more mass. As was argued at the beginning of this section, the assumption that in the neighborhood of the hiring decision the mass of workers increases with the individual productivity level is not unrealistic.[18]

Coming back to figure (5), the lower volatility of the separation rate reflects itself

[18] As a quantitative illustration, the calibration of firing costs taken up in the next section implies that the productivity threshold for unemployed workers increases by almost 15% compared to a situation with no firing costs. Given the distributional assumption, this implies that the fraction of unemployed workers with a lower productivity increases from 1.9% to 4.8%.

in smaller fluctuations of unemployment, employment and output. On the other hand vacancies are more volatile (but not enough to dominate the effect of separations to make employment more volatile). The higher vacancy volatility can be explained by examining the surplus of new matches

$$\begin{aligned} S_{i,t}^N =& W_{i,t}^N - U_t + J_{i,t}^N = \\ & z_t a_i - b + \beta(1-\rho_x) \int_{\widetilde{a}_{t+1}^E}^{\overline{a}} S_{t+1}^E - \frac{\eta}{1-\eta} \kappa \theta_t - \beta(1-\rho_x)\phi \end{aligned} \tag{39}$$

where $S_{t+1}^E = W_{i,t}^E - U_t + J_{i,t}^E + \phi$ is the existing match surplus. Equation (39) shows that with $\phi > 0$ for any level of idiosyncratic productivity a_i the match surplus is lower and thus more sensitive to aggregate fluctuations as discussed in Hornstein, Krusell, and Violante (2005). As mentioned earlier, Nash bargaining implies that the firms job value of a new match is proportional to the total surplus of the relationship. The job value then directly enters the vacancy posting condition and hence, vacancies become more sensitive to aggregate fluctuations.

Finally, higher firing costs make the job destruction margin relatively less attractive and thus firms start using hiring instead. This implies that the counterfactual positive relationship between unemployment and vacancies in the benchmark model is completely overturned (with $\phi = 0.2$ the correlation coefficient is -0.68).

6 Results

6.1 Calibrating firing costs

So far I have analyzed the effects of firing costs without an attempt to evaluate them quantitatively. This section provides a calibration of the firing costs. The Employment protection legislation index (EPL) published by the OECD is a comprehensive indicator and more precise than other alternatives[19]. It is a weighted average of indicators capturing protection of regular workers against individual dismissals, requirements for collective dismissals and regulation of temporary employment. However, one needs to translate this index into a suitable model parameter. Bentotila and Bertola (1990) provide estimates of firing cost for France, Germany, Italy and the UK in the period between 1975 and 1986. Assuming that the EPL is proportional to the estimates provided by Bentotila and Bertola, one can get an estimate of firing costs for the U.S., since EPL data is readily available for the above countries and the U.S. economy. I take the UK estimate as a benchmark assuming that its institutional environment is closest to that of the U.S. economy. The implied firing costs[20] are 4.47% of annual wage. Hence, firing costs are set to

[19]For instance compared to the hiring and firing costs calculated by the World Bank in its "Doing Business studies", the OECD indicator both covers a larger range of relevant aspects of LTC, and has more precise and differentiated sub-indicators.

[20]Using the "regular employment" EPL index.

$\phi = 4 * 0.0447 * \overline{w}^E = 0.179 * \overline{w}^E$ for a quarterly model, where $\overline{w}^E$ is the steady state wage for workers in existing employment relationships.

Using the above value for the firing costs I recalibrate the match efficiency parameter A, the flow cost of vacancies κ and the exogenous separation rate ρ_x to match the average job finding rate, unemployment rate and the vacancy filling rate. Although there is no longer a one to one relationship between separations and rejections, they are both still determined by the same distribution F. To discipline the properties of this distribution I change its standard deviation (σ^F) such that I match separation volatility in the data. The resulting parameter values are: $\sigma^F = 0.297$, $A = 0.594$, $\kappa = 0.119$ and $\rho^x = 0.059$. All the other parameters are as in the benchmark specification.

6.2 Impulse responses and business cycle statistics

Figure (6) shows the impulse response functions for this calibration. First, one can again see the effects of the distribution. The productivity threshold for new matches falls by slightly less than that for existing matches. However, the separation rate of new matches drops much more relative to the one for existing matches. This is due to the distribution assumption as discussed earlier. Second, vacancies now increase more strongly and remain positive. This means that the counterfactual positive sloping Beveridge curve disappears (correlation between vacancies and unemployment is -0.59). The correlation is weaker than in the data, but it can be strengthened by calibrating the matching elasticity μ to lower values.

Table (5) compares model standard deviations with those of the benchmark with zero firing costs and those in the US economy. The decoupling of the separation and rejection rate makes it possible to explain a greater portion of match efficiency fluctuations, while still fitting the separation rate fluctuations. The model calibrated in this way can explain 58% of the volatility found in the data.

Moreover, the model now comes closer to the data than the standard endogenous separations model in all respects. In the case of employment and the job finding rate, the model actually exaggerates the volatility slightly. Note that this is not in contradiction to the previous section that described how higher firing costs dampen responses of most of the endogenous variables. The values reported in the table are relative values to the volatility of output. It shows that output volatility falls relatively faster with higher firing costs than that of other variables.

Using other countries than the UK to calculate the firing costs one obtains slightly higher values for ϕ. As a robustness check, I redo the above experiment with $\phi = 0.25$ (last column of table (5)). The relative standard deviations of all variables increase even more. Under this calibration the model already exaggerates the volatility of match efficiency compared to what was found in the data. Similarly, vacancy volatility is too high and especially the volatility of the job finding rate is more than 1.5 times higher than in the data.

6.3 Model based match efficiency

The previous section showed that the model can account for a sizable portion of match efficiency variation. Another way to view this is to compare the estimated match efficiency with its model based counterpart. To this end I use data on real GDP (logged and detrended with a quadratic trend) and back-out the implied aggregate productivity shock. This is done by inverting the policy function obtained when solving the model. I then use this shock series to simulate the model.

Figure (7) compares the estimated match efficiency and the one implied by the model using the backed-out technology shock[21]. The model-based match efficiency series follows the estimated reasonably well (correlation coefficient 0.57), although it lacks in volatility. The latter is to be expected since the model underpredicts in this dimension. The GDP data used is longer than the estimated match efficiency series and thus gives a hint of its developments outside the empirical sample.

The model-based match efficiency experiences a substantial fall (over 5%) in the most recent recession and even captures the recovery in late 2009. However, the recent recession was not exceptional in terms of the severity of the match efficiency drop. Similar or even slightly larger falls occured in the 82 recession and also in the earlier ones in 1958 and 1960. The model-based match efficiency does not entirely capture the estimated falls in the recessions of the 70's and the 1991 recession. In other words, actual match efficiency was lower than the drop in real GDP would otherwise predict. In the case of the 91 recession it is not surprising since the economy experienced a jobless recovery. Output was picking up, while the job finding rate had not rebounded yet. Since the only shock driving the model is backed out from real GDP, such diverting dynamics cannot be captured by the model. Similarly the recessions in the 70's were not the typical downturns. The oil crises could have brought about structural change that was reflected relatively more in the labor market keeping unemployment higher (job finding rates lower) than would be expected given the fall in output.

Finally, I decompose the variance of the model-based job finding rate into contributions of match efficiency and labor market tightness as in section (3). The model predicts that 38% of the job finding rate fluctuations are driven by match efficiency (both for HP-filtered data with smoothing coefficient 1600 and 10^5). This is only slightly higher than the upper bound of 36% found in the data. This might seem counterintuitive considering that the model can explain only about 60% of the empirical variation in match efficiency. Remember, however, that the model also underpredicts the strength of the vacancy-unemployment correlation as well as their variation. Therefore, one cannot a priori say how well the model will do in terms of the job finding rate variance decomposition.

[21] For comparison the estimated match efficiency was detrended with a quadratic trend and adjusted for its mean.

7 Concluding remarks

This paper tries to further understand the determinants of job finding rate fluctuations. I relax the assumption of a constant matching function and construct an unobserved components model that allows one to obtain estimates of not only time variation in match efficiency, but also a new measure of vacancies for the U.S. economy.

Match efficiency is found to be procyclical and a quantitatively important determinant of job finding rate variation. At business cycle frequencies it is shown to account for 26-35% of fluctuations in the job finding rate. Recessions are thus periods when unemployed workers have a harder time finding jobs not only because vacancies are lower and there are more unemployed workers competing for them, but also because the matching process itself is less efficient.

A search and matching model with firing costs and an explicit distinction between newly hired workers and workers in existing employment relationships is shown to be able to account for about 60% of the observed match efficiency variation. With ex-post heterogeneous workers the job finding probability no longer depends only on the number of unemployed and vacancies, but also on whether or not the worker is productive enough to form a profitable match. The fraction of unemployed workers not suitable to form viable matches then increases in recessions, while the opposite holds for booms. Furthermore, positive firing costs make firms require a higher minimum productivity level from unemployed workers as a compensation for expected future dismisalls. This magnifies the effect of aggregate fluctuations on match efficiency as long as the skill distribution of the unemployed in the neighborhood of the hiring decision is such that the mass increases with the level of skills. I argue that such a property is not unreasonable.

One of the extensions that could strengthen the results and at the same time make the model more realistic is relaxing the assumption of intertemporal independence of the idiosyncratic shocks. Persistence of worker skills would lower the chances of separated workers finding a job in the following periods making the rejection rate both higher and more persistent. Given volatility of the rejection rate, a higher level would lead to larger fluctuations in match efficiency.

Finally, although vacancies are not the prime focus of the paper, the obtained empirical estimates are of separate interest. They provide a methodologically consistent vacancy indicator that dates back to the 1950's. The typically used alternative indicators are either too short (like the job openings series from the JOLTS database) or too crude (as in the case of the HWI) and thus not suitable for a comparison of labor market conditions between recent and older recessions. Therefore, the vacancy indicator in this paper deserves further attention. More detailed research and an extension of the dataset to include the most recent recession are left for the future.

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Appendix

A The Kalman filter

The state-space model is summarized by equations (1) and (2), which I rewrite here for convenience:

$$y_t = \Theta_{0,t} + \Theta_{1,t} s_t + \Theta_{2,t} x_t + \epsilon_t \tag{A.1}$$

$$s_t = \Phi_{0,t} + \Phi_{1,t} s_{t-1} + \eta_t \tag{A.2}$$

The Kalman filter recursions can then be written as

$$s_{t|t-1} = \Phi_0 + \Phi_1 s_{t-1|t-1} \tag{A.3}$$

$$P_{t|t-1} = \Phi_1 P_{t-1|t-1} \Phi_1' + Q \tag{A.4}$$

$$Z_t = \Theta_1 P_{t|t-1} \Theta_1' + R + \Theta_1 C + C' \Theta_1' \tag{A.5}$$

$$V_t = y_t - \Theta_0 - \Theta_1 s_{t|t-1} - \Theta_2 x_t \tag{A.6}$$

$$K_t = (P_{t|t-1} \Theta_1' + C) Z_t^{-1} \tag{A.7}$$

$$s_{t|t} = s_{t|t-1} + K_t V_t \tag{A.8}$$

$$P_{t|t} = P_{t|t-1} - K_t (\Theta_1 P_{t|t-1} + C') \tag{A.9}$$

where the subscript $t|t-1$ indicates a prediction of the variable for period t, using information available in period $t-1$. Similarly, $t|t$ is the update of the period t forecast, when period t information is revealed.

B Diagnostic tests

The assumption underlying the specified model is that the residuals are normally distributed with constant variance and no serial correlation. Following Durbin and Koopman (2001) one can apply diagnostic tests of these properties to the *standardized prediction errors* defined as:

$$e_t = V_t Z_t^{-1} \tag{B.1}$$

where it then follows that the standard deviation of e_t is approximately 1.

B.1 Serial correlation

One can use the Ljung-Box test to investigate the presence of serial correlation in the residuals. Denote the residual autocorrelation of order k as

$$r_k = \frac{\sum_{t=1}^{n-k}(e_t - \overline{e})(e_{t+k} - \overline{e})}{\sum_{t=1}^{n}(e_t - \overline{e})^2} \tag{B.2}$$

where $\overline{e}$ is the mean of the residuals. The Ljung-Box statistic is then

$$Q(k) = n(n+2)\sum_{l=1}^{k}\frac{r_l^2}{n-l} \tag{B.3}$$

which is $\chi^2(k-w+1)$ distributed, with w being the number of estimated hyperparameters (elements in the disturbance variance matrix). Table (**??**) shows the p-values of this test for different values of k. In all cases the null hypothesis of no serial correlation cannot be rejected.

B.2 Homoscedasticity

The assumption of constant variance can be tested with the following test statistic:

$$H(h) = \frac{\sum_{t=n-h+1}^{n} e_t^2}{\sum_{t=1}^{h} e_t^2} \tag{B.4}$$

where h is typically set to the nearest integer to $n/3$. The statistic then tests whether the variance in the first third of the sample is equal to that in the last third of the sample. This statistic is then $F(h,h)$ distributed. Table (**??**) tests homoscedasticity using the first and last quarter, third and half of the sample. In all three cases the null hypothesis that the two subsamples have the same variance cannot be rejected.

B.3 Normality

The assumption that the standardized prediction errors are normally distributed can be readily tested using the Jarcque-Berra test. The test statistic is defined as

$$JB = n\left(\frac{S^2}{6} + \frac{(K-3)^2}{24}\right) \tag{B.5}$$

where S denotes the skewness and K the kurtosis of the standardized prediction errors. The test statistic is $\chi^2(2)$ distributed. Table (**??**) shows that the assumption of normality is not violated.

C Robustness

C.1 Different state space representations

In the benchmark specification match efficiency was assumed to be an AR(1) process, while the process for vacancies was postulated to be a random walk. In this section I check the robustness of the results against two alternative specifications for the underlying states. First, I estimate the model assuming match efficiency follows a random walk, while keeping the specification of vacancies as in the benchmark model.[22] Second, I retain the AR(1) assumption on match efficiency, but I allow for a richer non-stationary structure for vacancies. Namely, I assume that the first difference of vacancies follows an AR(2) process. The level of vacancies can then be written as

$$v_t = (1 + \rho_1)v_{t-1} + (\rho_2 - \rho_1)v_{t-2} - \rho_2 v_{t-3} + \eta_t^v \tag{C.1}$$

Table (6) shows the estimated parameters for the benchmark model and the two alternative specifications. All specifications deliver very similar results. Figure (8) shows the Kalman smoothed states for the three specifications. As with the model parameters, the smoothed states are also very close to each other.

C.2 Estimating on subsamples and with different frequencies

Here I use two different subsamples to check whether the results are not driven just by a certain part of the data. The first subsample uses data after 1970 and the second data after 1985. Figure (9) shows the Kalman smoothed states for the subsamples together with the benchmark. Table (7) then shows the estimated parameter values. There are slight differences in the parameters, but they are also estimated with less precision as one discards more data points. Overall, the dynamics of the states are quite robust over the different samples.

Furthermore, virtually identical results are obtained using monthly frequencies. In this case the exogeneity tests suggest 4 lags as the appropriate instrument. This is also consistent with the quarterly tests.

D Endogeneity

A valid concern is that there are endogeneity problems in the first observation equation. Therefore, the model in the main text is estimated using lagged values of the regressor as an instrument, which is typically done in the literature. Such an instrument is valid only if there is no serial correlation in the residual. The Durbin-Watson statistic in a regression of the job finding probability on a constant and the estimated labor market tightness when using contemporaneous values (assuming constant matching function parameters) is 1.98.

[22]This specification makes it harder to identify the two states, because both have the same process. To help with this issue I use information from the benchmark for the starting values of the Kalman filter.

Similarly, the Breusch-Godfrey test for serial correlation cannot reject the null hypothesis of no autocorrelation in the residuals. In addition, the Hausmann test on exogeneity of instruments cannot reject the null hypothesis of exogenous instruments at the 40% level when the instrument is the first lag of unemployment.

E Steady state effect of firing costs

Assuming a uniform distribution over idiosyncratic productivity levels a and normalizing its lower bound to 0, the steady state threshold level for existing matches can be shown to be

$$\widetilde{a}^E = \frac{b + \frac{\eta}{1-\eta}\kappa\theta - \beta(1-\rho_x)\frac{\overline{a}}{2} - \phi(1-\beta(1-\rho_x))}{1-\beta(1-\rho_x)(1/2 + \frac{\phi}{(1-\eta)\overline{a}})} \tag{E.1}$$

where $\overline{a}$ is the upper bound of the uniform distribution. Since $1-\beta(1-\rho_x) > 0$ then for the threshold to fall with higher firing costs it must be that $1-\beta(1-\rho_x)(1/2+\frac{\phi}{(1-\eta)\overline{a}}) > 0$. This depends not only on the extent of the firing costs, but also on the width of the uniform distribution. It holds true as long as $\frac{\phi}{\overline{a}} < \frac{2-1}{2\beta(1-\rho_x)}(1-\eta)$. For example, assuming a tight distribution, where the upper bound is 1, then firing costs need to be smaller than 0.194. For comparison with the benchmark model, one needs to multiply this value by 2, since average idiosyncratic productivity is half of what it is in the main text.

F Log-linearized threshold equations

I log-linearize equations (37) and (38) around their steady states. "Bared" variables indicate steady states and "hatted" variables indicate log deviations from steady state values:

$$\widehat{\widetilde{a}}_t^E = \sum_{j=0}^{\infty}[\beta(1-\rho_x)\chi]^j \left(-\widehat{z}_{t+j} + \frac{\eta}{1-\eta}\frac{\kappa\overline{\theta}}{\overline{a}^E}(\widehat{v}_{t+j} - \widehat{u}_{t+j}) - \beta(1-\rho_x)\frac{G(\overline{a}^E)-\overline{a}^E}{\overline{a}^E}\widehat{z}_{t+1+j}\right) \tag{F.1}$$

$$\widehat{\widetilde{a}}_t^N = -\widehat{z}_{t+j} + \frac{\eta}{1-\eta}\frac{\kappa\overline{\theta}}{\overline{a}^N}(\widehat{v}_{t+j} - \widehat{u}_{t+j}) - \beta(1-\rho_x)\frac{G(\overline{a}^E)-\overline{a}^E}{\overline{a}^N}\widehat{z}_{t+1+j} + \beta(1-\rho_x)\chi\frac{\overline{a}^E}{\overline{a}^N}\widehat{\widetilde{a}}_{t+1}^E \tag{F.2}$$

where $\chi = 1+\phi\frac{f(\overline{a}^E)}{1-\eta} - \frac{f(\overline{a}^E)(G(\overline{a}^E)-\overline{a}^E)}{1-F(\overline{a}^E)}$ and $f(x)$ is the log-normal probability density function evaluated at x. The firing cost shows up in the parameter χ, which is part of the discount factor in equation (F.1). The variance of the threshold is the infinite sum of variances of all the components plus their covariances. Since the discount factor multiplies all of these components, the higher the firing costs, the higher the variance of the threshold. A similar argument goes through for the productivity threshold of newly matched workers.

G Tables and figures

Table 1: Parameter estimates

α	-0.671	
	(0.016)	
β	0.354	
	(0.009)	
ρ_α	0.719	
	(0.006)	
$10^3\ R$	1.00	-0.62
	-0.62	1.15
$10^3\ Q$	1.73	-0.07
	-0.07	3.32
serial independence	0.628	
homoscedasticity	0.129	
normality	0.266	

Note: standard errors in brackets and the reported values of diagnostic tests are p-values, where the null hypothesis is a satisfaction of the assumption. Details on the tests used are in the appendix.

Table 2: Contributions to job finding probability volatility

	β^A	β^θ
1^{st}-differenced	0.502	0.498
HP-filtered (1600)	0.346	0.658
HP-filtered (10^5)	0.259	0.742

Table 3: Parameter values

β	0.99	κ	0.137
γ	2	b	0.874
ρ_z	0.95	A	0.566
σ_z	0.007	ρ_x	0.042
μ	0.65	μ_F	0
η	0.65	σ_F	0.320

Table 4: Standard deviations relative to output volatility

	U.S. data	no firing costs
u	8.41	3.47
v	7.95	2.25
n	0.60	0.48
f	4.97	1.44
ρ	3.59	3.59
A	3.58	0.26

Table 5: Standard deviations relative to output volatility

		Model		
	U.S. data	$\phi = 0$	$\phi = 0.179$	$\phi = 0.25$
u	8.41	3.47	5.47	6.83
v	7.95	2.25	7.19	8.87
n	0.60	0.48	0.75	0.94
f	4.97	1.44	5.57	7.83
ρ	3.59	3.59	3.59	3.59
A	3.58	0.26	2.09	3.69

Table 6: Parameter estimates: different state space representations

	α $AR(1)$ v RW	α RW v RW	α $AR(1)$ Δv $AR(2)$
α	-0.671		-0.660
	(0.016)		(0.033)
β	0.354	0.321	0.352
	(0.009)	(0.050)	(0.033)
ρ_α	0.719		0.790
	(0.006)		(0.023)
σ_α	0.042	0.038	0.031
σ_v	0.058	0.060	0.060

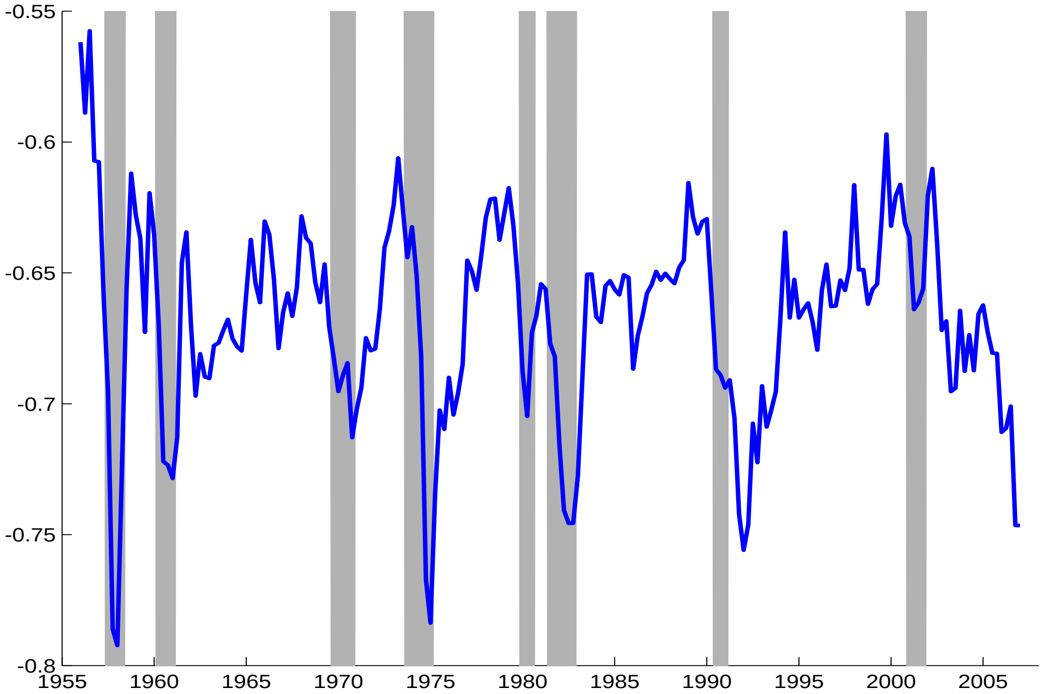
Note: standard errors in brackets.

Table 7: Parameter estimates: different subsamples

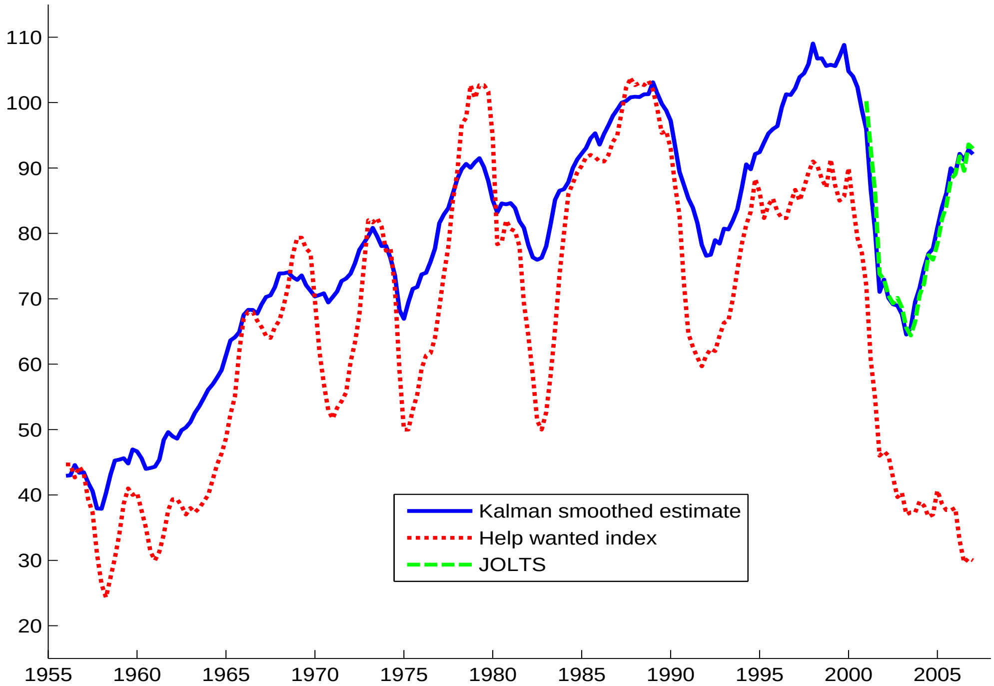
parameters/sample	Full	after 1970	after 1985
α	-0.671	-0.663	-0.659
	(0.016)	(0.033)	(0.044)
β	0.354	0.424	0.524
	(0.009)	(0.045)	(0.123)
ρ_α	0.719	0.804	0.960
	(0.006)	(0.133)	(0.064)
σ_α	0.059	0.054	0.073
$\sigma_v/\sigma_{\Delta HWI}$	0.91	0.85	0.98

Note: standard errors in brackets.

Figure 1: Kalman smoothed states: benchmark



(a) Match efficiency



(b) Vacancies

Figure 2: Job finding probability

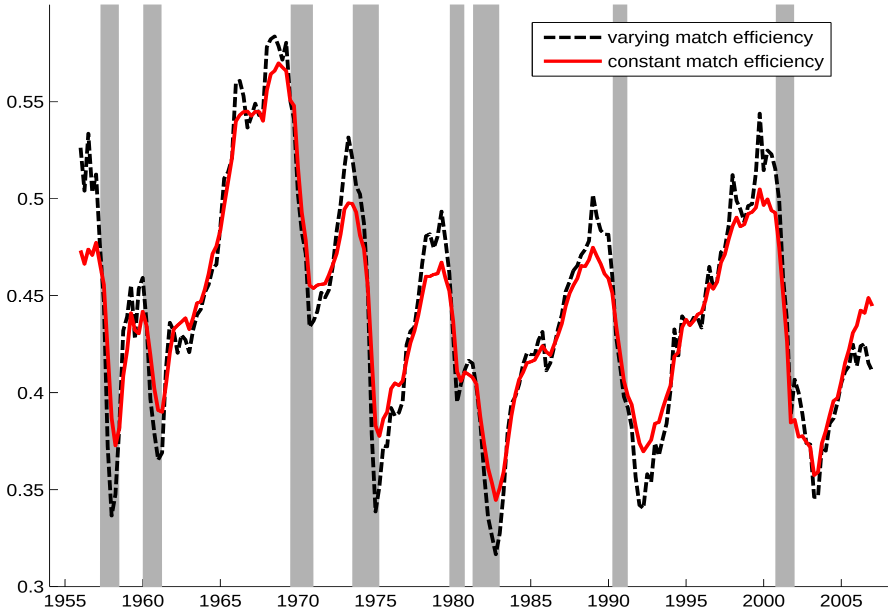


Figure 3: Decomposition of the cummulative drop in the job finding rate

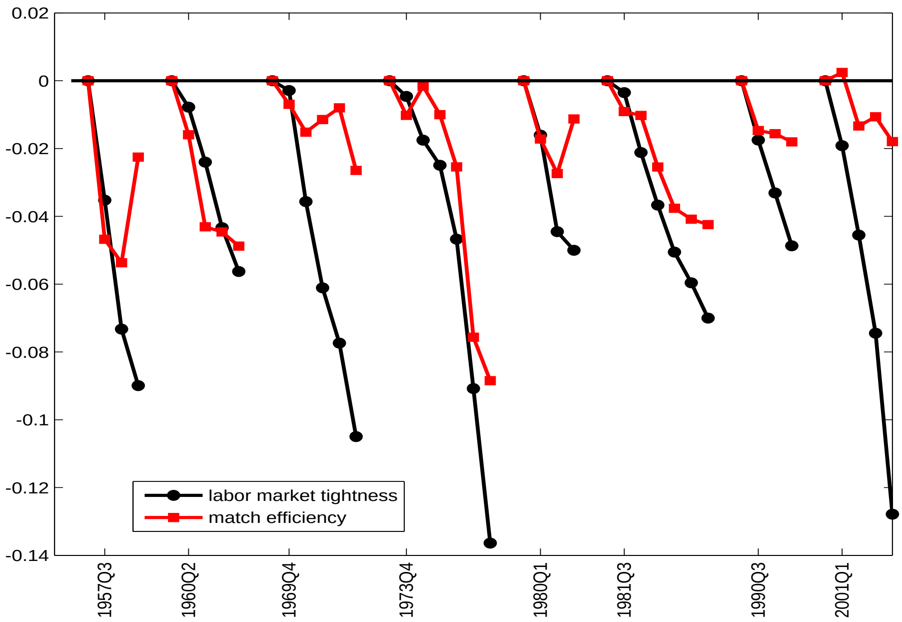


Figure 4: IRFs to a positive one standard deviation technology shock, $\phi = 0$

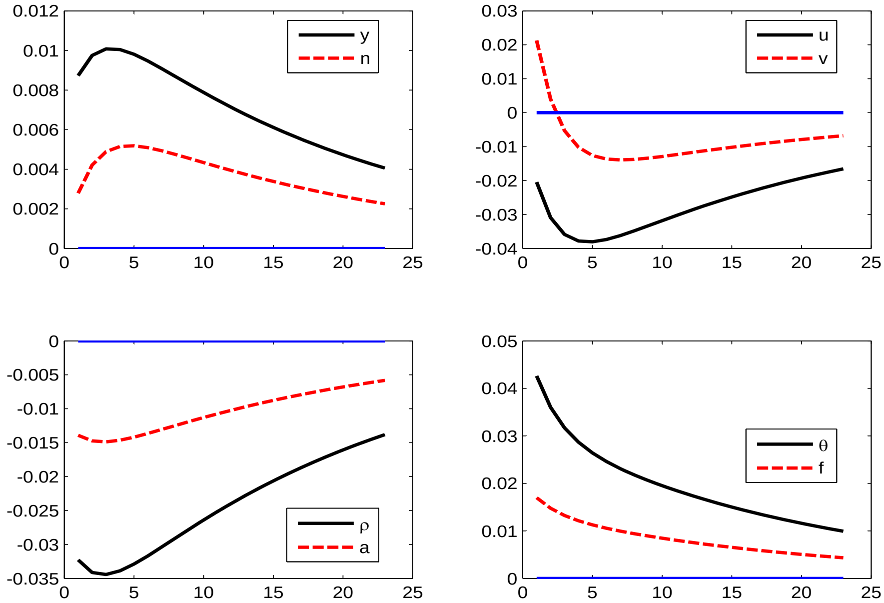


Figure 5: Standard deviations as a function of firing costs

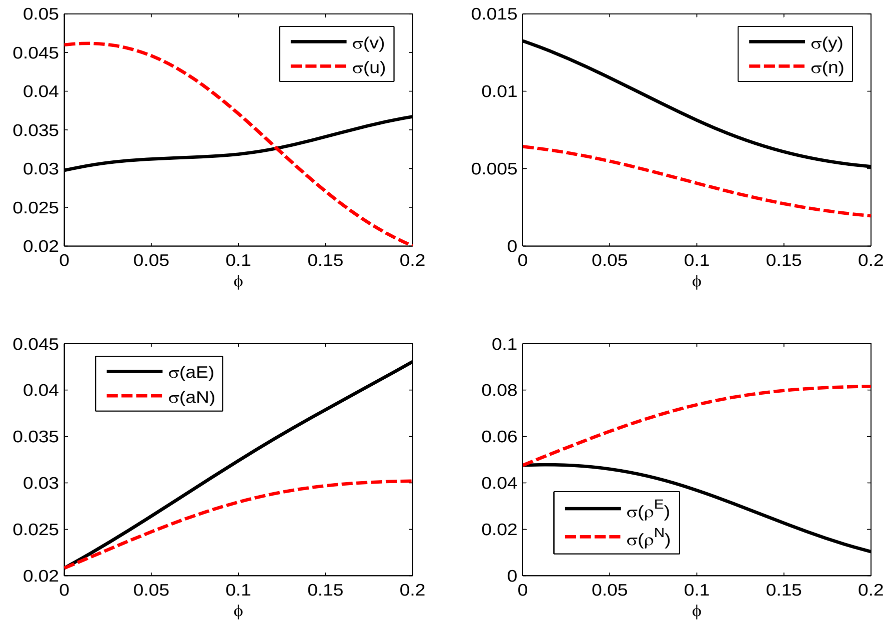


Figure 6: IRFs to a positive one standard deviation technology shock, $\phi = 0.179$

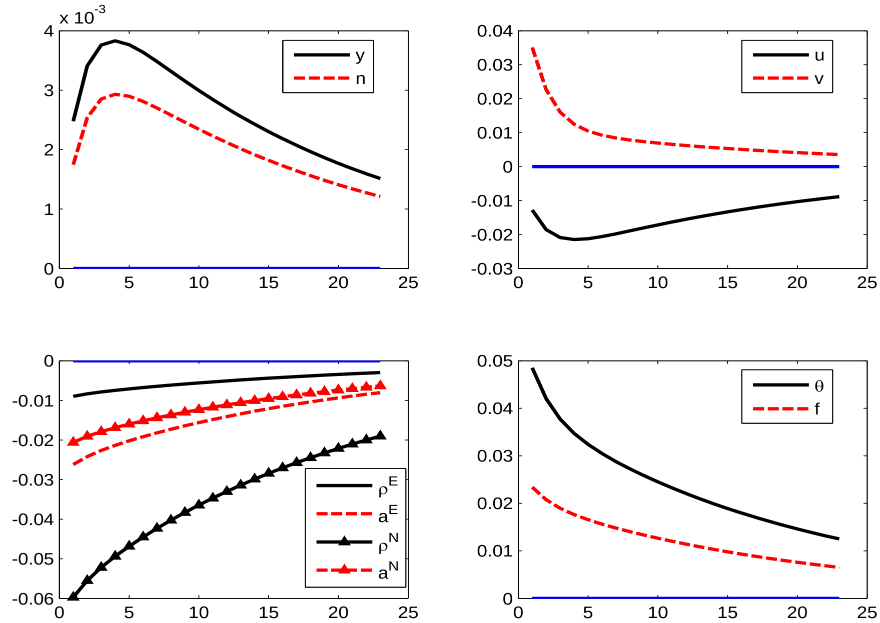


Figure 7: Estimated match efficiency and its model prediction

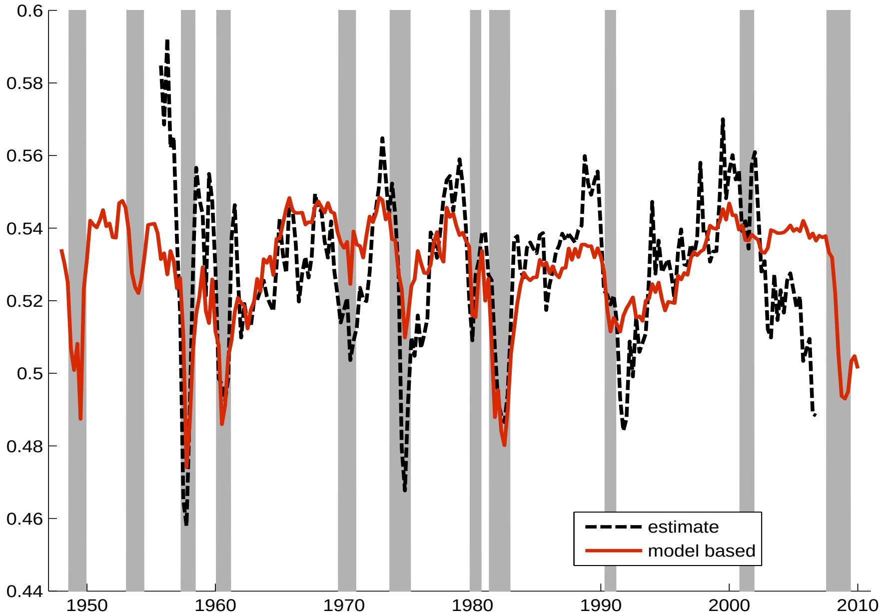
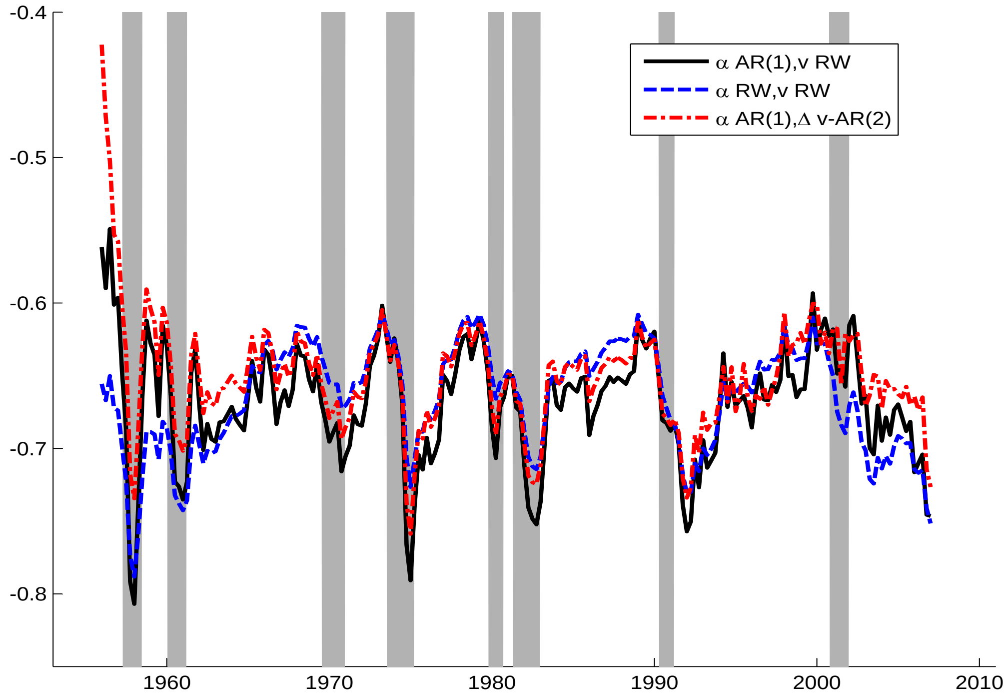
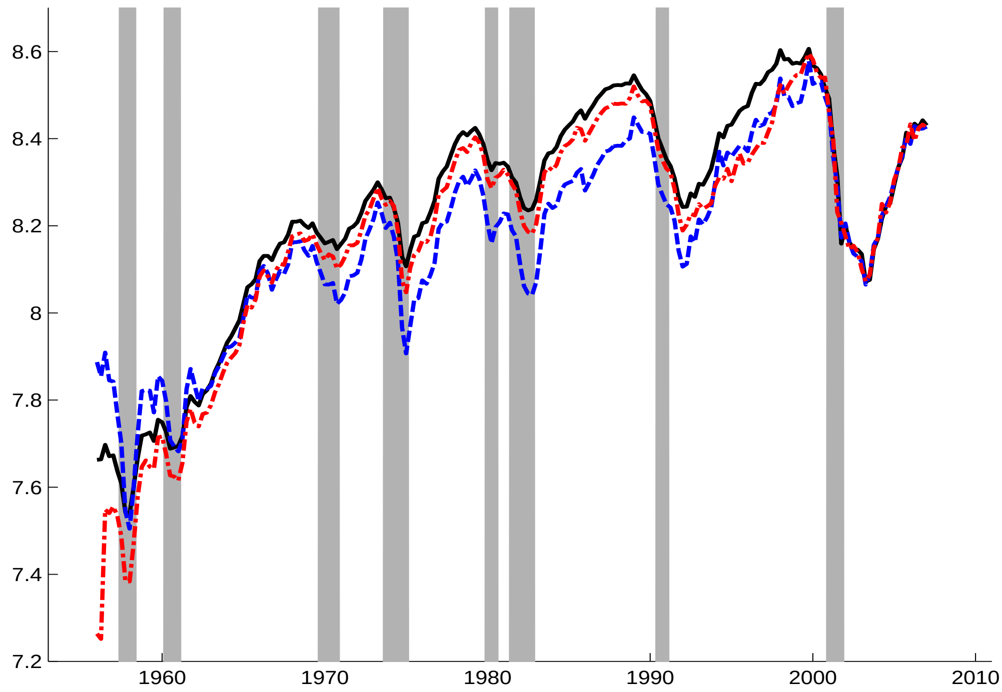


Figure 8: Kalman smoothed states: benchmark and RW specification

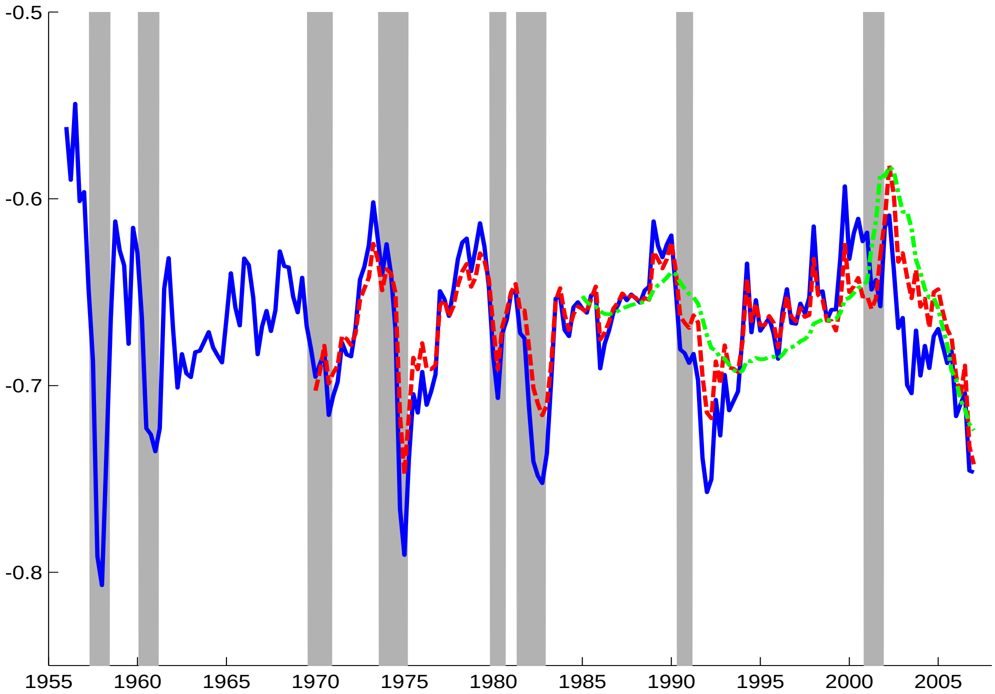


(a) Match efficiency

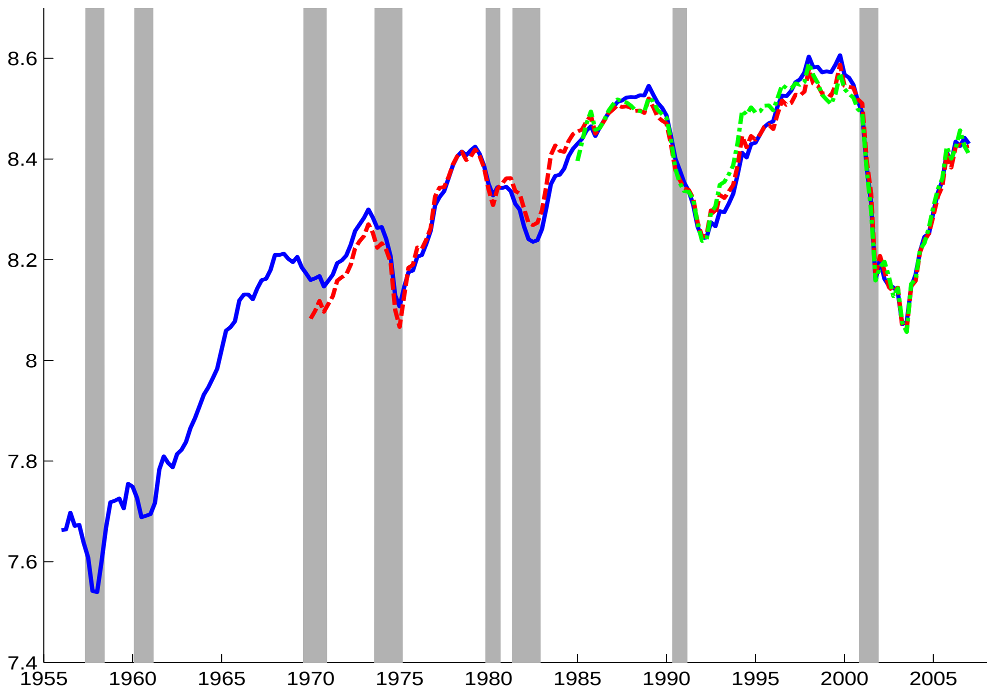


(b) Vacancies

Figure 9: Kalman smoothed states: benchmark and subsamples



(a) Match efficiency



(b) Vacancies