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Monetary Theory and Policy
Summer Term 2013

Problem Set 3

The Basic MIU model (Lecture 2) and Welfare Cost of Inflation in the Basic MIU model (Lecture 3)

Problem 1: Linearized dynamics
a) In the basic MIU model consider the example economy 2, as discussed in Lecture 2 on p. 45 f. Verify that on p. 47 the linearized equation (35), ie

$$\Leftrightarrow \quad m_{t+1} - m_1^* = \underbrace{[\sigma \frac{1+\theta}{\beta} + 1 - \sigma]}_{a_m > 1} \cdot (m_t - m_1^*) \tag{35}$$

can be derived from the original non-linear equation (34), ie

$$B(m_{t+1}) \equiv \frac{\beta}{1+\theta} \frac{1}{c^*} \cdot m_{t+1} = \frac{1}{c^*} m_t - \underbrace{m_t^{1-\sigma}}_{\phi_{m_t}(m_t) \cdot m_t} \equiv A(m_t) \tag{34}$$

if one uses a first-order Taylor expansion of (34) around the steady-state value

$$m_1^* = (\frac{1+\theta}{1+\theta-\beta} \cdot c^*)^{\frac{1}{\sigma}} > 0.$$

b) Use an appropriate graph to show why the linearized version does not capture important aspects of the global dynamics of the example economy.

Problem 2: Properties of utility functions:
Consider the CES utility function,

$$u(c_t, m_t) = [a c_t^{1-b} + (1-a) m_t^{1-b}]^{\frac{1}{1-b}}$$

assuming $0 < a < 1$, $b > 0$ (and $b \neq 1$), as discussed in Lecture 3.
a) Verify that the elasticity of substitution between consumption and real balances is given by $\frac{1}{b}$.
b) The parameter a captures the relative weight given to consumption in the utility function. Consider the steady-state version of

$$u_m(c_t, m_t) = \frac{i_t}{1+i_t} u_c(c_t, m_t),$$

and show that the steady-state ratio m^*/c^* decreases in a.
Moreover, show that m^*/c^* decreases in steady-state inflation.

Problem 3: Unique vs. multiple steady states in the basic MIU model
Consider, again, the CES utility function analyzed in Problem 2, ie

$$u(c_t, m_t) = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}}$$

Assume $b = 2$ (which is the value estimated by Lucas (2000)).
Discuss whether there exists a unique steady-state solution $m_1^* > 0$ or whether there exists a second hyperinflationary steady state with $m_2^* = 0$. Base your reasoning on equation (21) as derived in Lecture 2 on p. 19, ie

$$\frac{\beta}{1+\theta} u_c(c, m) \cdot m = [u_c(c, m) - u_m(c, m)] \cdot m.$$