Leopold von Thadden
Monetary Theory and Policy
Summer Term 2013

Problem Set 5

Extensions of the MIU model (Lecture 5)

Problem 1: Properties of utility functions:
Consider the utility function, assuming $0 < a < 1$, $b > 0$ (and $b \neq 1$), $\eta > 0$ (and $\eta \neq 1$), $\Phi > 0$ (and $\Phi \neq 1$), $\Psi > 0$,

$$u(c_t, m_t, 1 - n_t) = \frac{[ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1-\Phi}{1-b}}}{1 - \Phi} + \Psi \frac{(1 - n_t)^{1-\eta}}{1 - \eta} \tag{1}$$

Hint: Notice that for $\Phi = \Psi = 0$ this function turns into the special utility function

$$u(c_t, m_t) = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}} \tag{2}$$

as discussed in Lecture 3 and in PS 3 and 4.

a) Convince yourself that the more general function (1) preserves the properties of the special function (2) established on PS 3 and PS4.

b) Define

$$\widetilde{c} = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}}$$

Verify that the intertemporal elasticity of substitution in the composite consumption good $\widetilde{c}$ is given by $\frac{1}{\Phi}$.

c) Similarly, verify that the intertemporal elasticity of substitution in leisure is given by $\frac{1}{\eta}$.

Problem 2: Steady-state (non)-superneutrality of money under preferences which allow for a labour-leisure choice
Consider the steady-state discussion done in Lecture 5. Recall from p. 20f. that the pair of equations

$$\frac{U_l(\overline{\phi}n, m, 1-n)}{U_c(\overline{\phi}n, m, 1-n)} = \overline{w} \quad \text{and} \quad \frac{U_m(\overline{\phi}n, m, 1-n)}{U_c(\overline{\phi}n, m, 1-n)} = \frac{i}{1+i} = \frac{1+\theta-\beta}{1+\theta}$$

determines the steady-state values of the labour supply (n) and of real balances (m) and that money can be superneutral or not, depending on the structure of the utility function $u(c, m, 1-n)$.

a) Consider the utility function (1) from Problem 1.
a1) Argue why money is superneutral if $\Phi = b$.
a2) Show that $b > \Phi$ implies $u_{cm} > 0$.

b) Assume now instead that the utility function $u(c, m, 1-n)$ is multiplicatively separable in m, ie

$$u(c, m, 1-n) = v(c, 1-n) \cdot g(m)$$

Is money under such specification superneutral ?

Problem 3: Linearized equilibrium conditions
By means of example, consider from the system of equilibrium conditions established in Lecture 5 the following 5 equations

$$\begin{aligned} y_t &= e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha} \\ 1 + i_t &= (1 + r_t) \cdot E_t(1 + \pi_{t+1}) \\ \frac{1-a}{a}(\frac{m_t}{c_t})^{-b} &= \frac{i_t}{1 + i_t} \\ m_t &= (\frac{1+\theta_t}{1+\pi_t}) m_{t-1} \\ \lambda_t &= u_c(c_t, m_t, 1 - n_t), \end{aligned}$$

which correspond to the production function, the Fisher equation, the optimal choice of real balances, the law of motion of real balances, and the definition of the marginal utility of consumption. Verify the linearized versions of these 5 equations (as stated in the Appendix of Lecture 5), ie

$$\begin{aligned} \widehat{y}_t &= \alpha \widehat{k}_{t-1} + (1 - \alpha)\widehat{n}_t + z_t \\ \widehat{i}_t &= \widehat{r}_t + E_t \widehat{\pi}_{t+1} \\ \widehat{m}_t - \widehat{c}_t &= -(\frac{1}{b}) \frac{1}{i^{ss}} \widehat{i}_t \\ \widehat{m}_t &= \widehat{m}_{t-1} - \widehat{\pi}_t + u_t \\ \widehat{\lambda}_t &= \Omega_1 \cdot \widehat{c}_t + \Omega_2 \cdot \widehat{m}_t, \end{aligned}$$

where the last equation uses

$$\Omega_1 = (b - \Phi)\gamma - b, \quad \Omega_2 = (b - \Phi)(1 - \gamma), \quad \gamma = \frac{a\left(c^{ss}\right)^{1-b}}{a\left(c^{ss}\right)^{1-b} + (1-a)\left(m^{ss}\right)^{1-b}}.$$