Directed Search, Efficiency Wages and the Diamond Paradox*

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Abstract

In a labour market model with frictions where workers search for jobs and perfectly informed firms set wages, Diamond found equilibrium wages so low that workers cannot even recoup their search costs. Workers in theory then choose not to search at all but do search in practice (the Diamond paradox). We introduce effort and advertisements as choices for workers and firms, respectively. As such, neither resolves the Diamond paradox. The solution we propose combines efficiency wages with wage advertisements. As workers would not exert effort after a firm reneged on its advertisement, sufficiently patient firms can credibly advertise wages that make participation worthwhile for workers. By requiring a certain sign-up bonus or base wage, workers can ensure that only sufficiently patient firms participate. In equilibrium, workers then do engage in search and may earn positive rents. These results do not depend on a commitment assumption or on any form of competition.

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"If your wage is cut a bit, you'd start to misbehave in your job, but you wouldn't leave."

C. A. Pissarides[1]

1 Introduction

The most extreme inefficiency that can occur in a market where there are gains from trade is complete market breakdown. This is precisely the fate that Diamond's (1971) results imply for labour markets with frictions where perfectly informed firms set wages and job search is costly for workers. Paradoxically, real-world labour markets often broadly fit this description and yet do not break down. The resolutions of this paradox in the literature appear based on imperfectly informed firms or somehow ensure that firms compete directly for workers despite search frictions. Many other contributions, notably the directed search literature, circumvent the Diamond paradox by ad-hoc assumptions. In this paper, we propose a solution based on a combination of efficiency wages and advertisements.

The problem. Consider a labour market where firms set wages and job seekers can visit one firm at a time at some search cost. Once the job seeker has come to a firm, her search costs are sunk and do not affect the firm's behaviour. A perfectly informed firm then offers only the worker's reservation wage, that is, a wage as high as unemployment income.[2] Since the job seeker is thus not even reimbursed for the search costs, as a rational forward-looking agent she will choose to avoid these costs by not engaging in search in the first place. As this conclusion extends to any job seeker, there is then no functioning labour market. We refer to this result of theoretical market breakdown as the *Diamond paradox*.[3]

This result is a paradox because many labour markets in the real world do function although Diamond's (1971) setting seems to apply to them: firms set wages and have a good idea about reservation wages, job seekers incur search costs, and yet they engage in search. Hall and Krueger (2008) survey newly hired workers and report that roughly half of these workers were made a wage offer only after their new employer learnt their previous wage. In theory, however, Diamond's (1971) result has proven surprisingly robust, see for example Hopkins and Seymour (2002). Section 3.1 will demonstrate it in very simple terms within our model.[4]

[1]On April 9th, 2008 in Birmingham.

[2]Diamond's (1971) original model features buyers and sellers, with the finding that the seller can behave as a monopolist and set the profit-maximising price. Provided the seller is perfectly informed, the price is set equal to the buyer's valuation.

[3]In the literature, the degenerate wage distribution implied by Diamond's (1971) results is also often associated with the term "Diamond paradox". We use the definition given by Mortensen and Pissarides (1999): "The Diamond paradox is easily stated: there is no equilibrium in which exchange takes place if the cost of search is strictly positive. Instead, no workers participate." (See Mortensen and Pissarides (1999), p. 2609.) A more elaborate version of this definition is found on p. 2608.

[4]Simple demonstrations in different models can be found in Mortensen and Pissarides (1999) and in Gaumont et al. (2006).

Answers in the literature. An influential article by Burdett and Mortensen (1998) is often mentioned as a solution to the Diamond paradox. However, it is now known that their model of on-the-job search relies heavily on the assumption that firms do not know the employment status of job seekers visiting them; see Postel-Vinay and Robin (2002) and Stevens (2004). Carrillo-Tudela (2009) surveys empirical evidence against this assumption.

Another oft-cited response are efficiency wages, be it from the branch with heterogeneous workers, be it from the branch that models effort explicitly as a choice variable. From the first branch, particularly Albrecht and Axell (1984) has been associated with responses to the Diamond paradox (although they themselves do not mention the paradox). Yet it is well known now that neither this contribution nor many similar models resolve the paradox.[5] Essentially, Albrecht and Axell (1984) can explain why certain unmatched workers are willing to engage in search *given* that other unmatched workers do so. To solve the Diamond paradox, however, one has to show why these other unmatched workers search in the first place.

From the second branch of efficiency wage models, Shapiro and Stiglitz (1984) are sometimes considered a response to the Diamond paradox. In their model, firms have to pay wages that leave rents to workers because workers would otherwise shirk. However, we show in section 3.4 below that their model can only resolve the Diamond paradox using the assumption of imperfectly informed firms: only if firms are so ignorant of workers' effort that the probability of being caught shirking is sufficiently low, wages to induce effort will be high enough for workers to engage in search. In fact, we find that this kind of efficiency wages as such does not resolve the Diamond paradox either.

Directed search models have introduced wage advertisements into the sort of markets Diamond (1971) considered. Advertised wages allow workers to direct their search to firms advertising wages that at least reimburse them for their search costs. Workers will engage in this directed search - if and only if firms do not renege on their advertisement once a worker has come along. However, we demonstrate in section 3.2 below a clear incentive for firms to renege, so that workers would not engage in any search after all.

To avoid this result, virtually all directed search models assume this problem away by simply saying that firms commit to the wages (or contracts or mechanisms) they advertise.[6] By also leaving unclear how firms might be able to commit, the problem is effectively ignored, with the exception of contributions by Mortensen (2003) and Masters (2005) who explicitly acknowledge the problem. Mortensen (2003) then argues that firms' reputation concerns will lead them not to renege, but there is no room for reputation effects in Diamond's (1971) set-up, nor do real-world workers typically know about a firm's treatment of previous workers. Masters (2005) argues that firms can commit to paying

[5] See Mortensen and Pissarides (1999) and Gaumont et al. (2006).

[6] A contribution by Coles (2001) suggests that firms do not lower the wage ex post because they need to retain their workers. While this is similar in spirit to the ideas developed in this paper, Diamond's (1971) set-up implies that workers can match at most once and thus cannot gain from quitting a job.

the legal minimum wage if they are otherwise liable to heavy fines. While workers will participate if the minimum wage is sufficiently high, there are nevertheless many labour markets without binding minimum wages, and it remains unclear why workers should participate in these markets. In conclusion, models with just advertisements cannot be said to solve the Diamond paradox.[7]

A number of models avoid the Diamond paradox by allowing for multiple offers to workers as in Burdett and Judd (1983), for example. This way, it is ensured that firms compete at a stage where there are no search frictions for the worker. However, as a worker can only visit one firm at anyone time, this requires that the worker receives or already holds a (potential) job offer from a firm that she is not currently visiting. Then the question arises why firms should not renege on such offers once the worker has come to them. That is, again like advertisements, multiple offers require commitment by firms, without saying how firms can commit.

Ensuring commitment might seem like a purely theoretical problem; after all, workers in the real world do not take glossy advertisements literally. Yet this is easily incorporated into our view: such discrepancies may follow commonly known conventions, so that no-one is misled in equilibrium.[8] The problem arises only when workers encounter unpleasant surprises. A second objection might be that real-world firms rarely make a substantially lower wage offer than was indicated before. While that does not mean that firms have no incentive to do so, it seems that they typically do not follow this incentive. If it was better understood why not, this would point to a solution of the Diamond paradox.

Finally, certain labour market institutions trivially resolve the Diamond paradox. For example, unions, binding legal minimum wages, or laws requiring firms to reimburse search costs can ensure that workers engage in search. Apart from violating Diamond's (1971) original assumptions, however, such institutions can only partly explain why labour markets function: many functioning labour markets do not feature any of them.

In conclusion, while both Burdett and Mortensen (1998) and Shapiro and Stiglitz (1984) appear to depend critically on the assumption that firms are imperfectly informed, many other responses to the Diamond paradox seem to depend explicitly or implicitly on the assumption of commitment by firms. Given that neither assumption necessarily holds in real-world labour markets, the question arises whether the Diamond paradox can also be solved when firms are perfectly informed, as they expressly are in Diamond's (1971) original setting, and when they cannot commit.

[7] Our criticism of the commitment assumption is in no way qualified by the finding, due to Coles and Eeckhout (2003), that advertising a fixed wage and committing to it constitutes equilibrium behaviour: apart from this being only one of a continuum of equilibria emerging from Coles' and Eeckhout's model, it remains unclear how firms can commit.

[8] Formally, as long as advertisements can be related to firms' actual wage offers by a one-to-one function, the equilibrium will be the same as with truthful advertisements.

This paper. The solution we suggest in this paper does not violate Diamond's (1971) original assumptions. Instead, we extend the original setting in two notable ways. Firstly, we model the match between firm and worker explicitly as a repeated sequential game in which the firm sets the wage and the worker chooses effort. However, the Diamond paradox is not solved without the second extension, advertised wages (where we exclude competition through advertisements by assumption). This set-up reflects important features of many real labour markets. Hall and Krueger (2008) report from their survey that two thirds of newly hired workers were made a take-it-or-leave-it offer, and that a third had been informed about wages prior to visiting the firm.

Using these extensions, we model how advertisements fix workers' beliefs so that, if the firm reneges, the worker's credible retaliation will be not to work.[9] This may be thought of as "work to rule" or "white strike". Our model thus provides an endogenous explanation why firms do not renege: in equilibrium, sufficiently patient firms advertise wages that make workers' participation worthwhile and optimally choose not to renege on them. To ensure that firms are sufficiently patient, workers in our model require a sign-up bonus (or a fixed base wage) that credibly signals the firm's patience. Hence only sufficiently patient firms will engage in search in equilibrium.

The paper is organised as follows. Section 2 builds the base model. Section 3 demonstrates the Diamond paradox within our model and goes on to show that neither advertisements as such nor efficiency wages as such resolve it. Section 4, however, proposes a solution by combining advertisements and efficiency wages, under the assumption that firms are sufficiently patient. In this solution, workers do engage in search but do not earn rents. Section 5 drops the assumption of patient firms, shows that the market then unravels, and considers sign-up bonuses or base wages as a market response to this. Indeed, section 5 finds that this response will in equilibrium play much the same role as the assumption of patient firms, and that workers may even earn rents in equilibrium. Finally, section 6 concludes.

2 Base model

Our base model is essentially a labour-market version of Diamond's (1971) set-up. Time is discrete with an infinite horizon. The only agents are firms (possibly heterogeneous with respect to time preferences) and homogeneous workers. Each firm is thought of as an organisational unit that can employ at most one worker. Each worker is endowed with an undivisible unit of labour. Firms may match with workers in a marketplace, and matched agents leave the marketplace. There is a possibly large flow of firms with vacancies onto the marketplace and a corresponding flow of unemployed workers. While there are thus flows onto and out of the marketplace, a steady state is not necessary for our analysis.

[9]The idea that messages become relevant by fixing beliefs appears to be spreading, but the only other application in the context of search known to us is Menzio (2007).

Following MacLeod and Malcomson (1989), we think of any period t as being divided into subperiods t^0, t^1, and t^2. Consider a period of the matching process on the marketplace that we call period 0. In subperiod 0^0, new firms with vacancies and unemployed workers flow onto the marketplace. Let U_0 denote the number of workers in the marketplace in subperiod 0^1; this number is the sum of those who flowed in and of those who have remained in the marketplace from previous periods. Likewise, V_0 denotes the number of firms in the marketplace in subperiod 0^1. In 0^1, each firm in the marketplace sends one notification of a vacancy to a randomly chosen worker in the marketplace (provided that $U_0 \geq 1$), at explicit search costs $k_F > 0$ to the firm. For the case that a worker receives more than one such notification in subperiod 0^1, we make the following assumption:

Assumption 1 (No competition). *Among multiple communications from firms, the worker selects one randomly and ignores all others.*

Under this assumption, the expected number $C_0(U_0, V_0)$ of workers contacted through at least one notification follows from an urn-ball process:

$$C_0(U_0, V_0) = U_0 \left(1 - \left(1 - \frac{1}{U_0}\right)^{V_0}\right) \tag{1}$$

Then a worker's probability of being contacted in subperiod 0^1 is

$$\eta_0 \equiv \frac{C_0(U_0, V_0)}{U_0} = 1 - \left(1 - \frac{1}{U_0}\right)^{V_0}$$

Workers thus end up with at most one notification and in this case decide whether or not to visit the firm that notified them. Next, in subperiod 0^2, the worker may come to the firm at time-invariant costs $k_W > 0$. Such search costs may be thought of as writing applications or travel costs on the side of workers and recruitment costs on the side of firms. Those not visiting a firm may exit the marketplace and will otherwise begin a period like period 0 anew.

Once the worker has come to a firm at the end of period 0, period 1 will be an interview period for these two agents, and potentially a match. The interview takes place only in subperiod 1^0, as follows. The firm posts a wage w_1 under perfect information:

Assumption 2 (Diamond's assumptions). *Firms unilaterally set the wage and observe all relevant variables and parameters.*

If the worker rejects the wage offer, she will return to the marketplace. In this case, 1^1 and 1^2 will be like 0^1 and 0^2, respectively, without any possibility to recall previous offers. If the worker accepts the offer, worker and firm immediately exchange labour for the wage and exit the marketplace. We assume implicitly that the gains from trade are more than enough to reimburse both k_W and k_F. The matched agents do not play a role anymore in

1^1 and 1^2. An interview period can thus turn into either a match or into another period on the marketplace, depending on whether the worker accepts or not.

This structure corresponds to Diamond's (1971) original setting where sellers post prices under perfect information and buyers only learn the actual price after coming to the seller. Buyers then choose whether to buy at this price or continue searching. Diamond's setting is expressly intended for a single purchase of some consumer durable, so that there are no repeat purchases and issues around firms' reputation cannot arise. In our context, this translates into just a single match.

We now specify payoffs. All workers discount the future using the discount factor $\delta_W \in (0,1)$, while firms use discount factor $\delta_F \in (0,1)$. Positive payoffs can be earned only in matches. Let $\Pi_t(w_t)$ be the *value* to the firm of being in a match in period t that pays w_t to the worker. The corresponding value to a worker of being in the match is denoted $\Phi_t(w_t)$. Let $\Omega_{t+1}(w_{t+1})$ be the *expected* value to the worker of learning the actual wage w_{t+1} in an interview at the beginning of next period, having come to a firm. The value to the worker of being in the marketplace in period t is found as

$$\Gamma_t = (1-\eta_t)\delta_W \Gamma_{t+1} + \eta_t \max\left[\delta_W \Omega_{t+1}(w_{t+1}) - k_W, \delta_W \Gamma_{t+1}\right] \tag{2}$$

Equation (2) says that the worker is not contacted by any firm with probability $1-\eta_t$; when contacted, however, the worker will still not visit the firm if the expected payoff from doing so does not exceed $\delta_W \Gamma_{t+1}$. Whenever the worker does visit a firm and incurs the search costs, she subsequently obtains the expected value of learning the wage,

$$\Omega_{t+1}(w_{t+1}) = \int \max[\Phi_{t+1}(w_{t+1}), \Gamma_{t+1}] d\Lambda(w_{t+1}) \tag{3}$$

where $\Lambda(w_{t+1})$ is the known distribution of actual wage offers.[10] That is, she will reject the wage offer if she prefers to continue the period unmatched rather than matched at a wage w_{t+1}.

3 The Diamond paradox, advertisements, and effort

3.1 A simple demonstration of the Diamond paradox

Suppose a worker has come to a firm at the end of period t and is in an interview in period $t+1$. By rejecting the wage offer, she would obtain Γ_{t+1}. The firm seeks to minimise its wage costs by offering a wage that leaves the worker indifferent between accepting and rejecting (or an ϵ more to tip the balance), so that

$$\Phi_{t+1}(w_{t+1}) = \Gamma_{t+1}$$

[10] In all equilibria that we consider, $\Lambda(w_{t+1})$ consists of a single wage. Forward-looking workers can therefore easily infer it from the behaviour of individual firms.

in equation (3), implying

$$\delta_W \Omega_{t+1}(w_{t+1}) = \delta_W \Gamma_{t+1}$$

Then equation (2) becomes

$$\Gamma_t = (1-\eta_t)\delta_W \Gamma_{t+1} + \eta_t \max\left[\delta_W \Gamma_{t+1} - k_W, \delta_W \Gamma_{t+1}\right]$$
$$\Rightarrow \Gamma_t = \delta_W \Gamma_{t+1}$$

which means that the worker simply chooses not to visit the firm - after all, the only difference a visit makes is that the worker incurs the search costs k_W. Since exactly the same logic obtains in the next period, the worker again decides against the visit, hence $\Gamma_{t+1} = \delta_W \Gamma_{t+2}$, and so on. The worker always continues to search, never matches, and never earns any payoff, so that $\Gamma_t = \Gamma_{t+1} = \Gamma_{t+2} = \ldots = 0$. This argument generalises to any worker and any interview period. Note that it does not depend on the value of η_t, so that even a single worker without competition from others (i.e. $\eta_t = 1$) would obtain a zero payoff.

In conclusion, no worker actually engages in search, as the search costs would never be recouped. This result of an inactive market as the unique equilibrium when there are search costs (or entry costs) is the Diamond paradox. The root cause of the Diamond paradox are thus search costs that are incurred ex ante and become sunk costs by the time a worker visits a firm, so that the firm has no reason to reimburse the worker for these costs.

3.2 Do advertisements solve the Diamond paradox?

If search costs that are sunk lie at the heart of the Diamond paradox, wage advertisements appear promising as a remedy: if firms advertise wages to workers before they visit a firm, this may bridge the step that makes these costs sunk costs. Workers would then only visit firms whose advertised wage offer suggests that they are reimbursed for their costs. Then only such wage offers attract applicants, which forces firms to advertise such wage offers. This argument, however, critically depends on advertisements being truthful, so that no firm reneges on its advertised wage offer and actually makes a different offer once a worker has come along.

We can formally evaluate this argument by replacing the mere notifications about vacancies in our set-up by advertisements that include a precise wage figure, denoted $\tilde{w}$. To focus on advertised wages as such, we apply assumption 1 also here, so that workers select an advertisement randomly and firms therefore cannot compete in advertisements. Suppose that workers take $\tilde{w}$ at face value. Then we have *directed search* at least in the sense that workers can direct their search away from firms whose $\tilde{w}$ would not reimburse workers for search costs. Denote by $\Omega_{t+1}(w_{t+1}|\tilde{w}_{t+1})$ the expected value to the worker of learning the offered wage in an interview, given that $\tilde{w}_{t+1}$ has been advertised. Since workers take advertisements at face value,

$$\Omega_{t+1}(w_{t+1}|\tilde{w}_{t+1}) = \max[\Phi_{t+1}(\tilde{w}_{t+1}), \Gamma_{t+1}]$$

The best outcome for the firm is that the worker comes to the firm and then works at the lowest acceptable wage. At time t, the firm's optimal choice of $\tilde{w}_{t+1}$ therefore ensures

$$\begin{aligned} \delta_W \Omega_{t+1}(w_{t+1}|\tilde{w}_{t+1}) - k_W &\geq \delta_W \Gamma_{t+1} \\ \Leftrightarrow \quad \delta_W \max[\Phi_{t+1}(\tilde{w}_{t+1}), \Gamma_{t+1}] - k_W &\geq \delta_W \Gamma_{t+1} \end{aligned} \tag{4}$$

Equation (4) can only possibly be satisfied if $\Phi_{t+1}(\tilde{w}_{t+1}) > \Gamma_{t+1}$. By contrast, the firm's optimal choice of w_{t+1} sets, as before,

$$\Phi_{t+1}(w_{t+1}) = \Gamma_{t+1}$$

which implies

$$\Phi_{t+1}(\tilde{w}_{t+1}) > \Phi_{t+1}(w_{t+1}) \quad \Rightarrow \quad \tilde{w}_{t+1} > w_{t+1}$$

In words, the firm's optimal behaviour is *time inconsistent*: it would like to induce worker participation through advertisements, but once workers participate it reneges on these advertisements. Rational workers would be aware of this inconsistency, would therefore distrust firms' advertisements, and would not participate. The result is still the Diamond paradox. Therefore, the numerous contributions that simply assume firms commit to their advertisements essentially assume the Diamond paradox away.

3.3 Do efficiency wages solve the Diamond paradox?

In this section, we introduce workers' effort instead of firms' advertisements into the base model. In order to elicit effort from workers during the match, firms might pay efficiency wages, and we ask whether this resolves the Diamond paradox.[11] We extend the base model by viewing the match as repeated and dynamic productive interaction between worker and firm, rather than a one-off exchange transaction. Production technology is identical across firms and uses labour as the only input. The worker produces perfectly measurable, yet not verifiable, output $p(e_t)$ per period during the match, where the continuous $e_t \geq 0$ denotes effort in period t. Let the price for output be 1, so that $p(e_t)$ is also the monetary value of output. From exerting effort, a worker incurs costs $c(e_t)$ measured as the monetary value of her disutility from effort. We assume that these costs are convex, that output per worker is concave, and that there are gains from trade:

Assumption 3 (Regularity conditions). *Output per worker $p(\cdot)$ satisfies $p'(e) > 0$,*

[11]Models with effort constitute one major branch of efficiency wage models. The other branch, as in Weiss (1980), features differential reservation wages, which we mentioned in the context of Albrecht and Axell (1984).

$p''(e) < 0$, and $p(0) = 0$, while the costs of effort $c(\cdot)$ satisfy $c'(e) > 0$, $c''(e) > 0$, and $c(0) = 0$. Finally, $\exists\ e > 0$ such that

$$\frac{\min[\delta_W, \delta_F][p(e) - c(e)]}{1 - \min[\delta_W, \delta_F]} > k_W + k_F \tag{5}$$

Equation (5) says that both the firm's and the worker's search costs can be recouped by the present discounted value of match production over the infinite horizon, where we discount to the period 0 just before the match because search costs are incurred in that period. Next, the interview period changes as follows. In 1^0, the firm does not only make a wage *offer*, denoted $\hat{w}_1 \geq 0$, but also demands a level of e_1, which is not modelled explicitly. The worker accepts or rejects. If she rejects, she will return to the marketplace as before, so that 1^1 and 1^2 will be like 0^1 and 0^2. If she accepts, the match is concluded and neither worker nor firm can return to the marketplace in 1^1. Instead, the worker then freely chooses effort e_1 and produces for the firm in 1^1. The firm pays the actual wage w_1 only in 1^2.

As we want to model repeated interaction, we introduce a period of a continued match as the third category of periods. While the interview period can turn into the first period of a match, any subsequent periods of the match fall into this third category. Therefore suppose a match was concluded in 1 and consider 2. In 2^0, the firm decides whether to continue the match and, if so, makes the wage offer $\hat{w}_2 \geq 0$. If the firm decides to terminate the match, let $\hat{w}_2 = 0$. Given the wage offer, the worker in 2^1 decides whether to continue the match and, if so, chooses effort e_2 and produces. Again $e_2 = 0$ in case the worker terminates the match. If neither agent terminated the match, wage w_2 is paid in period 2^2; otherwise the match simply ends. Let us summarise the three categories of periods we distinguish:

Period 0: on the marketplace
0^0: new workers and firms flow in
0^1: firms contact workers and incur k_F; the contacted decide whether to visit a firm
0^2: workers who so decided come to a firm and incur k_W
Period 1: interview and potentially first period of a match
1^0: firm offers wage $\hat{w}_1$; worker accepts or rejects
1^1: workers who accepted work; others return to search and pass a subperiod like 0^1
1^2: workers who accepted receive w_1; others pass a subperiod like 0^2
Period 2: second period of a match
2^0: firm decides whether to continue; if so, offers $\hat{w}_2$
2^1: worker decides whether to continue; if so, works
2^2: whenever match was continued, w_2 is paid

The match lasts forever unless either side chooses to terminate it. While agents may terminate the match, they cannot return to a previous stage. As before, only a single match is thus possible for each agent. Note that each period of a match constitutes an extensive game where agents move sequentially in subperiods. The match is thus a potentially infinite repetition of this extensive stage game. In each match period t, their actions $\hat{w}_t \in \mathbb{R}_+$, $e_t \in \mathbb{R}_+$, and $w_t \in \mathbb{R}_+$ combine to an outcome $(\hat{w}_t, e_t, w_t) \in \mathbb{R}^3_+$. The histories in subperiods t^0, t^1, and t^2 can then be expressed as

$$\begin{aligned} h(t^0) &= \{(\hat{w}_1, e_1, w_1), (\hat{w}_2, e_2, w_2), \ldots, (\hat{w}_{t-1}, e_{t-1}, w_{t-1})\} \\ h(t^1) &= h(t^0) \cup \{\hat{w}_t\} \\ h(t^2) &= h(t^1) \cup \{e_t\} \end{aligned}$$

with $h(1^0) = \emptyset$. The firm's preference relation on $\mathbb{R}^3_+$ is represented by its payoff function $\pi : \mathbb{R}^3_+ \to \mathbb{R}$ (i.e. its profit function) so that a weak preference for outcome $(\hat{w}_t, e_t, w_t)$ over $(\hat{w}'_t, e'_t, w'_t)$ implies

$$\pi(\hat{w}_t, e_t, w_t) \geq \pi(\hat{w}'_t, e'_t, w'_t)$$

The worker's preference relation on $\mathbb{R}^3_+$ is similarly represented by a payoff function $\phi : \mathbb{R}^3_+ \to \mathbb{R}$. We specify these payoff functions in a very natural way as

$$\pi(\hat{w}_t, e_t, w_t) = p(e_t) - w_t \quad \text{and} \quad \phi(\hat{w}_t, e_t, w_t) = w_t - c(e_t)$$

The firm's preferences in the infinitely repeated game are defined on the set of infinite sequences of stage game outcomes and satisfy weak separability. Concretely, they are given by δ-discounting, i.e. an infinite sequence of payoffs $(\pi(\hat{w}_t, e_t, w_t))_{t=1}^{\infty}$ is weakly preferred to an infinite sequence $(\pi(\hat{w}'_t, e'_t, w'_t))_{t=1}^{\infty}$ if

$$\sum_{t=1}^{\infty} \delta_F^{t-1} [\pi(\hat{w}_t, e_t, w_t) - \pi(\hat{w}'_t, e'_t, w'_t)] \geq 0$$

and equivalently for the worker's preferences. Next, let us recursively define $\Pi_t(e_t, w_t)$ as the value to the firm of being in a match in period t:

$$\Pi_t(e_t, w_t) = \max[p(e_t) - w_t + (1 - \alpha_t)\delta_F \Pi_{t+1}(e_{t+1}, w_{t+1}), 0] \tag{6}$$

where α_t is the probability that the worker terminates the match before period $t + 1$. Equation (6) says that a firm in a match can choose to continue the match, earning $p(e_t) - w_t$ in t.[12] Then the firm will enter the same situation in period $t + 1$ provided the worker does not terminate the match, and will otherwise be unmatched at the beginning of period $t + 1$, earning 0 forever. Alternatively, the firm can itself terminate the match.

[12]The formulation of equation (6) encompasses the case that the worker terminates the match already in period t: then $e_t = p(e_t) = 0$ and $\alpha_t = 1$.

Similarly, let $\Phi_t(e_t, w_t)$ be the value to the worker of being in a match in period t:

$$\Phi_t(e_t, w_t) = \max[w_t - c(e_t) + (1 - \beta_t)\delta_W \Phi_{t+1}(e_{t+1}, w_{t+1}), 0] \tag{7}$$

where β_t is the probability that the firm terminates the match before period $t + 1$.

We first focus on the stage game in some period t of a match between a firm and a worker. Analysing this game in isolation means treating it like a one-shot game without any repetition. Then the relevant payoffs are just the payoffs from period t (not counting for the moment any sunk costs). In analysing the stage game, which is an extensive game, we employ the concept of a subgame perfect equilibrium (SPE). We find that the unique SPE of the stage game is degenerate, a now well-known result (see Malcomson (1999)) that is proven by backward induction.

Lemma 1 (One-shot game). *In the unique SPE of the one-shot game,* $e_t^* = w_t^* = 0$ *(and* $\hat{w}_t^*$ *indeterminate).*

Proof. In period t^2, the firm takes equilibrium actions $\hat{w}_t^*$ and e_t^* as given because they have been taken in periods t^0 and t^1. As $\pi(\hat{w}_t, e_t, w_t|\hat{w}_t, e_t)$ is then unambiguously falling in w_t, the firm optimally sets $w_t^* = 0$. By backward induction, the worker in subperiod t^1 knows she will receive a wage 0 in any case, and thus maximises $\phi(\hat{w}_t, e_t, w_t|\hat{w}_t, w_t = 0)$ by choosing $e_t^* = 0$. All this is independent of the firm's choice of $\hat{w}_t$ in t^0, so that $\hat{w}_t$ may take any value in $\mathbb{R}_+$ in this equilibrium. □

We are thus left with a situation of no production:

$$\pi(\hat{w}_t^*, e_t^*, w_t^*) = \phi(\hat{w}_t^*, e_t^*, w_t^*) = 0$$

Turning to the SPE of the infinitely repeated game in matches, the following SPE results when players in a match maximise their payoffs period by period:

Lemma 2 (Worst SPE). $\exists$ *a SPE of the infinitely repeated game such that* $e_t^* = w_t^* = 0$ *(and* $\hat{w}_t^*$ *indeterminate) in every period t forever and hence* $\Phi_t(e_t^*, w_t^*) = \Pi_t(e_t^*, w_t^*) = 0$.

Proof. By definition, a SPE must also be a NE because any game is a proper subgame of itself. The SPE of the stage game identified in lemma 1 is therefore also a NE, and the infinite repetition of a NE of the stage game must be a SPE of the infinitely repeated game. It remains to verify that the stage game is in fact repeated infinitely under these circumstances. As $e_t = 0$ in every period t while the match lasts, no surplus is ever generated so that $\Phi_t(e_t^*, w_t^*) = \Pi_t(e_t^*, w_t^*) = 0$, yet terminating the match also only carries a payoff of 0 for each player. Hence neither player prefers to terminate the match, $\alpha_t^* = \beta_t^* = 0\ \forall t$, and the stage game is repeated infinitely. □

Hence a situation of no production each period constitutes a SPE. To prepare later results, we note that neither lemma 1 nor lemma 2 depend at all on the value of the firm's discount factor δ_F. Of course, this equilibrium is unsatisfactory for the players, given that, by assumption 3, a different feasible choice of e_t would lead to a positive surplus that could then be shared in a Pareto-improving way. The firm can achieve this as follows. Consider a match where the firm pays w_t only in return for a certain choice of e_t. The worker will not choose e_t unless she expects the wage to at least cover $c(e_t)$, hence $w_t \geq c(e_t)$ would be necessary. To simplify the notation, we denote by $b_t \geq 0$ any extra wage payments over and above $c(e_t)$. That is, our notation splits up the total wage w_t paid in subperiod t^2 into $c(e_t)$ and b_t:

$$b_t \equiv w_t - c(e_t)$$

Yet $c(e_t)$ and b_t should not be thought of as base wage and bonus, as they are both at the firm's discretion and paid ex post, if at all. As the firm can set b_t, it decides how the surplus is allocated among players. The lowest b_t the firm might promise is $b_t = 0$. Yet no more is needed when the firm *credibly* promises to pay $c(e_t)$ if the worker chooses a specific e_t: then the worker obtains a payoff 0 in any case and is thus indifferent between the firm's offer and not working at all. Because the firm could always pay some ϵ greater than but very close to 0 in order to tip the balance, we henceforth suppose that the worker accepts the firm's offer in case of indifference. As the firm can freely choose which e_t to reward, it will choose a (time-invariant) level we denote by $\bar{e}$:

$$\bar{e} \equiv \arg\max_e [p(e) - c(e)] \tag{8}$$

In other words, the firm seeks to maximise the surplus generated by the worker because the firm will appropriate all of this surplus. The next lemma claims that $e_t = \bar{e}$ and $b_t = 0$ can indeed arise as equilibrium actions in each period of the infinitely repeated game, provided the firm is patient enough.

Lemma 3 (Firm-optimal SPE). $\exists\ \delta_F^* < 1$ *such that for all* $\delta_F \in [\delta_F^*, 1)$*, the profile* $(\hat{w}_t, e_t, w_t) = (c(\bar{e}), \bar{e}, c(\bar{e}))$ *in every period* t *is a SPE of the infinitely repeated game. The worker and the firm respectively obtain payoffs* $\Pi_t(e_t^*, w_t^*) > 0$ *and* $\Phi_t(e_t^*, w_t^*) = 0$.

Proof. Let players use *grim trigger strategies* as follows. In each period t^0, the firm offers $\hat{w}_t = c(\bar{e})$ and demands $e_t^* = \bar{e}$ if there has been no earlier deviation by either side. In each period t^1, the worker chooses $e_t = \bar{e}$ unless there was a deviation. In each period t^2 the firm pays $c(\bar{e})$ unless there was a deviation. If there was a deviation by either side, the worker instead chooses $e_t = 0$, the firm chooses $w_t = 0$ (with $\hat{w}_t$ indeterminate), and likewise in every subsequent period. That is, players switch to the SPE in lemma 2 in case of a deviation.[13] As noted before, any SPE must also be a NE, and therefore the

[13]The only rational direction of a deviation here is downward. If, however, the firm was for some reason nice enough to suddenly pay more, this would also be treated as a deviation. Such anomalies arise

threat of switching is credible. These threats can thus sustain a SPE with $e_t^* = \bar{e}$ and $w_t = c(\bar{e})$ if both players prefer not to deviate, given that the threats would be carried out. The worker will obtain a payoff 0 if there is never a deviation, but would also obtain only 0 after a deviation, and hence does not have an incentive to deviate. As to the firm, it also obtains 0 in the SPE in lemma 2. With $e_t^* = \bar{e}$, the firm here obtains the entire maximised surplus when there is no deviation. From assumption 3, we know

$$p(\bar{e}) - c(\bar{e}) > 0 \tag{9}$$

so that $\Pi_t(e_t^*, w_t^*) > 0$ here. However, the firm can make a short-term gain of $c(\bar{e})$ by deviating (i.e. paying nothing at all). The loss sustained from switching to the SPE in lemma 2 will outweigh the short-term gain if

$$c(\bar{e}) \leq \delta_F \Pi_{t+1}(e_{t+1}^*, w_{t+1}^*) = \sum_{t=1}^{\infty} \delta_F^t [p(\bar{e}) - c(\bar{e})]$$

where, following the one-deviation principle, we do not consider a further deviation. This condition holds with equality at δ_F^*:

$$c(\bar{e}) = \frac{\delta_F^* [p(\bar{e}) - c(\bar{e})]}{1 - \delta_F^*} \quad \Rightarrow \quad \delta_F^* = \frac{c(\bar{e})}{p(\bar{e})} < 1$$

where the last inequality follows from equation (9). That is, δ_F^* is the lowest discount factor the firm may have for this SPE to be sustainable. Hence both players do not deviate if the firm is sufficiently patient. Finally, $\Pi_t(e_t^*, w_t^*) > 0$ and $\Phi_t(e_t^*, w_t^*) = 0$ imply that players also do not terminate the match, so that $\alpha_t^* = \beta_t^* = 0$, $\forall t$. □

An important insight for the next section is obtained by generalising lemma 3:

Corollary 1 (SPE with higher wages). *For every $b_t \in [0, p(\bar{e}) - c(\bar{e}))$, $\exists\ \delta_F^* < 1$ such that for all $\delta_F \in [\delta_F^*, 1)$, the profile $(\hat{w}_t, e_t, w_t) = (c(\bar{e}) + b_t, \bar{e}, c(\bar{e}) + b_t)$ in every match period t is a SPE of the infinitely repeated game, where each δ_F^* is determined by*

$$c(\bar{e}) + b_t = \delta_F^* \Pi_{t+1}(e_{t+1}^*, w_{t+1}^*), \quad b_t \in [0, p(\bar{e}) - c(\bar{e})) \tag{10}$$

The worker and the firm respectively obtain payoffs $\Pi_t(e_t^*, w_t^*) \geq 0$ and $\Phi_t(e_t^*, w_t^*) \geq 0$.

Proof. The proof of lemma 3 can be adapted to every $b_t \in [0, p(\bar{e}) - c(\bar{e}))$. □

To illustrate, suppose for a moment that firms' δ_F just satisfies equation (10) if, say,

$$b_t = \frac{1}{2} [p(\bar{e}) - c(\bar{e})]$$

frequently in this kind of SPE.

There is in fact a continuum of SPE in which firms pay some b_t between (and including) 0 and $(1/2)\,[p(\bar{e}) - c(\bar{e})]$ while workers choose $e_t = \bar{e}$. In any such SPE with $b_t > 0$, firms pay more than the minimum needed to induce workers to choose $\bar{e}$, but if a firm unilaterally reduces b_t, this will be a deviation from equilibrium play. The worker would respond to the deviation by henceforth choosing $e_t = 0$, which is credible because this action is in line with a NE. This threat sustains a $b_t > 0$ in SPE provided δ_F is high enough. In principle, even a b_t very close to $p(\bar{e}) - c(\bar{e})$ may be sustained if δ_F is very close to 1, but $b_t = p(\bar{e}) - c(\bar{e})$ would mean that the gain to the firm from not deviating is 0 while there is always a short-term gain from deviating.

It is not crucial for the proof of corollary 1 that play moves to the worst SPE (see lemma 2) after a deviation. It is crucial, however, that play moves to some SPE in which the worker works sufficiently less so that this SPE is worse for the firm. Any such SPE with lower e_t will be equivalent, after an appropriate normalisation, to the worst SPE. In fact, we think of the worst SPE as "work to rule" or "white strike" instead of literally zero work.[14]

The firm's ability to set wages in the Diamond paradox allows us to predict which SPE among those in lemma 3 and corollary 1 would in fact be played in the match: the firm will select the firm-optimal SPE with $b_t = 0$. In this equilibrium, the worker only obtains a payoff 0. Since $k_W > 0$, the forward-looking worker avoids k_W by not participating at all and the Diamond paradox results. Hence *efficiency wages as such do not solve the Diamond paradox.*

3.4 Implications for Shapiro and Stiglitz (1984)

How can our conclusion of the last section be reconciled with the view that efficiency wages in Shapiro and Stiglitz (1984) solve the Diamond paradox? In their model, firms pay indeed higher wages, equivalent to $b_t > 0$ here. Yet their result is driven by their assumption of imperfect information for the firm: it does not always observe the worker's effort. To see this, suppose a worker who "shirks" by choosing $e_t = 0$ is only caught with probability $q < 1$ (thus far, our model has implicitly assumed $q = 1$). Then the firm-optimal equilibrium in lemma 3 is no longer feasible: by choosing $e_t = \bar{e}$ as before, a worker would obtain payoff 0, but by choosing $e_t = 0$ in anyone period she will in expectation obtain a strictly positive payoff $(1 - q)c(\bar{e})$. That is, she will be laid off if caught but will otherwise be paid a wage $w_t = c(\bar{e})$ that compensates her for effort she has never made. To ensure that the worker chooses $e_t = \bar{e}$ under these circumstances, the payoff from working needs to outweigh the expected payoff from shirking. Thus the

[14] To skeptics of game theory, the move all the way to the worst SPE might seem mechanical: can there not be a "compromise" SPE in which the firm pays some lower but still positive b_t and the worker works? While such an objection might be made to a great many SPE in the literature, the response appears particularly simple in this paper. Given that it deviates, a profit-maximising firm would want to pay the smallest b_t that is still positive, which does not exist: for any small but positive b_t, one can find a lower b_t that is also still positive. Hence a "compromise" SPE does not exist.

no-shirking condition is

$$b_t + \delta_W \Phi_{t+1}(e_{t+1}, w_{t+1}) \geq (1-q)[c(\bar{e}) + b_t + \delta_W \Phi_{t+1}(e_{t+1}, w_{t+1})]$$

To limit wage costs as far as possible while still incentivising workers, firms will pay the lowest b_t that satisfies this condition, denoted b^{NSC}. It is determined as the b_t at which the condition holds with equality, so that the worker's payoff from working would be the same as from shirking. Therefore, $\Phi_{t+1}(e_{t+1}, w_{t+1})$ is found as the value of earning b^{NSC} forever, $b^{NSC}/(1-\delta_W)$. With this, we can solve the no-shirking condition for b^{NSC}:

$$b^{NSC} = (1-\delta_W)\frac{1-q}{q}c(\bar{e})$$

Firms will thus pay a wage that reimburses workers for their search costs if and only if the value of earning b^{NSC} over the entire match duration weakly exceeds k_W:[15]

$$\frac{\delta_W b^{NSC}}{1-\delta_W} \geq k_W \quad \Leftrightarrow \quad \delta_W \frac{1-q}{q} c(\bar{e}) \geq k_W$$

This will hold for q low enough but cannot hold for $q = 1$. Hence the assumption of imperfectly informed firms drives the finding of higher wages in Shapiro and Stiglitz (1984). While the imperfections they assume appear much more plausible to us than the information imperfections assumed by Burdett and Mortensen (1998), it remains an assumption that violates Diamond's (1971) original set-up and that does not always hold in reality, as effort can be observed well in many industries (e.g. construction). Instead we seek a solution to the Diamond paradox that adds to or refines Diamond's (1971) settings but does not violate them.

4 Advertised efficiency wages

While previous sections found that neither advertisements alone nor efficiency wages alone solve the Diamond paradox, this section and the next argue that a combination of the two solves it. To this end, we now introduce advertisements into the extended set-up with effort. We would argue that firms in the real world advertise both the total wage they offer and the effort they expect. Of course, such advertisements imply an advertised b, denoted $\tilde{b}$, as the advertised wage less the disutility of the expected effort. We drop the time subscripts here because we found in the previous sections that b_t is time-invariant in all SPE of interest. Since workers in our model only care about b, we can pretend for simplicity that firms' only advertisement is $\tilde{b}$. Firms make advertisements in period 0^1,

[15]Note that here we discount wages paid in a match period to the period preceding the match, as the worker incurs k_W and makes her decision to engage in search in this period.

so that $\tilde{b}$ is a history before the first wage offer $\hat{w}_1$ is made:

$$h(1^0) = \{\tilde{b}\} \quad \Rightarrow \quad h(t^0) = \{\tilde{b}, (\hat{w}_1, e_1, w_1), (\hat{w}_2, e_2, w_2), \ldots, (\hat{w}_{t-1}, e_{t-1}, w_{t-1})\}$$

As all communications by firms are now advertisements,[16] by assumption 1 a worker selects an advertisement at random. How the worker responds to the selected advertisement depends on her beliefs: let $\mu(b = \tilde{b}|\tilde{b})$ denote the subjective probability that the SPE with $b = \tilde{b}$ will be played during the match when $\tilde{b}$ has been advertised. We call an advertisement $\tilde{b}$ *potentially truthful* if there exists a SPE in which the firm pays $b = \tilde{b}$. Let us further define $\underline{b}$ and $\bar{b}$ such that

$$\frac{\delta_W \underline{b}}{(1-\delta_W)} = k_W \quad , \quad \frac{\delta_F[p(\bar{e}) - c(\bar{e}) - \bar{b}]}{1-\delta_F} = k_F \tag{11}$$

so that every $b \in [\underline{b}, \bar{b}]$ would allow both the worker and the firm to recoup their respective search costs.[17] By assumption 3, $\underline{b} < \bar{b}$ and each firm is thus prepared to pay a $b \in [\underline{b}, \bar{b}]$ if necessary. For reasons that will become clear, we also assume for this section that firms are sufficiently patient to pay at most some $B \in [\underline{b}, \bar{b}]$ during the match:

Assumption 4 (Patient firms). *Firms' common δ_F just satisfies equation (10) when $b = B$, for some $B \in [\underline{b}, \bar{b}]$.*

Recall from corollary 1 that there are SPE in matches with $b > 0$, but we found no reason why the firm should not select the SPE with $b = 0$. This changes in the current set-up with advertisements. In particular, the model has a *perfect Bayesian equilibrium* (PBE) with all firms paying $b = \underline{b}$ in each match period, as follows.

Proposition 1 (PBE with worker participation). *$\exists$ a PBE of the model with advertisements in which*

(i) each firm chooses the advertisement $\tilde{b} = \underline{b}$

(ii) workers trust all potentially truthful advertisements: $\mu(b = \tilde{b}|0 \le \tilde{b} \le B) = 1$ and $\mu(b = \tilde{b}|\tilde{b} > B) = 0$

(iii) the SPE profile $(\hat{w}_t, e_t, w_t) = (c(\bar{e}) + \underline{b}, \bar{e}, c(\bar{e}) + \underline{b})$ is played, $\forall t$ of a match

(iv) workers engage in search and match at the first opportunity.

[16] Our implicit assumption that no firm strictly prefers to send mere notifications of vacancies is without loss of generality: mere notifications never attract any workers (see section 3.3), while we show below that advertisements may attract workers.

[17] As before, we discount earnings in a match period to the period preceding the match, since agents incur k_W and k_F in this period.

Proof. Given workers' beliefs as specified under *(ii)*, workers who select an advertisement $\tilde{b} < \underline{b}$ would trust it and consequently would not visit this firm. Advertisements $\tilde{b} > B$ are not trusted. Thus only advertisements $\tilde{b} \in [\underline{b}, B]$ attract workers.

Given their beliefs, workers visiting a firm treat it as a deviation from the SPE they expected when a firm reneges on its advertisement. Let players use grim trigger strategies analogous to those in the proof of lemma 3. If a firm reneges, play therefore moves to the SPE in lemma 2, giving both players payoff 0. Firms will thus be strictly better off if a SPE as in corollary 1 is played with $b \in [\underline{b}, B]$ because, by equation (11), the firm obtains a payoff $k_F > 0$ even with $B = \bar{b}$. By assumption 4, the firm is also patient enough to sustain a SPE with $b \in [\underline{b}, B]$. Hence, given workers' reactions to reneging, firms strictly prefer not to renege.

Among all combinations of some $b \in [\underline{b}, B]$ and its truthful advertisement $\tilde{b}$ that still attracts workers, profit-maximising firms will choose the one with the lowest b. Hence the SPE with $b = \underline{b}$ is always played and preceded by the truthful advertisement $\tilde{b} = \underline{b}$. As all firms advertise and pay $\underline{b}$, there is nothing to gain from visiting a second firm, and the worker would thus match at the first opportunity and incur k_W only once. By equation (11), the payoffs from the SPE with $b = \underline{b}$ allow workers to exactly recoup k_W. In terms of equation (2), we thus have

$$\delta_W \Omega_{t+1}(w_{t+1} | \tilde{b} = \underline{b}) = k_W$$

Now suppose

$$\delta_W \Gamma_{t+1} > \delta_W \Omega_{t+1}(w_{t+1} | \tilde{b} = \underline{b}) - k_W = 0 \tag{12}$$

in equation (2). Then $\Gamma_t = \delta_W \Gamma_{t+1}$ and likewise in all subsequent periods. Yet if the worker thus never visits a firm and never earns any payoffs, equation (12) cannot hold. Hence

$$\delta_W \Omega_{t+1}(w_{t+1} | \tilde{b} = \underline{b}) - k_W \geq \delta_W \Gamma_{t+1}$$

so that workers engage in search. Since $\underline{b} < \bar{b}$, firms also recoup their search cost. As workers are homogeneous, firms also match at the first opportunity.

Finally, analogous grim trigger strategies would sustain every other SPE with $b \in [0, B]$, so that workers correctly believe that they can trust all potentially truthful advertisements (while they would only visit firms advertising at least $\underline{b}$). As firms are not sufficiently patient to pay some $b > B$, workers also correctly believe that such SPE will be played with 0 probability. □

This cheap-talk PBE can be viewed as one of the SPE in corollary 1 extended by advertisements and by the beliefs such advertisements can induce. The key result is that workers engage in search. Advertisements are crucial here because they serve as a benchmark against which a deviation by the firm is defined. Once advertisements have fixed workers' beliefs, the same credible "punishment" for a deviating firm in a SPE in lemma 3 now also applies to firms reneging on their advertisements. Intuitively, one could say that the firm loses the worker's goodwill if it reneges on its advertisement. Then the firm prefers not to renege and actually pays the advertised wage. This wage must be sufficient to reimburse the worker for her search costs because she would otherwise not have visited the firm with this advertisement.[18]

As an illustration, consider what would happen if a firm tried to play the firm-optimal SPE in lemma 2 where $b = 0$. If the firm advertises $\tilde{b} = 0$ (or any other $\tilde{b} < \underline{b}$), no worker will visit. The same holds for advertisements greater than B. If the firm advertises some $b \in [\underline{b}, B]$, workers will expect a SPE with this b. Then any attempt by the firm to play the firm-optimal SPE with $b = 0$ is seen by the worker as a deviation. Concretely, when the worker has to choose e_1, she can compare the advertised $\tilde{b}$ to the wage offer $\hat{w}_1$, having also been told that effort level $\bar{e}$ is expected. If $\hat{w}_1 < c(\bar{e}) + \tilde{b}$, the firm reneges on the advertisement and the worker treats this as a deviation from the SPE in which $\hat{w}_1 = c(\bar{e}) + \tilde{b}$ would have been equilibrium play.[19] Consequently, the worker chooses $e_t = 0$ in every period t of the match, leading to the SPE in lemma 2. The firm-optimal SPE is no longer feasible, and the firm can only choose between outcomes with payoff 0 (when no worker visits or when the SPE in lemma 2 results) and some SPE with $b \in [\underline{b}, B]$.

In conclusion, this section has formalised the idea that workers who are confronted with a unilateral reduction of the promised wage can react by "work to rule", working as little as possible. This reaction is consistent with equilibrium play in a subgame-perfect equilibrium and the threat of this reaction therefore induces firms not to renege on their advertisements. Truthful advertisements in turn will only attract workers if the wages reimburse workers for their search costs, so that workers do engage in search. Since workers by assumption select one firm's notification randomly, competition does not play a role in obtaining this result. Imperfect information does not play any role here either, nor does wage dispersion: with $w_t = c(\bar{e}) + \underline{b}$ in every period t, the equilibrium wage distribution consists of a single wage. Wage dispersion would be generated in our model if we relaxed assumption 1 to allow for a minimum of competition, for example as in Halko et al. (2008). What does play a role for the results in this section is that firms are sufficiently patient by assumption 4, and the next section addresses this issue.

[18] Any babbling equilibrium would not fix workers' beliefs about the SPE that is played in the match. Instead, workers would believe that any given firm will choose the firm-optimal SPE with a certain probability, regardless of the advertisement. If firms can thus renege without consequences, all firms will do so. As workers' beliefs must be correct in a PBE, the only beliefs workers may have in the babbling equilibrium are $\mu(b = \tilde{b} | \tilde{b} > 0) = 0$. Hence workers prefer not to participate, so that the babbling equilibrium leads to the Diamond paradox, in contrast to the PBE in proposition 1.

[19] $\hat{w}_1 = c(\bar{e}) + \tilde{b}$ is equilibrium play in the SPE with $b = \tilde{b}$ but also in the worst SPE where $\hat{w}$ is indeterminate (see lemma 2). However, the advertisement leads the worker to believe that the former SPE is being played when choosing e_1, which is in this context advantageous to the firm.

5 The issue of firms' patience

This section explores what happens if assumption 4 fails. We first show that this can lead to market breakdown and then consider sign-up bonuses or fixed base wages as an institution to prevent market breakdown. In this context, we find that wages may rise even above the level that exactly compensates workers for their search costs. However, a second finding is that firms will prefer inefficiently low levels of effort in order to limit this rise in wages. To begin, let us drop assumption 4 and adopt instead the following assumption:

Assumption 4' (Patient and impatient firms). *Firms' discount factors are uniformly distributed on the open unit interval, $\delta_F \sim \mathbf{U}(0,1)$. A firm's respective δ_F is its private information.*[20] *Instead of equation (5), henceforth it is only required that $\exists\ e > 0$ such that*

$$\frac{\delta_W[p(e) - c(e)]}{1-\delta_W} > k_W + k_F$$

Now there are also firms that are too impatient to pay $\underline{b}$. (Throughout this section, we refer to those firms that deviate at a given b simply as impatient firms, and to the other firms as patient firms.) If these impatient firms truthfully advertised a $\tilde{b} < \underline{b}$, they would not attract any applicants. If they advertise some $\tilde{b} > \underline{b}$ but then renege already in their choice of $\hat{w}_1$, they will obtain a payoff 0. However, they can advertise some $\tilde{b} > \underline{b}$, thereby attract applicants, then pretend to play the SPE with $b = \tilde{b}$ by choosing $\hat{w}_1$ accordingly, and ultimately pay nothing after the worker exerted effort $e_1 = \bar{e}$. From then on, the SPE in lemma 2 would result, but such firms would already have obtained strictly positive payoffs this way.

One might think that it is sufficient for all patient firms to raise their b so far that workers still just obtain $\delta_W\underline{b}/(1-\delta_W)$ in expectation and thus just recoup their search costs in expectation. A worker will only obtain a payoff $-c(\bar{e})$ in the first period of a match with an impatient firm. With δ_F^* here referring to the δ_F of the firm that is indifferent between deviating and paying $\underline{b}$, patient firms' b would have to satisfy the following equation:

$$\frac{\delta_W\underline{b}}{1-\delta_W} = (1-\delta_F^*)\frac{\delta_W b}{1-\delta_W} - \delta_F^*\delta_W c(\bar{e}) \tag{13}$$

where $1-\delta_F^*$ and δ_F^* are the probabilities of drawing a patient and an impatient firm, respectively. However, if patient firms raise b, this will affect δ_F^*, and we have the following result:

Proposition 2 (Market unravelling). *Patient firms' attempts to raise b in response to the presence of impatient firms will result in all firms becoming impatient, so that $\delta_F < \delta_F^*$, $\forall \delta_F \in (0,1)$. Irrespectively of what patient firms do, workers thus do not engage in search.*

[20] Note that none of Diamond's (1971) original assumptions is violated if we let workers be imperfectly informed about firms' characteristics. Diamond's set-up only relies on firms having perfect information about workers' characteristics.

Proof. Recall that δ_F^* is determined as the δ_F of the firm that is indifferent between deviating and not deviating from the SPE in the match. Discounting to the first match period here for simplicity, δ_F^* is thus determined by

$$p(\bar{e}) - \frac{k_F}{\delta_F^*} = \frac{p(\bar{e}) - c(\bar{e}) - b}{1 - \delta_F^*} - \frac{k_F}{\delta_F^*} \quad \Leftrightarrow \quad \delta_F^* = \frac{c(\bar{e}) + b}{p(\bar{e})} \tag{14}$$

so that δ_F^* increases linearly with b. Next, we solve equation (13) for b and find the first as well as the second derivative of b with respect to δ_F^* as, respectively,

$$\frac{\underline{b} + (1 - \delta_W)c(\bar{e})}{(1 - \delta_F^*)^2} > 0 \quad \text{and} \quad \frac{2[\underline{b} + (1 - \delta_W)c(\bar{e})]}{(1 - \delta_F^*)^3} > 0$$

Hence b is convex in δ_F^*. Any attempt to raise b above $\underline{b}$ will raise δ_F^*, then the patient firms would have to raise b again, and so on. Since both the positive effects of b on δ_F^* and of δ_F^* on b are stable or increasing, δ_F^* always continues to rise, so that all firms eventually have $\delta_F < \delta_F^*$. Whether initially patient firms attempt to raise b or not, equation (13) is thus never satisfied, and workers consequently do not engage in search. $\square$

Essentially, the presence of impatient firms necessitates a higher b to satisfy equation (13), which leads in turn to a still higher δ_F^* and a still higher b, at an increasing rate. Hence the market unravels. The key link is that more firms will prefer to deviate in the match if they have to pay a higher b when not deviating.

However, a more desirable outcome might be achieved in a situation where patient firms manage to differentiate themselves from impatient firms. We will show below that patient firms can indeed differentiate themselves by paying a one-off *sign-up bonus* σ. This may be paid in the interview in period 1^0 upon acceptance by the worker and thus before 1^1 when the worker chooses effort. That is, we let histories be

$$\begin{aligned} h(1^0) &= \{\tilde{b}\} \\ h(1^1) &= \{\tilde{b}, \sigma, \hat{w}_1\} \\ h(t^0) &= \{\tilde{b}, \sigma, (\hat{w}_1, e_1, w_1), (\hat{w}_2, e_2, w_2), \ldots, (\hat{w}_{t-1}, e_{t-1}, w_{t-1})\} \end{aligned}$$

Provided the sign-up bonus is sufficiently high, all firms will either prefer not to deviate from the SPE in the match or not engage in search altogether. Note that such a sign-up bonus may in reality be a contracted fixed base wage in period 1. In any case, it functions here as a costly signal of being patient, just as education is a costly signal of having high ability in Spence (1973). In order to maintain our definition of b as the part of the wage over and above the costs of effort, we suppose that patient firms recoup σ by a small reduction of b throughout the match (which is in this context expected by the worker and therefore not treated as a deviation). Our equations below therefore do not mention σ as a cost to a patient firm.[21]

Patient firms can only differentiate themselves through σ if impatient firms do not want to mimic patient firms. In other words, there must not be any incentive for a firm in equilibrium to engage in search, pay σ, and then deviate during the match. In particular, this needs to hold for the firm with the lowest δ_F that still engages in search and pays σ. Profit-maximisation implies that this firm will be indifferent between deviating and not deviating; otherwise the patient firms could reduce σ without any consequences. This firm's δ_F is therefore δ_F^*:[22]

$$p(e) - \frac{k_F}{\delta_F^*} - \sigma = \frac{p(e) - c(e) - b}{1 - \delta_F^*} - \frac{k_F}{\delta_F^*} \tag{15}$$

All firms with higher δ_F strictly prefer not to deviate. In any equilibrium where patient firms differentiate themselves by paying σ, a firm that does not pay σ is treated as an impatient firm, thus fails to make any worker exert effort, and prefers not to engage in search so as to avoid k_F. Since the firm with $\delta_F = \delta_F^*$ is the firm with the lowest δ_F that still engages in search and pays σ, it must be indifferent between doing so and not engaging in search:

$$\frac{p(e) - c(e) - b}{1 - \delta_F^*} - \frac{k_F}{\delta_F^*} = 0 \tag{16}$$

Finally, in any match period, firms demand the (same) effort that maximises their share of production:

$$\max_e p(e) - c(e) - b \tag{17}$$

Apart from allowing a worker to identify patient firms, a sign-up bonus may also affect workers' behaviour in more subtle ways. While workers cannot collect several sign-up bonuses here because they match at most once in our model, they might prefer to just receive the sign-up bonus without working. That is, also workers might now be too impatient: the present discounted value to them of receiving b forever in the match might be less than σ. For workers to exert effort, we need

$$\sigma - \frac{k_W}{\delta_W} \leq \frac{b}{1 - \delta_W} - \frac{k_W}{\delta_W} \tag{18}$$

If $\underline{b}$ satisfies this condition, profit-maximising firms will not pay more than that. However, if equation (18) is satisfied only at a $b > \underline{b}$, then the necessity to pay σ will imply a minimum for b that exceeds $\underline{b}$. Firms will nevertheless pay this minimum for b to induce the worker to work (but profit-maximising firms would not pay more than this). Importantly, this logic would explain *why firms pay more than needed to reimburse workers for search costs*. Note that this is not just the logic of efficiency wages, as it is only the firm's need

[21]Note that, in equilibrium, firms whose rents are insufficient to recoup σ will not pay σ in the first place. We ensure that there exist patient firms who can afford to pay σ in equilibrium; see equation (26).

[22]In equations (15) through (18), we discount to the first match period for simplicity.

to signal its patience that gives rise to σ and then potentially to a higher b.[23]

An equilibrium in which patient firms differentiate themselves from impatient firms would have to satisfy equations (15) through (18). Equilibrium values, however, differ depending on whether or not $\underline{b}$ satisfies equation (18). Let us first suppose that equation (18) is not satisfied at $\underline{b}$. The following proposition claims that an equilibrium exists for a range of parameter values. We analyse this signalling equilibrium as a PBE, which requires beliefs: let $\nu(\delta_F \geq \delta_F^*|\sigma)$ be a worker's subjective probability that a firm is patient, given that it pays sign-up bonus σ.

Proposition 3 (PBE with sign-up bonus and $b^* > \underline{b}$). *$\exists$ a PBE of the model with $\delta_F \sim \mathbf{U}(0,1)$ in which*

(i) each firm that engages in search pays a sign-up bonus $\sigma^* = \frac{c(e^*)-k_F}{\delta_W}$

(ii) workers' beliefs about δ_F are $\nu(\delta_F \geq \delta_F^*|\sigma \geq \sigma^*) = 1$ and $\nu(\delta_F \geq \delta_F^*|\sigma < \sigma^*) = 0$

(iii) workers and only those firms with $\delta_F \geq \delta_F^*$ engage in search, where

$$\delta_F^* = \frac{\delta_W k_F}{\delta_W p(e^*) - c(e^*) + k_F}$$

(iv) all firms that engage in search advertise and pay

$$b^* = \frac{1-\delta_W}{\delta_W}\left[c(e^*) - k_F\right] > \underline{b}$$

(v) workers trust all potentially truthful advertisements:
$\mu(b = \tilde{b}|0 \leq \tilde{b} < p(\bar{e}) - c(\bar{e})) = 1$ and $\mu(b = \tilde{b}|\tilde{b} \geq p(\bar{e}) - c(\bar{e})) = 0$

(vi) the SPE profile $(\hat{w}_t, e_t, w_t) = (c(e^*)+b^*, e^*, c(e^*)+b^*)$ is played, $\forall t$ of a match where firms demand a level $e^* < \bar{e}$ that satisfies (primes denoting derivatives)

$$\delta_W p'(e^*) = c'(e^*)$$

provided that parameter values satisfy $k_W < c(e^*) - k_F < \delta_W[p(e^*) - k_F]$.

Proof. We find the equilibrium values by solving equations (15) through (18) simultaneously. Note first that equations (15) and (16) together imply

$$p(e) - \frac{k_F}{\delta_F^*} - \sigma = 0 \tag{19}$$

[23] Beaudry (1994) uses a signalling game in a similar way to explain why, in a principal-agent context with effort and asymmetric information about job characteristics, even a perfectly informed principal might leave an agent with rents.

We solve for δ_F^* as

$$\delta_F^* = \frac{k_F}{p(e) - \sigma} \tag{20}$$

We next rearrange equation (16) as

$$p(e) - c(e) - b = k_F \frac{1 - \delta_F^*}{\delta_F^*} \tag{21}$$

Substituting for δ_F^* from equation (20) leads to

$$\begin{aligned} p(e) - c(e) - b &= k_F \frac{p(e) - \sigma - k_F}{k_F} \\ \Leftrightarrow \quad \sigma^* &= c(e) - k_F + b \end{aligned} \tag{22}$$

With $b = (1 - \delta_W)\sigma$ as implied by equation (18) we obtain

$$\sigma^* = \frac{c(e) - k_F}{\delta_W} \quad \text{and consequently} \quad b^* = \frac{1 - \delta_W}{\delta_W} \left[c(e) - k_F \right] \tag{23}$$

This b^* exceeds $\underline{b}$ whenever

$$\begin{aligned} \frac{1 - \delta_W}{\delta_W} \left[c(e) - k_F \right] &> \underline{b} = \frac{1 - \delta_W}{\delta_W} k_W \\ \Leftrightarrow \quad c(e) - k_F &> k_W \end{aligned} \tag{24}$$

Workers will thus correctly believe to face a patient firm if and only if at least σ^* is paid as a sign-up bonus, and will subsequently exert effort only in this case. They correctly believe that they can trust advertisements including those $\tilde{b}$ arbitrarily close (but not equal or greater) to $p(\bar{e}) - c(\bar{e})$: by corollary 1, such SPE might be played by firms with sufficiently high δ_F. Profit-maximising firms, however, will pay the lowest possible b, i.e. b^* because anything lower would violate equation (18), so that workers would deviate. As play would move to the worst SPE as in the proof of proposition 1 if a firm reneged on its advertisement, all firms thus advertise and pay b^*. That workers then engage in search and match at the first opportunity (as do firms) can be shown like in the proof of proposition 1. Next, using the result for σ^* in equation (20) gives us

$$\delta_F^* = \frac{\delta_W k_F}{\delta_W p(e) - c(e) + k_F} \tag{25}$$

To ensure that there exist patient firms prepared to pay σ^*, we require that $\delta_F^* < 1$, or

$$\delta_W k_F < \delta_W p(e) - c(e) + k_F \tag{26}$$

Because $\delta_W k_F > 0$ this condition, if satisfied, will also imply $\delta_F^* > 0$. The requirements on parameter values in equations (24) and (26) combine to

$$k_W < c(e) - k_F < \delta_W[p(e) - k_F]$$

Finally, the level of e demanded by firms satisfies equation (17). Employing the result for b^*, we rewrite this equation as

$$\max_e p(e) - \frac{1}{\delta_W} c(e) + \frac{1-\delta_W}{\delta_W} k_F$$

Then the first-order condition with respect to e leads to

$$\delta_W p'(e^*) = c'(e^*)$$

Recall that $\bar{e}$ would equate $c'(e)$ with $p'(e)$. Since $\delta_W < 1$, e^* equates $c'(e)$ with something lower. By the convexity of $c(e)$, this equality must occur at some $e < \bar{e}$. $\square$

The equilibrium in proposition 3 thus has the distinctive feature that firms pay more than needed to reimburse workers. In the presence of sufficiently high sign-up bonuses, workers would otherwise keep the bonus but would not work. It appears that this logic in and of itself proposes a different solution to the Diamond paradox, without the need for advertisements. This solution would, however, be limited to certain parameter values, while we show below that firms will pay at least $\underline{b}$ also for other parameter values, thanks to advertisements. This solution would further presuppose that impatient firms exist and operate in the market, which might depend strongly on the legal and institutional context. Finally, the last result is also interesting in its own right: here firms have an incentive to demand a lower level of e so as to limit b^*. This creates an inefficiency because $\bar{e}$ would maximise the surplus from match production.

Now suppose that $\underline{b}$ does satisfy equation (18). The logic of proposition 1 implies that firms will then advertise and pay $\underline{b}$ instead of a lower b that just satisfies equation (18), so that equation (18) can be slack in equilibrium. The equilibrium in this case has the same structure as before but different values:

Proposition 4 (PBE with sign-up bonus and $\underline{b}$). *$\exists$ a PBE of the model with $\delta_F \sim$ $\mathbf{U}(0,1)$ in which*

(i) each firm that engages in search pays a sign-up bonus $\sigma^* = c(e) - k_F + \underline{b}$

(ii) workers' beliefs about δ_F are $\nu(\delta_F \geq \delta_F^* | \sigma \geq \sigma^*) = 1$ and $\nu(\delta_F \geq \delta_F^* | \sigma < \sigma^*) = 0$

(iii) workers and only those firms with $\delta_F \geq \delta_F^*$ engage in search, where

$$\delta_F^* = \frac{k_F}{p(e) - c(e) + k_F - \underline{b}}$$

(iv) all firms that engage in search advertise and pay $\underline{b}$

(v) workers trust all potentially truthful advertisements:
$\mu(b = \tilde{b} | 0 \leq \tilde{b} < p(\bar{e}) - c(\bar{e})) = 1$ *and* $\mu(b = \tilde{b} | \tilde{b} \geq p(\bar{e}) - c(\bar{e})) = 0$

(vi) the SPE profile $(\hat{w}_t, e_t, w_t) = (c(\bar{e}) + \underline{b}, \bar{e}, c(\bar{e}) + \underline{b})$ *is played,* $\forall t$ *of a match*

provided that parameter values satisfy $-\frac{1-\delta_W}{\delta_W} k_W \leq c(e) - k_F \leq k_W$.

Proof. Equation (22) can be derived exactly as in the proof of proposition 3, with the only difference that $b = \underline{b}$ is already determined. Here therefore

$$\sigma^* = c(e) - k_F + \underline{b} \tag{27}$$

If the sign-up bonus is at least σ^*, again workers will correctly believe to face a patient firm. Plugging the result for σ^* back into equation (20) gives

$$\delta_F^* = \frac{k_F}{p(e) - c(e) + k_F - \underline{b}} \tag{28}$$

Since, by assumption 4',

$$p(e) - c(e) > \frac{1 - \delta_W}{\delta_W} k_W = \underline{b} \tag{29}$$

we conclude that $\delta_F^* \in (0, 1)$ for any parameter values. However, we require $\sigma^* \geq 0$ and recall that the PBE in proposition 3 applies whenever equation (24) holds. Hence the PBE in proposition 4 will only exist if parameter values satisfy

$$-\frac{1 - \delta_W}{\delta_W} k_W \leq c(e) - k_F \leq k_W \tag{30}$$

That workers correctly believe they can even trust $\tilde{b}$ arbitrarily close to $p(\bar{e}) - c(\bar{e})$ follows from corollary 1 and the proof of proposition 1 as before. Also by the same logic as before, workers engage in search and match at the first opportunity. During the match, firms demand the level of e determined by equation (17). As $\underline{b}$ does not depend on e, this level is $\bar{e}$ here. □

The overall intuition for propositions 3 and 4 is that the firm has to gain the worker's trust by offering a σ it will only pay if it is patient. What sustains these PBE is thus the loss of trust in the firm that would occur if less than σ^* was paid. The core logic here is thus a very simple signalling game. Propositions 3 and 4 together show for a connected range of parameter values that, when there are impatient firms, a sign-up bonus can fix the problem of market unravelling. While a sign-up bonus therefore *can* fix the problem by separating firms, the question remains whether this necessarily happens whenever a sign-up bonus exists. The following corollary provides an affirmative answer:

Corollary 2 (Uniqueness). *Suppose we require that beliefs are consistent with probability 0 of strictly dominated and equilibrium-dominated actions occurring. Then* σ^* *in proposition 3 is the unique* σ *in equilibrium for any equilibrium in the range of parameter values for which proposition 3 holds, and likewise for* σ^* *in proposition 4.*

Proof. We only sketch the proof here because structurally identical proofs have been provided in similar contexts. The following argument applies equally to propositions 3 and 4. Essentially, any $\sigma > \sigma^*$ is a strictly dominated action for an impatient firm, so that the worker should believe to face a patient firm when any such σ is paid. Hence patient firms need not pay more than σ^* to credibly signal their patience. Then only the separating equilibrium with σ^* remains among the separating equilibria. Patient firms can and will break every pooling equilibrium by choosing a σ that would be an equilibrium-dominated action if chosen by an impatient firm, so that workers identify such deviants as patient firms. □

Under the conditions on beliefs in corollary 2, a sign-up bonus thus necessarily separates patient firms from impatient firms. Hence, the existence of a sign-up bonus effectively plays the same role as the assumption of patient firms we had made before proposition 1. The equilibria in propositions 3 and 4 thus represent the PBE in proposition 1 extended by a signalling game instead of assumption 4. In conclusion, it is not necessary that all firms be patient to arrive at the finding that there are equilibria in which workers engage in search.

This section has thus shown why impatient firms undermine the PBE in proposition 1 and has argued that a sign-up bonus (or base wage) necessarily fixes this problem for a connected range of parameter values. For the part of this range where $k_W < c(e^*) - k_F$, proposition 3 establishes in addition that all firms advertise and pay a b^* strictly above $\underline{b}$, so that workers earn strictly positive rents. By our assumptions 1 and 2, none of these results rely on competition between firms or on imperfectly informed firms.

6 Conclusions

We hope this paper furthers our understanding why forward-looking job seekers participate in labour markets at all. Our analysis suggests that advertisements and efficiency wages are both important parts of an explanation. Including these elements in a model with search frictions and perfectly informed firms that set wages, we have found equilibria in which workers participate and may even earn rents. In these equilibria, all firms pay the same wage. Contrary to what is sometimes believed, solving the Diamond paradox is not necessarily the same as generating wage dispersion. Unlike responses to the Diamond paradox in the literature, the solution proposed here does not somehow rely on imperfectly informed firms or their ability to commit. By relying on advertising and sign-up bonuses or base wages instead, our solution rather emphasises the role of labour market institutions. While binding minimum wages or unions have also been identified as solutions to the Diamond paradox, these latter institutions do not appear as widespread in real-world labour markets as advertising and base wages.

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