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Monetary Theory and Policy
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Problem Set 2

The Basic MIU model

Problem 1: Intertemporal budget constraint of the representative household in the decentralized MIU model

Suppose the aggregate production

$$Y_t = F(K_{t-1}, N)$$

is of constant returns to scale (CRTS). For simplicity, the labour force is assumed to be constant, ie $N_t = N$, $\forall t \geqslant 0$. Moreover, assume that households receive their factor income from competitive factor markets which pay labour and capital according to their marginal products.

Let w_t denote the wage rate paid in period t and assume perfect substitutability between real bonds and physical capital such that $1 + r_{t-1} = 1 + f_k(k_{t-1}) - \delta$.

a) Show that the **flow budget constraint** of the household derived in the Lecture Notes, ie

$$f(k_{t-1}) + \tau_t + (1-\delta)k_{t-1} + (1 + r_{t-1})b_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + b_t + m_t$$

can be rewritten as

$$w_t + \tau_t + (1 + r_{t-1})(k_{t-1} + b_{t-1}) + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + b_t + m_t$$

b) Show that the transversality condition

$$\lim_{t\to\infty} \beta^t \lambda_t x_t = 0 \qquad x = k, b, m$$

implies that the flow budget constraint can be transformed into the **intertemporal budget constraint**

$$(1 + r_{-1})\psi_{-1} + \sum_{t=0}^{\infty} Q_t(w_t + \tau_t) = \sum_{t=0}^{\infty} Q_t(c_t + \frac{i_t}{1+i_t} m_t),$$

using

$$\begin{aligned} Q_0 &= 1 \quad \text{and} \quad Q_t = \prod_{j=0}^{t-1} \frac{1}{1+r_j} \quad \forall t \geqslant 1 \\ (1 + r_{-1})\psi_{-1} &\equiv (1 + f_k(k_{-1}) - \delta)\frac{K_{-1}}{N} + \frac{(1 + i_{-1})B_{-1} + M_{-1}}{P_0 N} \end{aligned}$$

Interpret the intertemporal budget constraint.