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Monetary Theory and Policy
Summer Term 2013

Problem Set 4

Welfare Cost of Inflation in the Basic MIU model (Lecture 3)

Problem 1: Properties of utility functions:
Consider, again, the CES utility function,

$$u(c_t, m_t) = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}} \tag{1}$$

assuming $0 < a < 1$, $b > 0$ (and $b \neq 1$), as discussed in Lecture 3.
Using

$$u_m(c_t, m_t) = \frac{i_t}{1+i_t} u_c(c_t, m_t),$$

derive the elasticity of money demand with respect to the level of the interest rate i_t (ie $\eta_{m,\, i}$) and verify that for small values of i the two elasticities $\eta_{m,\, \frac{i}{1+i}}$ and $\eta_{m,\, i}$ are approximately equal.

Problem 2: Partial equilibrium estimates of welfare costs of inflation under log-log and semi-log specifications
Lecture 3 argues on p. 21 that in line with Lucas (2000) partial equilibrium welfare gains from a permanent reduction of the short-term nominal interest rate from a level i_1 of 10% to a level i_0 of 0% can be evaluated if one uses the consumer surplus expression

$$\int_0^{i_1} \widetilde{m}(x)dx - i_1 \widetilde{m}(i_1), \quad \text{with: } \widetilde{m} = \frac{m}{y}.$$

a) Consider the *log-log specification of money demand*

$$\frac{m}{y} = \widetilde{m} = A \cdot i^{-\eta}$$

and confirm

$$\int_0^{i_1} \widetilde{m}(x)dx - i_1 \widetilde{m}(i_1) = A \cdot \frac{\eta}{1-\eta} i_1^{1-\eta}.$$

b) Let $A = 0.05$ and assume $\eta = 0.5$.

b1) Confirm the result that a permanent reduction of i from 10% to 0% leads to a welfare gain of about 1.6% of annual income.
b2) Confirm the result that a permanent reduction of i from 10% to 3% implies that a residual welfare gain of about 0.9% of annual income remains unexploited.

c) Consider the *semi-log specification of money demand*

$$\frac{m}{y} = \widetilde{m} = \widetilde{A} \cdot e^{-\xi i}$$

and confirm

$$\int_0^{i_1} \widetilde{m}(x)dx - i_1\widetilde{m}(i_1) = \frac{\widetilde{A}}{\xi} \cdot [1 - e^{-\xi \cdot i_1}(1 + \xi \cdot i_1)]$$

d) Let $\widetilde{A} = 0.35$ and assume $\xi = 7$.
d1) Confirm the result that a permanent reduction of i from 10% to 0% leads to a welfare gain of about 0.8% of annual income.
d2) Confirm the result that a permanent reduction of i from 10% to 3% implies that a residual welfare gain of about 0.1% of annual income remains unexploited.

e) Discuss the implications of b) and d).

Problem 3: General equilibrium estimates of welfare costs of inflation
Lecture 3 uses on p. 27 f. the following utility function

$$u(c, m) = \frac{1}{1-\sigma}\{[c \cdot \varphi(\frac{m}{c})]^{1-\sigma} - 1\}, \quad \text{with: } \varphi(\frac{m}{c}) = \frac{1}{1 + A^2 \cdot \frac{c}{m}} \tag{2}$$

to discuss welfare costs of inflation from a general equilibrium perspective, in the spirit of Lucas (2000).

a) Show that the utility function (2) can be obtained from the utility function (1) discussed in Problem 1, using $b = 2 \Leftrightarrow \eta_{m, \frac{i}{1+i}} = 0.5$, via monotonic transformations.

b) Use the utility function (1) and verify for $b = 2$ the welfare measure established in Lecture 3, ie

$$w(\frac{i}{1+i}) = A \cdot \sqrt{\frac{i}{1+i}}$$