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Mathematical Methods, Part 1
Winter 2015/16

Problem Set 4

Problem 4.1. Consider a planner's problem in the neo-classical growth model

$$\max_{\{c_t,k_{t+1}\}_{t\geq 0}} \left\{ \sum_{t=0}^{\infty} \beta^t \log(c_t) \right. \tag{1}$$

subject to (2)

$$\left. c_t + k_{t+1} \leq A k_t^{\alpha}, \quad A > 0, 0 < \alpha < 1, \ k_0 \text{ given} \right\} \tag{3}$$

(a) Write down the Bellman equation.

(b) Solve the Bellman equation by 'guess and verify' to obtain the policy function. Hint: You may follow the steps below.

(i) Guess the value function of the form $V(k) = A + B\log(k)$.

(ii) Derive the FOC using the guessed value function.

(ii) Plug the FOC into the Bellman equation again and match the parameters.

(c) Compute the fixed point(s) of the policy function and analyze their local stability.

(d) Sketch the policy function in a diagram and compare with the one obtained in Problem 1.2.

Problem 4.2. Now consider the planner's problem with *habit persistence* in her utility function.

$$\max_{\{c_t,k_{t+1}\}_{t\geq 0}} \left\{ \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \gamma \log(c_{t-1}) \right. \tag{4}$$

subject to (5)

$$\left. c_t + k_{t+1} \leq A k_t^{\alpha}, \quad A > 0, 0 < \alpha < 1, 0 \leq \gamma \leq 1, \ k_0 \text{ given} \right\} \tag{6}$$

(a) Write down the Bellman equation. Hint: There are two state variables, k and c_{-1}.

(b) Solve the Bellman equation by 'guess and verify' to obtain the policy function. Hint: You may follow the steps as in Problem 4.1. And you should guess $V(k) = E + F\log(k) + G\log(c_{-1})$.

(c) Compare the policy function with the one obtained in Problem 4.1. Interpret the result economically.

Enjoy!

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