

Problem Set 1

First-order linear difference equations

Problem 1: First-order linear difference equations

Consider the first-order linear difference equation

$$x_{t+1} = (1 + r) \cdot x_t + a \quad \text{with: } r \neq 0, 1 + r > 0 \quad (1)$$

Think of (1) as a law of motion governing a bank account which offers a constant real interest rate $r \neq 0$ on the (beginning of period) balances x_t and which is subject to a constant deposit ($a > 0$) or withdrawal ($a < 0$) per period.

a) General solution

Verify that

$$x_t = c \cdot (1 + r)^t - \frac{a}{r} \quad (2)$$

is a general solution of (1), with unknown coefficient c .

b) Backwardlooking stability

Let $r < 0$ and assume that the initial balance in $t = 0$ is given (predetermined) by $x_0 > 0$. Derive the definite solution of (2) and state the value of $\lim_{T \rightarrow \infty} x_T$.

c) Forwardlooking stability

Let $r > 0$ and assume that in $t = 0$ the starting balance can be flexibly adjusted in order to satisfy the terminal condition $\lim_{T \rightarrow \infty} x_T = -\frac{a}{r}$. Derive the definite solution of (2).