Marten Hillebrand, Hoang Van Khieu **Mathematical Methods, Part 1**
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Problem Set 3

Problem 3.1. Consider the decision problem with infinite horizon studied in Chapter 2 in class. Unless stated otherwise, all assumptions and the notation introduced there continue to hold. Specifically, assume that period utility u is of the form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \tag{1}$$

where we assume $\sigma > 0$ (by now you should be aware of the technical issues arising if $\sigma > 1$).

(a) Set up the consumer's decision problem.

(b) Compute the solution to the decision problem using a Lagrangian approach.

(c) Interpret the optimal decision economically and compare it to the solution from Problem 1.1.

Problem 3.2. Consider the decision problem with finite time horizon $T = 2$ studied in class. Assume that period utility u is of the form $u(c) = \ln(c)$. Solve the decision problem by recursive methods following the steps below.

(i) State the decision problem in $t = 1$ given $W_1 > -E_1$.

(ii) Compute the solution as functions $C_1(W_1)$ and $S_1(W_1)$.

(iii) Define and compute the value function V_1 and verify that $V_1'(W_1) = u'(W_1 - S_1(W_1))$ [Equation (19)].

(iv) Write down the decision problem in $t = 0$ as a *one-stage problem* involving the value function V_1.

(v) Compute the solutions c_0^* and s_0^* and verify that they are the same as with the Lagrangian approach used in Problem 1.1 when $\sigma = 1$.

(vi) Define $W_1^* = e_1 + R_1 s_0^*$. Compute the solutions c_1^* and s_1^* using the function C_1 and S_1 obtained in (ii).

(vii) Bonus: You may want to generalize the previous procedure to $T > 2$ by following the steps above. To compute the functional form of the value functions, you should apply an induction argument.

Enjoy!

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