Optimal Wage Policy with Endogenous Search Intensity*

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Abstract

Firms that counter outside offers face the moral hazard problem of rent seeking on-the-job search. When choosing a wage policy firms have to trade off the loss due to this moral hazard problem with the gain from a lower quitting probability. Given that step contracts provide the optimal wage tenure profile to reduce the quitting rate for all wage policies, the value of employment increases with tenure. Thus, firms can condition their wage policy on the value of employment a competing firm will offer. Since low productivity firms do not gain from matching an outside offer, they never counter an outside offers. High productivity firms, however, generally match outside offers of less productive firms, but do not match outside offers of equally or more productive firms.

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1 Introduction

Universities counter outside offers to prevent their professors from changing to other universities. In other labor markets, however, firms do not match outside offers. It is more common to condition a job offer on an applicant's characteristics to ensure that he is willing to accept the job. Sometimes firms, however, don't condition on a worker's reservation wage and find it optimal to take the risk of being turned down. The search literature considers wage policies where firms either condition or not condition their offer on a worker's characteristics and where firms either counter outside offers or don't. The question which wage policy is privatly optimal for firms is addressed in this paper.

In on-the-job search models based on Burdett and Mortensen (1998) firms commit not to match outside offers and to pay the same wage irrespective of how much workers' value their current job. Postel-Vinay and Robin (2002a,b) in contrary assume that firms condition their wage offers on the workers' current job value and react to outside offers. Given these two wage policy options a firm has to trade off the effect of a wage policy on the recruitment and retention probability and on the search effort of its workers. Matching outside offers ensures that a firm is successful in retaining a worker as long as it is more productive than the competing firm. The prospect of an expected promotion, if a worker contacts another firm, however, increases workers' incentive to search for another job. This moral hazard problem of rent seeking on-the-job search makes it attractive for firms to commit not to counter outside offers. However, in the Burdett-Mortensen model firms that commit not to counter outside offers also commit not to condition their offers on workers' value of employment. Unconditional wage offers, however, limit the chances that a worker will accept an offer. Thus, a firm has to trade off the higher recruitment and retention probability, if it is willing to counter outside offers, with the higher on-the-job search effort that induces such a wage policy.

Besides these two classes of models two additional combinations are generally possible: (a) Firms that commit not to counter outside offers but condition on the workers' characteristics and (b) firms that counter outside offers but commit not to condition their wages on the workers' characteristics. Firms of type (b) that make unconditional wage offers but counter outside offers are not able to recruit as many workers as PVR firms. At the same time they can only make the same profits per matched worker as PVR firms. Thus, such a wage policy must be suboptimal. Type (a) firms make the same profit per matched worker as BM firms. In addition, however, they recruit workers employed at BM firms. Thus, they recruit with the same probability as PVR firms, but do not face the same moral hazard problem. This makes the wage policy where firms (i) make conditional outside offers and (ii) make not to counter offers to equally or more productive firms optimal.

Wage policies might also differ with respect to their time structure. Stevens (2004), Burdett and Coles (2003, 2007) and Carrillo-Tudela (2009b) have shown that a wage tenure contract is optimal in an on-the-job search environment where a firms can commit to offer the same wage contract to all workers and not to counter outside offers. Wage tenure contracts are optimal, if labor laws or credit constraints prohibit contracts where workers pay an entry fee on accepting the job and are paid the marginal product during employment. For risk neutral workers the optimal shape of a wage tenure contract is a step contract with a wage equal to the minimum wage in the beginning and the marginal product after a specified period (see Stevens, 2004). Optimality arises because wage tenure contracts reduce the quitting rate of workers. In the present paper I show that step contracts are also optimal for firms that counter outside offers and condition their wage contracts on how much workers' value their current job, because step contracts increase a worker's value of employment at the fastest possible rate and therefore reduce the rent seeking search effort to a minimum. Furthermore, step contracts offered by firms that counter outside offers have a longer time to promotion than step contracts offered by firms that do not match outside offers.

Having shown the optimal shape of the wage tenure contract for different types of wage policies, I first investigate under which conditions BM and PVR wage policies coexist. BM firms that don't counter outside offers and that don't condition their wage offers on how much employed workers' value their current job exist, if the minimum wage is high enough to reduce the rent that firms with a matching wage policy can extract from their workers. A high minimum wage reduces the rent that a firm with a matching wage policy can extract from its workers because matching outside offers gives workers a higher value of employment as not matching outside offers. This requires a lower minimum wage for firms that counter outside offers to be able to extract the whole rent from their employees. A high minimum wage, therefore, increases the return of a wage policy that does not counter outside offers relative to a firm that matches outside offers. However, PVR firms that counter outside offers and condition their wage contracts on how much workers' value their current job always exist, because they can recruit unemployed as well as employed workers while firms that do not match outside offers can only recruit unemployed workers.

I then proceed to analysing the optimal wage policy. Since making conditional wage offers is the optmal hiring strategy, I investigate at which part of an employment spell it is optmal for a firm to counter outside offers. If all firms are equally productive, matching outside offers is never optimal, since commiting to counter outside offers induces outside firms to offer a wage equal to the marginal product. Furthermore, since workes employed at firms that match outside offers search more and therefore quit at a higher rate, matching outside offers of equally or more productive firms is never optimal. The dominance of the wage policy, where firms do not match outside offers, no longer hold for high productive firms. If high productive firms do not counter outside offers of low productivity firms, they loose workers although matching the outside offers of low productivity firms would still generate a positive profit for high productivity firms. Thus, more productive firms might benefit from a type-m wage policy, if their workers encounter a less productive firm.

The paper is related to several other papers. Assuming that recalling the last employer is not possible, Postel-Vinay and Robin (2004) compare the effects of the wage policy used in their 2002 papers with a type (b) wage policy. Thus, in their setting firms cannot commit to post unconditional wage offers. Postel-Vinay and Robin (2004), therefore, focus on the retention decision and disregard the recruitment aspect of the wage policy. They also assume constant wages and don't analyse the optimal time structure of wage contracts. Related to the present analysis is also Moscarini (2008). He investigates whether reputation effects can support the assumptions of the on-the-job search model by Burdett and Mortensen (1998). Moscarini (2008) does, however, not analyze which wage policy firms choose, if they deviate from the Burdett-Mortensen assumptions. Related to the present analysis is also a paper by Carrillo-Tudela (2009a), who compares firm's profits between the original Burdett-Mortensen framework, an adjusted Burdett-Mortensen framework, where firms condition their wage offers on the employment status of workers, and the framework by Postel-Vinay and Robin (2002a,b). In contrast to the present analysis, Carrillo-Tudela (2009a) compares the profits of firms across different frameworks without considering which wage policy firms would choose, if different wage policies are allowed to coexist.

The paper is structured as follows. Section 2 presents the framework. Section 3 analyses the optimal wage contracts offered by BM and PVR firms. Section 4 characterizes the equilibrium for homogenous firms and presents the condition under which BM and PVR wage policies coexist. Section 5 anaylses the optimal wage policy. Section 6 concludes.

2 The Framework

The model has an infinite horizon, is set in continuous time and concentrates on steady states. Workers and firms are infinitely lived and risk neutral. A worker's stay in the labor market is exponentially distributed at rate δ. The exit rate also serves as discount rate for workers and firms. The economy is populated by a unit mass of homogenous workers that face a unit mass of firms.

Firms and workers are brought together through a sequential and undirected search process. Unemployed and employed workers meet a firm at rate $\lambda+s$, where s is the search intensity chosen by a worker given the search cost function $c(s) = s^{(1+1/\alpha)}/(1+1/\alpha)$. Workers that search with intensity $s > 0$ search actively. While being unemployed a worker receives unemployment benefits b. If a firm offers a wage contract that the worker values at least as much as unemployment, then the worker accepts the offer. Workers also search on-the-job. A worker will change employer, if the outside firm offers a higher value of employment than the incumbent firm. A worker will quit into unemployment, if the value of unemployment exceeds the value of employment.

All firms observe whether the worker they meet is employed or unemployed. If a worker is employed, firms observe the worker's current value of employment, the productivity and the wage policy of the firm that competes for the same worker. Firms can choose between two wage policies. Type-m firms condition their wage contract $w^m(t, E)$ on the worker's current value of employment E and commit to counter outside offers. Type-m firms, therefore, behave like in Postel-Vinay and Robin (2002a,b) with the only difference that the offered wage can vary with tenure t. Type-n firms commit to a wage tenure contract ex-ante and don't condition their wage contract on a worker's value of employment like in Burdett and Mortensen (1998). Furthermore, wages can vary with tenure t like in Stevens (2004) and Burdett and Coles (2003). In section 5 we allow firms to mix both wage policies and to change the wage policy over the employment spell of a worker.

All firms are assumed to have the same productivity $p > b$. Let γ denote the fraction of firms that choose type n. Denote by E_0^n the workers expected lifetime utility, if the worker accepts the wage tenure contract $w^n(.)$ and uses an optimal quitting strategy in the future. Type-n firms might offer wage tenure contracts $w^n(.)$ that differ in the value of employment E_0^n. The distribution of wage contracts offered by type-n firms is denoted by $F(E_0^n)$ with support $\left[U, \overline{E}_0^n\right]$. The upper bound $\overline{E}_0^n = p/\delta$ is given by the wage-tenure contract that pays a wage equal to the marginal product p from the beginning.

Furthermore, I assume that all firms have to pay a wage that is at least as high as the minimum wage $\underline{w}$, with $\underline{w} \leq b$. If I assumed no minimum wage $\underline{w}$, firms would offer fee-contracts to all workers such that workers pay up-front for their job and are paid their marginal product while being employed (see Stevens, 2004). Since workers would have no incentive to search actively, fee contracts solve the moral hazard problem. The paper concentrate on an economy, where workers are unable to finance a fee-contract.

3 Workers' and firms' strategies

3.1 Workers' search strategy

Workers encounter a type-n firm at rate $[\lambda + s]\gamma$. An unemployed worker will only accept the offered value of employment E_0^n, if it is not lower than the value of being unemployed U. At rate $[\lambda + s](1-\gamma)$ workers encounter a type-m firm. Type-m firms offer workers a value of employment $E^m(w^m(., U))$. Since firms irrespective of their type will only attract a worker, if the offered wage contract ensures that an unemployed worker gets at least the value of unemployment, we can write the value of being unemployed as,

$$\begin{aligned}\delta U &= \max_s \left\{ b + [\lambda + s]\gamma \int_U^{\overline{E}^n} [E_0^n - U]\, dF(E_0^n) \right. \\ &\quad \left. + [\lambda + s](1-\gamma)[E^m(w^m(., U)) - U] - c(s) \right\}. \end{aligned} \tag{1}$$

A worker with tenure t that is employed at a type-n firm values employment according to the following Bellman equation, i.e.

$$\begin{aligned} \delta E^n\left(t|w^n\left(.\right)\right) &= \max_s \Big\{ w^n\left(t\right) + \dot{E}^n\left(t|w^n\left(.\right)\right) - c\left(s\right) \\ &\quad + \left[\lambda + s\right]\gamma \int_{E^n\left(t|w^n\left(.\right)\right)}^{\overline{E}^n} \left[E_0^n - E^n\left(t|w^n\left(.\right)\right)\right] dF\left(E_0^n\right) \\ &\quad + \left[\lambda + s\right]\left(1-\gamma\right)\left[E^m\left(w^m\left(., E^n\left(t|w^n\left(.\right)\right)\right)\right) - E^n\left(t|w^n\left(.\right)\right)\right]\Big\}. \end{aligned} \tag{2}$$

If $E^n\left(t|w^n\left(.\right)\right) < U$ for some tenure t, the worker's optimal strategy is to quit into unemployment. Given the current value of employment $E^n\left(t|w^n\left(.\right)\right)$ a worker employed at a type-n firm at tenure t will choose the search intensity such that the marginal cost equals the gain from searching, i.e.

$$\begin{aligned} s\left(E^n\left(t|w^n\left(.\right)\right)\right)^{\frac{1}{\alpha}} &= \gamma \int_{E^n\left(t|w^n\left(.\right)\right)}^{\overline{E}^n} \left[E_0^n - E^n\left(t|w^n\left(.\right)\right)\right] dF\left(E_0^n\right) \\ &\quad + \left(1-\gamma\right)\left[E^m\left(w^m\left(., E^n\left(t|w^n\left(.\right)\right)\right)\right) - E^n\left(t|w^n\left(.\right)\right)\right]. \end{aligned} \tag{3}$$

The convexity of the search cost function implies that the search intensity $s\left(E^n\left(t|w^n\left(.\right)\right)\right)$ decreases with a higher value of employment. Unemployed workers chose their search intensity similarly with $E^n\left(t|w^n\left(.\right)\right)$ being replaced by U.

Workers employed at a type-m firm at wage $w^m\left(., E\right)$ will similarly receive outside offers. If a worker meets another type-m firm, Bertrand competition between firms ensures that the worker will receive a wage equal to the marginal product p. The value of being employed at a wage equal to the marginal product is given by $E\left(p\right) = p/\delta$. If a worker meets a type-n firm, he is offered a value of employment E_0^n from the distribution $F\left(E_0^n\right)$. If the outside offer is higher than the current value of employment, i.e. $E_0^n > E^m\left(t|w^m\left(., E\right)\right)$, the worker will ask the current employer to match the outside offer and the current employer will offer the worker a wage contract $E^m\left(w^m\left(., E_0^n\right)\right) = E_0^n$ (plus epsilon). The value of being employed at a type-m firm at tenure t is, therefore, given by

$$\begin{aligned} \delta E^m\left(t|w^m\left(., E\right)\right) &= \max_s \Big\{ w^m\left(t, E\right) + \dot{E}^m\left(t|w^m\left(., E\right)\right) - c\left(s\right) \\ &\quad + \left[\lambda + s\right]\gamma \int_{E^m\left(t|w^m\left(., E\right)\right)}^{\overline{E}^n} \left[E_0^n - E^m\left(t|w^m\left(., E\right)\right)\right] dF\left(E_0^n\right) \\ &\quad + \left[\lambda + s\right]\left(1-\gamma\right)\left[E^m\left(p\right) - E^m\left(t|w^m\left(., E\right)\right)\right]\Big\}. \end{aligned} \tag{4}$$

The search intensity of workers employed at type-m firms at the wage $w^m(t, E)$ is chosen such that the marginal cost equals the gain from searching, i.e.

$$s\left(E^m\left(t|w^m(.,E)\right)\right)^{\frac{1}{\alpha}} = \gamma \int_{E^m(t|w^m(.,E))}^{\overline{E}^n} \left[E_0^n - E^m\left(t|w^m(.,E)\right)\right] dF\left(E_0^n\right) + (1-\gamma)\left[E^m(p) - E^m\left(t|w^m(.,E)\right)\right]. \tag{5}$$

The convexity of the search cost function again implies that the search intensity is decreasing in the value of employment $E^m(t|w^m(.,E))$. Furthermore, comparing the search intensity of workers with the same value of employment at different types of firms, i.e.$E^m(t|w^m(.,E)) = E^n(t|w^n(.))$, implies that workers employed at type-m firms search more than workers employed at type-n firms, since the gain from meeting a type-n firm is the same at both types of firms, but the gain from meeting a type-m firm is higher for workers employed at type-m firms, i.e. $E(p) > E^m(t|w^m(.,E))$.

3.2 Type-n firms' wage policy

If a type-n firm meets a worker, the firm offers a take-it-or-leave-it wage tenure contract to the worker. Since workers will only accept the wage contract, if the offered value of employment E_0^n is high enough to attract a worker, the firm has to take into account that a higher value of employment increases the probability of hiring a worker. When making the offer E_0^n the firm also accounts for the fact that a higher value of employment decreases the worker's expected gain from searching for better jobs and, therefore, reduces the quitting probability of a worker. Thus, a type-n firm has to trade off that a higher value of employment reduces profits, but increases the hiring rate and the staying probability of a worker.

Since a firm will only attract a worker, if it offers a value of employment that exceeds the value of unemployment, all type-n firms offer a wage-tenure contract with $E_0^n \geq U$. Denote the steady state number of unemployed workers by u and the steady state number of workers employed at type-n firms at a value of employment equal or less than E^n by $L^n(E^n)$. Then a type-n firm offering a value of employment E_0^n will hire workers at rate,

$$h^n\left(E_0^n\right) = \left[\lambda + s(U)\right] u + \int_U^{E_0^n} \left[\lambda + s\left(E^n\right)\right] dL^n\left(E^n\right). \tag{6}$$

The quitting probability of an employed worker at tenure t depends on the search intensity of the worker and the probability that the worker meets a firm that offers him a higher value of employment, i.e. $(1-\gamma)+\gamma\left[1-F\left(E^{n}\left(t|w^{n}(.)\right)\right)\right]$. Thus, for a type-$n$ firm that offers a wage-tenure contract $w^{n}(.)$ the value of employing a worker with tenure t is given by the following differential equation, i.e.

$$J^{n}\left(t|w^{n}(.)\right)=\frac{p-w^{n}(t)+\dot{J}^{n}\left(t|w^{n}(.)\right)}{\delta+\left[\lambda+s\left(E^{n}\left(t|w^{n}(.)\right)\right)\right]\left[1-\gamma F\left(E^{n}\left(t|w^{n}(.)\right)\right)\right]}. \tag{7}$$

A type-n firm maximizes its expected profit,

$$\Pi^{n}\left(E_{0}^{n},w^{n}(.)\right)=\max_{E_{0}^{n},w^{n}(.)}h^{n}\left(E_{0}^{n}\right)J^{n}\left(0|w^{n}(.)\right), \tag{8}$$

by chosing the optimal wage tenure profile $w^{n}(.)$ and the value E_{0}^{n} of employment that is offered to contacted workers.

The optimization problem of the firm can be decomposed. Conditional on the offered value of employment E_{0}^{n} to newly hired workers, one can determine the optimal wage tenure contract $w^{n}(.)$. Similar to Stevens (2004) the optimal wage tenure contract is a step contract where workers are paid $\underline{w}$ until the time $T^{n}\left(E_{0}^{n}\right)$ to promotion. After promotion they are paid the marginal product p. The time to promotion $T^{n}\left(E_{0}^{n}\right)$ is chosen such that the value of employment equals E_{0}^{n}, i.e. $E^{n}\left(0|\underline{w},T^{n}\left(E_{0}^{n}\right)\right)=E_{0}^{n}$. This is formally shown in the following Lemma.

Lemma 1: *Given an $E_{0}^{n}\in[U,E(p))$ a step contract $\left(\underline{w},T^{n}\left(E_{0}^{n}\right)\right)$ is optimal for type-n firms.*

Proof: See Appendix.

The proof of Lemma 1 relies on showing that for any $E_{0}^{n}\in\left[U,E^{n}(p)\right)$ a step contract maximizes the expected value of employing a newly hired worker $J^{n}\left(0|w^{n}(.)\right)$ by minimizing the probability that the worker quits. The quitting probability is minimized by taking the worker's search and quitting incentives away at the fastest possible way. Since a worker has no incentive to change jobs, if he earn his marginal product, the quitting probability is minimized by paying the worker the marginal product as soon as possible, i.e. as soon as the firm has extracted all the surplus of the match. The best way for a firm to extract all the surplus, i.e. to maximize $J^{n}\left(0|w^{n}(.)\right)$, is to offer him a lowest possible wage $\underline{w}$ in the beginning and to commit to paying him the marginal product p at the tenure $T^{n}\left(E_{0}^{n}\right)$ such that the worker is just willing to accept the offer, i.e. $E^{n}\left(0|\underline{w},T^{n}\left(E^{n}\right)\right)=E_{0}^{n}$.

To simplify the analysis consider the following re-normalisation. The type-n firm offering the lowest value of employment will post a step contract that makes unemployed workers indifferent between accepting employment or staying unemployed. Define the offered time to promotion T_u^n as $E^n(0|\underline{w}, T_u^n) = U$.[1] Following Burdett and Coles (2003) we denote the wage contract $(\underline{w}, T_u^n)$ as the baseline salary scale. Thus, all type-n firms offer a time to promotion $T^n(E_0^n) \leq T_u^n$. Given the definition of the baseline salary scale, let τ denote the position of a type-n firm on the baseline salary scale with $\tau \in [-T_u^n, 0]$. Note, that $\tau < 0$ and that firms that offer a position $\tau' > \tau$ offer a value of employment $E^n(\tau') > E^n(\tau)$. Equivalently, the longer a worker is employed at a type-n firm the higher is the position on the baseline salary scale.

Given the optimal wage tenure profile for a given value of employment E_0^n that firms offer to newly hired workers, firms have to decide on the optimal value of employment E_0^n that trades off a higher hiring rate $h(E_0^n)$ with a lower value of employing a worker $J^n(0|w^n(.))$.

Firms will find it optimal to offer wage contracts that are acceptable for unemployed workers. Thus, all unemployed workers that meet a firm will become employed. The measure of unemployed workers in the economy is, therefore, given by $u = \delta / [\delta + \lambda + s(U)]$. The number of workers employed at type-n firms at a value $E^n(\tau)$ or lower is denoted by $L^n(E^n(\tau))$. Since unemployed workers start by definition at the position $\tau = -T_u^n$ on the baseline salary scale, it follows that $L^n(E^n(-T_u^n)) = [\lambda + s(U)]\gamma u$. $L^n(E^n(0)) - L^n(E^n(\tau))$ denotes the number of workers employed at type-n firms with a position on the baseline salary scale no lower than τ. Workers exit the pool $L^n(E^n(0)) - L^n(E^n(\tau))$ of workers either because of they exit the labor market (at rate δ) or because they are matched with a type-m firm (at rate $[\lambda + s(E^n(\tau'))](1-\gamma)$). Workers enter this group either because they are hired from other type-n firms (at rate $[1 - F(E^n(\tau))]\gamma \int_{-T_u^n}^{\tau} [\lambda + s(E^n(\tau'))] dL^n(E^n(\tau'))$) or because their tenure evolves, i.e. $dL^n(E^n(\tau))/d\tau$. Equating in- and outflows determines the steady state differential equation for the number of workers employed at the position τ on the baseline salary scale,

[1] As long as some type-m firms exist, i.e. $\gamma < 1$, the time to promotion is finite for any $\underline{w} \leq b$. Since $\delta U \geq b$, it follows that for $\underline{w} < b$, a finite time to promotion T_u^n exists such that $E^n(0|\underline{w}, T_u^n) = U$. If $\underline{w} = b$, any wage contract offered by type-m firms offers a value of employment $E^m(w^m(., U)) > U$, since workers are paid their marginal product, if they meet another type-m firm, i.e. $E^m(p) > E^m(w^m(., U))$. Since $E^m(w^m(., U)) > U$, it follows that $\delta U > b$, which ensures a finite time to promotion T_u^n exists such that $E^n(0|\underline{w}, T_u^n) = U$.

i.e.

$$\frac{dL^n\left(E^n\left(\tau\right)\right)}{d\tau}+\left[1-F\left(E^n\left(\tau\right)\right)\right]\gamma\int_{-T_u^n}^{\tau}\left[\lambda+s\left(E^n\left(\tau'\right)\right)\right]dL^n\left(E^n\left(\tau'\right)\right) \tag{9}$$
$$=\int_{\tau}^{0}\left[\delta+\left[\lambda+s\left(E^n\left(\tau'\right)\right)\right]\left(1-\gamma\right)\right]dL^n\left(E^n\left(\tau'\right)\right).$$

Note, that type-n firms are not able to hire any worker employed at a type-m firm, since type-m firms successfully counter any offer made by type-n firms.

Given the pool of unemployed and employed workers that type-n firms can recruit from, they chose the value of employment E_0^n, or equivalently the starting position τ on the baseline salary scale, that maximizes expected profits. Like in Stevens (2004) type-n firms find it optimal to offer a step contract $(\underline{w},-T_u^n)$ that extracts the whole rent from unemployed workers to all workers that are contacted.

Proposition 1: *All type-n firms offer the same step contract $(\underline{w},-T_u^n)$ to all contacted workers.*

Proof: See Appendix.

Type-n firms have no incentive to deviate from the step contract that offers workers the value of being unemployed. A marginal decrease in the time to promotion does – in contrast to a marginal increase in the wage in Burdett and Mortensen (1998) – not lead to a significantly higher number of workers, since all workers employed for some small time already have a higher value of employment. Thus, the number of workers that are willing to accept a slightly lower time to promotion is only marginal. This makes a deviation from the mass point where firms extract the maximum rent from unemployed workers unprofitable. Thus, type-n firms hire only unemployed workers, i.e. $\left[\lambda+s\left(U\right)\right]u$.

3.3 Type-m firms' wage policy

Since type-m firms take the value of employment into account when offering a wage contract, their optimization problem reduces to maximizing the value of employing a newly hired workers $J^m\left(0|w^m\left(.,E\right)\right)$ conditional on the worker's current value of employment $E\in\left[U,E\left(p\right)\right)$ and the level of the minimum wage $\underline{w}$. The hiring rate $h^m\left(E\right)$ of workers with a value of employment E is given by, $h^m\left(U\right)=\left[\lambda+s\left(U\right)\right]u$ for unemployed workers and by $h^m\left(E^n\left(\tau\right)\right)=\left[\lambda+s\left(E^n\left(\tau\right)\right)\right]\dot{L}^n\left(E^n\left(\tau\right)\right)$ for workers employed at the position τ on the baseline salary scale at type-n firms. Note, that no worker is hired from another type-m firms, since the incumbent type-m firm counters the outside offer by offering the marginal product.

Workers leave the firm either because workers exit the labor market (at rate δ) or because workers meet another type-m firm and are paid the marginal product (at rate $[\lambda + s(E^m(t|w^m(.,E)))](1-\gamma)$). If employed workers meet a type-n firm, they are offered the value of being unemployed, i.e. $E^n(-T_u^n) = U$, which they decline. Thus, for a type-m firm the value of employing a worker at tenure t is given by,

$$J^m(t|w^m(.,E)) = \frac{p - w^m(t,E) + \dot{J}^m(t|w^m(.,E))}{[\delta + [\lambda + s(E^m(t|w^m(.,E)))](1-\gamma)]}. \tag{10}$$

The expected profit of a type-m firm is then given by,

$$\begin{aligned} \Pi^m &= [\lambda + s(U)]\, u \max_{w^m(.,U)} J^m(w^m(.,U)) \\ &\quad + \int_{-T_u^n}^{0} [\lambda + s(E^n(\tau))] \max_{w^m(.,E^n(\tau))} J^m(w^m(.,E^n(\tau)))\, dL^n(E^n(\tau)). \end{aligned} \tag{11}$$

subject to $w^m(t, E^n(\tau)) \geq \underline{w}$ for all $t \geq 0$.

Conditional on $E \in [U, E(p))$ we can determine the optimal wage tenure contract $w^m(.,E)$. Similar to a wage tenure contract at type-n firms a step contract is defined such that workers are paid $\underline{w}$ until the time to promotion $T^m(E)$ or until workers meet another type-m firm. Thereafter, they are paid the marginal product p. The time to promotion need not be finite. If paying the minimum wage until workers meet another type-m firm, ensures a value of employment that is higher than their current value of employment, type-m firms will offer a constant minimum wage as shown in Lemma 2 below. The value of being employed at a type-m firm paying a constant minimum wage is according to equation (4) given by $E^m(\underline{w})$ with $\dot{E}^m(\underline{w}) = 0$.

Lemma 2: *Given an $E \in [U, E^m(\underline{w})]$ a constant minimum wage is optimal for type-m firms. Given an $E \in (E^m(\underline{w}), E(p))$ a step contract $(\underline{w}, T^m(E))$ is optimal for type-c firms.*

Proof: See Appendix.

Step contracts are optimal for type-m firms, because they decrease the search cost incured by employed workers. Paying a constant wage implies that employed workers search until they meet another type-m firm. The associated search cost reduces the match surplus. With the step contract workers initially search with the same intensity as with a constant wage contract (since the have the same value of employment). However, a worker employed at a step contract decreases his search intensity the closer he gets to the time of promotion. Thus, the overall search cost associated with a step contract is lower than the overall search cost associated with a constant wage contract. If type-m firms are able to extract this additional match surplus from workers, step contracts are optimal for type-c firms. However, if the minimum wage is so high that the firm cannot extract all the surplus type-m firms find it optimal to offer a constant wage. Thus, only if the minimum wage is high enough, i.e.

$$E^m(\underline{w}) \geq U \Longleftrightarrow \underline{w} \geq \underline{W} = b - \left[\lambda + \frac{s(E^m(\underline{W}))}{1+\alpha}\right](1-\gamma)\frac{p-b}{\delta}, \qquad (12)$$
$$\text{where} \quad s(E^m(\underline{W}))^{\frac{1}{\alpha}} = (1-\gamma)[p-b]/\delta,$$

type-m firms offer a constant minimum wage contract to unemployed workers and workers employed at type-n firms at $E^n(\tau) \leq E^m(\underline{w})$.

Since workers employed at type-m firms are promoted then they meet another type-m firm, the time to promotion for a worker with the same value of employment E is longer at a type-m firm than at a type-n firms.

Lemma 3: *Given an $E \in [U, E(p))$ the time to promotion at a type-m firm is longer than at a type-n firm, i.e. $T^m(E) > T^n(E)$.*

Proof: See Appendix.

A worker employed at a type-m firm with time to promotion T values his employment higher than a worker employed at a type-n firm with the same time to promotion, i.e. $E^n(T) < E^m(T)$ for all $T \in [0, T_u^n]$, because workers employed at type-m firms are in addition to workers, who are employed at type-n firms, promoted, if they meet another type-m firm. Thus, the value of employment E can only be identical across types, if the time to promotion is longer at type-m firms.

This also explains, why type-m firms are more vulnerable to a rise in the minimum wage $\underline{w}$ than type-n firms. Both firms will react to an increase in the minimum wage by increasing the time to promotion in order to extract the maximum rent possible from unemployed workers. Since type-m firms are offering a longer time to promotion than type-n firms, a rise in the minimum wage sooner restricts their ability to extract the whole rent from unemployed workers.

The baseline salary scale also re-normalizes the time to promotion at type-m firms. If the minimum wage is not binding, i.e. $\underline{w} < \underline{W}$, type-$m$ firms offer step contracts with a finite time to promotion. Workers employed at type-n firms at a position τ on the baseline salary scale are offered by type-m firms a step contract $(\underline{w}, \hat{\tau}(\tau))$ that gives the same value of employment (plus epsilon). Thus the function $\hat{\tau}(\tau)$ is defined such that $E^m(\hat{\tau}(\tau)) = E^n(\tau)$. Lemma 3 implies $d\hat{\tau}(\tau)/d\tau > 1$ for all $\tau \in [-T_u^n, 0]$. If the minimum wage is binding, i.e. $\underline{w} \geq \underline{W}$, type-$m$ firms will offer workers with $E \in [U, E^m(\underline{w}))$ a constant minimum wage, i.e. an infinite time to promotion. Workers with a value of employment $E \in [E^m(\underline{w}), E^m(p))$ are offered step contracts. In case of a binding minimum wage $\underline{w} \geq \underline{W}$, define $\tau_{\underline{w}}$ such that workers employed at type-n firms value their job as high as workers employed at type-m firms with a constant minimum wage contract, i.e. $E^m(\underline{w}) = E^n(\tau_{\underline{w}})$. Thus, $\hat{\tau}(\tau) = -\infty$ for $\tau \leq \tau_{\underline{w}}$ and $\hat{\tau}(\tau)$ is defined by $E^m(\hat{\tau}(\tau)) = E^n(\tau)$ for $\tau > \tau_{\underline{w}}$.

4 Equilibrium characterization

In equilibrium workers chose their search intensity optimally according to equations (3) and (5). They accept only wage contracts that offer them a value of employment no lower than the value of unemployment and employed workers only change employers, if they are offered a higher value of employment. In equilibrium the number of unemployed workers u and the number of workers employed at type-n and at type-m firms need to be consistent with steady state turnover.

Since firms can always make positive profits, i.e. $\Pi^n(\underline{w}, -T_u^n) = \Pi^n > 0$ and $\Pi^m > 0$, if they offer the level of unemployment benefits for ever, all firms irrespective of their type have to make the positive profit in equilibrium. Furthermore, both types of firms only coexist in equilibrium, if

$$\Pi^n = \Pi^m > 0, \tag{13}$$

for all type-m firms and all type-n firms.

4.1 Workers' equilibrium payoffs

Since all type-n firms offer in equilibrium the same value of employment, workers employed at type-n firms do not gain from meeting another type-n firm. Workers employed at type-n

firms only gain from searching actively, if type-m firms are constrained by the minimum wage, i.e. $\underline{w} \geq \underline{W}$, and if workers' current value of employment is below $E^m(\underline{w})$, i.e. $E^n(\tau) \in [U, E^m(\underline{w}))$. The search intensity chosen by workers employed at type-n firms is, therefore, given by,

$$s\left(E^n(\tau)\right)^{\frac{1}{\alpha}} = \begin{cases} (1-\gamma)\left[E^m(\underline{w}) - E^n(\tau)\right] & \text{for } -T_u^n \leq \tau \leq \tau_{\underline{w}} \text{ and } \underline{w} \geq \underline{W}, \\ 0 & \text{for } \tau > \tau_{\underline{w}} \text{ or } \underline{w} < \underline{W}, \end{cases} \tag{14}$$

where $\tau_{\underline{w}}$ is defined as $E^m(\underline{w}) = E^n(\tau_{\underline{w}})$. Thus, workers employed at type-n firms at a position $\tau \leq \tau_{\underline{w}}$ search actively, while workers employed at a position $\tau > \tau_{\underline{w}}$ stop searching. Type-n wage policy of "not to condition" the job offer on the worker's value of employment and not to match outside offers, therefore, eliminates the moral hazard problem of rent seeking on-the-job search, only if type-m firms are not constrained by the minimum wage.

Since all type-n firms offer the step contract $(\underline{w}, -T_u^n)$, $\overline{F}(\tau) = 0$ for all $\tau < -T_u$ and $\overline{F}(\tau) = 1$ for all $\tau \geq -T_u^n$. Thus, the value of employment at a type-n firm at the position τ on the baseline salary scale is given by,

$$\delta E^n(\tau) = \begin{cases} \underline{w} + \dot{E}^n(\tau) + \left[\lambda + \dfrac{s\left(E^n(\tau)\right)}{1+\alpha}\right] & (1-\gamma)\left[E^m(\underline{w}) - E^n(\tau)\right] \\ & \text{for } -T_u^n \leq \tau \leq \tau_{\underline{w}} \text{ and } \underline{w} \geq \underline{W} \\ \underline{w} + [p - \underline{w}]\, e^{\delta\tau} & \text{for } \tau_{\underline{w}} < \tau < 0 \text{ or } \underline{w} < \underline{W}, \\ p & \text{for } \tau \geq 0. \end{cases} \tag{15}$$

Workers employed at type-m firms that earn less than their marginal product always search actively, since they gain from meeting another type-m firm. The search intensity of workers being employed at a constant minimum wage contract $E^m(\underline{w})$ is given by,

$$s\left(E^m(\underline{w})\right)^{\frac{1}{\alpha}} = (1-\gamma)\left[E(p) - E^m(\underline{w})\right]. \tag{16}$$

Workers employed at type-m firms with a step contract will search with intensity $s\left(E^m(\tau)\right)$, where

$$s\left(E^m(\tau)\right)^{\frac{1}{\alpha}} = (1-\gamma)\left[E(p) - E^m(\tau)\right] \quad \text{for} \quad \tau < 0, \tag{17}$$

and $s\left(E^m(\tau)\right) = 0$ for $\tau \geq 0$.

The value of employment at a type-m firm depends on the pervious value of employment and the type of firm that was last contacted. If the minimum wage is restrictive, i.e.

$\underline{w} \geq \underline{W}$, workers employed at a constant minimum wage contract at type-m firms value their job according to the following Bellman equation,

$$\delta E^m(\underline{w}) = \underline{w} + \left[\lambda + \frac{s(E^m(\underline{w}))}{1+\alpha}\right](1-\gamma)[E(p) - E^m(\underline{w})]. \tag{18}$$

The value of being employed at a type-m firm at the position $\tau \in (-\infty, 0)$ on the baseline salary scale is given by,

$$\delta E^m(\hat{\tau}(\tau)) = \underline{w} + \dot{E}^m(\hat{\tau}(\tau)) + \left[\lambda + \frac{s(E^m(\hat{\tau}(\tau)))}{1+\alpha}\right](1-\gamma)[E(p) - E^m(\hat{\tau}(\tau))]. \tag{19}$$

Workers with a position $\tau \geq 0$ on the baseline salary scale and workers that met with another type-m firm are paid the marginal product, i.e. $\delta E(p) = p$.

4.2 Equilibrium profits

Since all type-n firms offer step-contracts that attract only unemployed workers, the hiring rate of type-n firms is given by λu, if the minimum wage is so low that unemployed workers do not gain from searching, i.e. $\underline{w} \leq \underline{W}$, and $[\lambda + s(U)]u$, if the minimum wage is restrictive for type-m firms, i.e. $\underline{w} > \underline{W}$.

For a type-n firm the value $J^n(\tau)$ of employing a worker at the position τ on the baseline salary scale can be obtained by solving the differential equation (7). Workers with a value of employment $E^n(\tau) \geq E^n(\tau_{\underline{w}})$ do not search actively for a job, i.e. $s(E^n(\tau)) = 0$. The value of employing a worker at a position $\tau > \tau_{\underline{w}}$ is therefore given by,

$$J^n(\tau) = \frac{[p - \underline{w}]\left[1 - e^{[\delta + \lambda(1-\gamma)]\tau}\right]}{\delta + \lambda(1-\gamma)}, \tag{20}$$

for $\tau_{\underline{w}} < \tau < 0$ or $\underline{w} \leq \underline{W}$ and $J^n(\tau) = 0$ for $\tau \geq 0$. Workers employed at a value of employment $E^n(\tau) < E^n(\tau_{\underline{w}})$ search actively. The value of employing a worker at a position $\tau \leq \tau_{\underline{w}}$ is therefore given by,

$$J^n(\tau) = [p - \underline{w}] \int_{\tau}^{0} e^{-\int_{\tau'}^{0}[\delta + [\lambda + s(E^n(\tau''))](1-\gamma)]d\tau''} d\tau' \tag{21}$$

for $-T_u^n \leq \tau \leq \tau_{\underline{w}}$ and $\underline{w} > \underline{W}$.

The expected profit of a type-n firm that offers the wage tenure contract $(\underline{w}, -T_u)$ is then given by,

$$\Pi^n = \begin{cases} \lambda u \left[p - \underline{w}\right] \dfrac{1 - e^{-[\delta + \lambda(1-\gamma)]T_u^n}}{\delta + \lambda(1-\gamma)} & \text{for } \underline{w} \leq \underline{W}, \\ \left[\lambda + s(U)\right] u \left[p - \underline{w}\right] \displaystyle\int_{-T_u^n}^{0} e^{-\int_{\tau'}^{0}[\delta + [\lambda + s(E^n(\tau''))](1-\gamma)]d\tau''} d\tau' & \text{for } \underline{w} > \underline{W}. \end{cases} \tag{22}$$

If the number of type-m firms increases, i.e. $1-\gamma$ increases, expected profits of a type-n firms decrease, because the likelihood that an employed worker contacts a type-m firm increases. This effect is more severe, if the minimum wage is binding, i.e.$\underline{w} > \underline{W}$, since the associated gain from meeting a type-m firm increases the search intensity of workers employed at type-n firms. In addition the gain from searchin also increases the search intensity of unemployed workers and therefore reduces the pool of workers a type-n firm can hire from.

By conditioning their wage offer on the worker's value of employment type-m firms are able to attract unemployed workers and workers that are employed at type-n firms at a position $\tau < 0$ on the baseline salary scale. Workers employed at other type-m firms, however, do not change employers.

The value of employing a worker depends on the level of the minimum wage and on the value of employment that a type-m firm has to offer in order to persuade a worker to accept the offer. If the minimum wage is not binding, i.e. $\underline{w} \leq \underline{W}$, type-$m$ firms can offer step contracts. For a type-m firm the value of employing a worker at the position τ of the baseline salary scale is given by,

$$J^m(\hat{\tau}(\tau)) = \left[p - \underline{w}\right] \int_{\hat{\tau}(\tau)}^{0} e^{-\int_{\tau'}^{0}[\delta + [\lambda + s(E^m(\tau''))](1-\gamma)]d\tau''} d\tau', \tag{23}$$

for $\underline{w} \leq \underline{W}$. The value of employing a worker that is offered a constant minimum wage contract is given by,

$$J^m(\underline{w}) = \frac{p - \underline{w}}{\delta + \left[\lambda + s(E^m(\underline{w}))\right](1-\gamma)}. \tag{24}$$

Given the respective hiring probabilities the expected profit of a type-m firm is given by,

$$\Pi^m = \begin{cases} \lambda u J^m(-T_u^m) + \int_{-T_u^n}^{0} \lambda J^m(\hat{\tau}(\tau))\, dL^n(E^n(\tau)) & \text{for } \underline{w} \leq \underline{W}, \\ \left[\left[\lambda + s(U)\right] u + \int_{-T_u^n}^{\tau_{\underline{w}}} \left[\lambda + s(E^n(\tau))\right] dL^n(E^n(\tau))\right] J^m(\underline{w}) & \\ \quad + \int_{\tau_{\underline{w}}}^{0} \lambda J^m(\hat{\tau}(\tau))\, dL^n(E^n(\tau)) & \text{for } \underline{w} > \underline{W}. \end{cases} \tag{25}$$

If the number of type-m firms increases, the expected profits of a type-m firm decrease for two reasons. First, the probability that a type-m firm has to pay a worker the marginal product increases, since the likelihood that an employed worker meets a type-m firm increases directly due to the larger number of type-m firms and indirectly due to an increase in the search intensity. Second, type-m firms are less likely to meet and hire workers employed at type-n firms. While the first effect is also present at type-n firms, the second effect only affects type-m firms, since type-n firms only recruit unemployed workers.

4.3 Condition for the existence of both types

Comparing the expected profits of type-n and type-m firms reveals that type-m firms will always exist. The reason for the robust existence of type-m firms is that the moral hazard problem of rent seeking on-the-job search decreases with the number of type-n firms in the market, because gains from searching actively are only high, if the chances of meeting a type-m firm are high. Thus, the loss due to the rent seeking search of employed workers is small, if the number of type-m firms is small. In addition, the advantage of the type-m wage policy of being able to hire workers from type-n firms increases with the number of type-n firms. Thus, if there was no type-m firm in the market, it would be profitable for a type-n firm to change to a type-m wage policy, because it could recruit employed as well as unemployed workers and would not face the moral hazard problem.

Type-n firms might not exist, if high search cost reduce the moral hazard problem of rent seeking on-the-job search to such an extend, that the advantage of the type-m wage policy – to be able to recruit from type-n firms – dominates the disadvantage of the increased on-the-job search intensity of workers. If search costs are low enough and the search intensity and quitting rate at type-m firms high enough, the type-n wage policy becomes profitable. As type-n firms enter the market, they increase the hiring possibilities and the expected profits of type-m firms. Thus, type-n firms will only enter until the fraction of type-n firms is high enough to ensure equal profits for both wage policies. This is stated in the first part of Proposition 2.

Proposition 2: *(i) Type-m firms always exist. For any given* $\underline{w} < b$ *there exists an* $\alpha^* > 0$ *such that for any* $\alpha \in (0, \alpha^*)$ *only type-m firms exist and for any* $\alpha \in [\alpha^*, \infty)$ *type-n and type-m firms coexist.*

(ii) For any given $\alpha > 0$ there exists a $\underline{w}^* \in (\underline{W}, b)$ such that for $\underline{w} \in [\underline{w}^*, b]$ type-n and type-m firms coexist.

Proof: See Appendix.

Type-m and type-n firms also coexist, if the minimum wage is high enough to restrict the expected profits of type-m firms to such an extend that they cannot extract the whole rent from unemployed workers. Type-m firms are more restricted by a high minimum wage than type-n firms, since the commitment to counter outside offers reduces the expected rent that type-m firms can extract over a certain period of time. This is also the reason why type-n firms can offer a step contract while type-m firms have to pay a constant minimum wage contract, if $\underline{w} \geq \underline{W}$.

To understand the second part of Proposition 2 suppose that only type-m firms exist at a high minimum wage $\underline{w} \geq \underline{W}$, where type-$m$ firms offer constant minimum wage contracts. Since only type-m firms exist, firms only hire unemployed workers. Thus, type-m firms have no hiring advantage. At the same time a type-n wage policy leads to a lower quitting rate than a type-m wage policy. As the minimum wage increases time to promotion at type-n firms also increases such that the contract offered by type-n firms is almost identical to the constant minimum wage contract offered by type-m firms with the important difference that type-m firms' commitment to counter outside offers leads to higher quitting rates and therefore lower profits.

5 Optimal wage policy

Firms that make unconditional wage offers but counter outside offers are not able to recruit as many workers as firms that make conditional offers. At the same time they can only make the same profits per matched worker as type-m firms. Thus, such a wage policy must be suboptimal.

Now consider a wage policy where firms condition their offer on the contacted worker's value of employment and match outside offers and denote it as type-c wage policy. Such a wage policy guarantees the same profit per matched worker as type-n wage policy, but it also ensure the same hiring rate as a type-m wage policy. A type-c wage policy therefore generates higher profits than a type-n wage policy. Thus, the following analysis investigates at which point of an employment spell a type-m or a type-c wage policy is optimal.

5.1 Workers' search intensity

In order to show that a type-c wage policy always dominates a type-m wage policy, if firms are homogenous, I start with assuming that firms either use a type-c or a type-m wage policy. Unemployed workers encounter a type-c firm at rate $[\lambda + s]\,\gamma$ and a type-m firm at rate $[\lambda + s]\,(1-\gamma)$, A worker will only accept the offered value of employment $E^j\left(w^j\left(.\right), U\right)$ with $j \in \{m, c\}$, if it is not lower than the value of being unemployed U. Since firms can only make profits, if they employ somebody, they offer a wage contract that ensures that unemployed workers get at least the value of unemployment. Thus, an unemployed worker chooses the search intensity s such that the value of being unemployed is maximized, i.e.,

$$\begin{aligned}\delta U &= \max_s \{b + [\lambda + s]\,\gamma\,[E^c\left(w^c\left(.\right), U\right) - U] \\ &\quad + [\lambda + s]\,(1-\gamma)\,[E^m\left(w^m\left(.\right), U\right) - U] - c\left(s\right)\}\,.\end{aligned} \tag{26}$$

If a worker employed at a type-c firm is contacted by another non-matching firm, he will be offered the same value of employment plus epsilon, i.e. $E^c\left(w^c\left(.\right), E^c\left(t|w^c\left(.\right)\right)\right) = E^c\left(t|w^c\left(.\right)\right) + \varepsilon$ with $\varepsilon > 0$. Thus, employed workers do not gain anything, if they meet a type-c firm. If an employed worker contacts a matching firm, he might – depending on the level of the minimum wage – be offered the same value of employment (plus epsilon) or a higher value (if the type-m firm is constraint by the minimum wage). Since workers quit into unemployment, if the value of employment falls below the value of unemployment, firms will only offer wage contracts that ensure $E^c\left(t|w^c\left(.\right)\right) \geq U$ at all t. A worker that is employed at a type-c firm at tenure t and wage tenure contract $w^c\left(t\right)$ chooses the search intensity s that maximizes the value of being employed, i.e.,

$$\begin{aligned}\delta E^c\left(t|w^c\left(.\right)\right) &= \max_s \Big\{w^c\left(t\right) + \dot{E}^c\left(t|w^c\left(.\right)\right) - c\left(s\right) \\ &\quad + [\lambda + s]\,(1-\gamma)\,[E^m\left(w^m\left(.\right), E^c\left(t|w^c\left(.\right)\right)\right) - E^c\left(t|w^c\left(.\right)\right)]\Big\}\,.\end{aligned} \tag{27}$$

Given the current value of the wage tenure contract $E^c\left(t|w^c\left(.\right)\right)$ a worker employed at a type-c firm will choose the search intensity such that the marginal cost equals the gain from searching for another job, i.e.,

$$s\left(E^c\left(t|w^c\left(.\right)\right)\right)^{\frac{1}{\alpha}} = (1-\gamma)\,[E^m\left(w^m\left(.\right), E^c\left(t|w^c\left(.\right)\right)\right) - E^c\left(t|w^c\left(.\right)\right)]\,. \tag{28}$$

Since the gain from meeting another type-c firm is zero, only the potential gains from meeting a type-m firm induce the worker to search. The convexity of the search cost function implies that the search intensity $s\left(E^c\left(t|w^c\left(.\right)\right)\right)$ decreases with a higher value of employment. The search intensity chosen by unemployed workers is given by the same equation with $E^c\left(t|w^c\left(.\right)\right)$ being replaced by U.

Workers employed at a type-m firm at tenure t will similarly receive outside offers. If a worker meets another firm, the optimal strategy of the outside firm is to offer a wage equal to the marginal product p regardless its type, since it knows that any offer below the marginal product will be matched by the incumbent firm. The value of being employed at a type-m firm at tenure t with a wage tenure contract $w^m\left(.\right)$ is, therefore, given by

$$\begin{aligned} \delta E^m\left(t|w^m\left(.\right)\right) &= \max_s \Big\{ w^m\left(t\right) + \dot{E}^m\left(t|w^m\left(.\right)\right) - c\left(s\right) \\ &\quad + \left[\lambda + s\right]\left[E\left(p\right) - E^m\left(t|w^m\left(.\right)\right)\right]\Big\}. \end{aligned} \tag{29}$$

The search intensity of workers employed at type-m firms at a value of employment $E^m\left(t|w^m\left(.\right)\right)$ is, therefore, given by,

$$s\left(E^m\left(t|w^m\left(.\right)\right)\right)^{\frac{1}{\alpha}} = \left[E\left(p\right) - E^m\left(t|w^m\left(.\right)\right)\right]. \tag{30}$$

The convexity of the search cost function again implies that the search intensity is decreasing in the value of employment $E^m\left(t|w^m\left(.\right)\right)$. Furthermore, comparing the search intensity of workers with the same value of employment at different types of firms, i.e.$E^m\left(t|w^m\left(.\right)\right) = E^c\left(t|w^c\left(.\right)\right)$, implies that workers employed at type-m firms search more than workers employed at type-c firms, since any firm contact leads to a wage equal to the marginal product.

5.2 Wage-tenure profiles

Since type-c firms do not match outside offers, they will not be able to retain a worker that contacts another firm. The quitting rate of a worker with tenure t is, therefore, given by $\delta + \lambda + s\left(E^c\left(t|w^c\left(.\right)\right)\right)$. Thus, the value of employing a worker at tenure t is given by the following differential equation, i.e.,

$$J^c\left(t|w^c\left(.\right)\right) = \frac{p - w^c\left(t\right) + \dot{J}^c\left(t|w^c\left(.\right)\right)}{\delta + \lambda + s\left(E^c\left(t|w^c\left(.\right)\right)\right)}. \tag{31}$$

If a type-c firm meets a worker at another type-c firm, it maximizes expected profits by choosing the optimal wage tenure contract $w^c(.)$ conditional on the value of employment E^c. Similar to type-n firms the optimal wage tenure contract for type-c firms is a step contract $(\underline{w}, T^c(E^c))$. Thus, Lemma 1 still holds with type-n firms replaced by type-c firms.

Type-m firms loose profitable workers either because workers exit the labor market (at rate δ) or because workers are paid their marginal product after meeting another firm (at rate $\lambda + s(E^m(t|w^m(.)))$). Note, that all firms that meet a worker employed at a type-m firm will offer the marginal product irrespective of their type. Thus, for a type-m firm the value of employing a worker at tenure t is given by,

$$J^m(t|w^m(.)) = \frac{p - w^m(t) + \dot{J}^m(t|w^m(.))}{\delta + \lambda + s(E^m(t|w^m(.)))}. \tag{32}$$

The optimal wage tenure contract offered by type-m firms is like in section 3.3 given by a step contract $(\underline{w}, T^m(E^m))$, if the minimum wage is not binding. Therefore, Lemma 2 is still valid. Thus, the level of the minium wage, above which the minium wage is binding, is given by

$$E^m(\underline{w}) \geq U \Longleftrightarrow \underline{w} \geq \underline{W}' = b - \left[\lambda + \frac{s(E^m(\underline{W}'))}{1+\alpha}\right]\frac{p-b}{\delta},$$
$$\text{where} \quad s(E^m(\underline{W}'))^{\frac{1}{\alpha}} = [p-b]/\delta.$$

Since all contacts of workers employed at a type-m firms lead to an immediate promotion, workers search even more and aggravate the moral hazard problem. Thus, type-m firms are now more constrained compared to the previous analysis. They, therefore, pay a constant minimum wage contract at even lower minimum wages.

Workers employed at type-m firms are promoted then they meet another type-m firm, while workers employed at type-c firms are not promoted. Thus, similar to the previous anaylsis workers employed at type-m firms with the same time to promotion T as workers at a type-c firm value their employment contract higher, i.e. $E^c(T) < E^m(T)$. This in turn implies that the time to promotion for a worker with the same value of employment E is longer at a type-m firm than at a type-c firm. Therefore, Lemma 3 still holds with type-n firms replaced by type-c firms.

5.3 Expected profits

Type-c firms can not only attract unemployed workers, but also workers employed at other type-c firms by offering them a higher value of employment (plus epsilon). They are not able to hire any worker employed at type-m firms, since type-m firms successfully counter any offer made by type-c firms. The hiring rate of unemployed workers is given by $[\lambda + s(U)]\, u$ and the hiring rate of workers employed at a position τ on the baseline salary scale of type-c firms is given by $[\lambda + s(E^c)]\, \dot{L}^c(E^c)$. The expected profit of a type-c firm can, therefore, be written as

$$\Pi^c = [\lambda + s(U)]\, uJ^c(U) + \int_{-T_u^c}^{0} [\lambda + s(E^c)]\, J^c(E^c)\, dL^c(E^c)\,. \tag{33}$$

Since the hiring rate of employed workers is affected by the fraction of type-c firms in the market, a higher fraction of type-m firms in the market decreases the hiring rate and ceteris paribus expected profits of type-c firms. If the minimum wage is not binding for type-m firms, i.e. $\underline{w} < \underline{W}'$, unemployed workers and workers employed at type-c firms have no incentive to search actively, i.e. $s(U) = s(E^c) = 0$. Thus, the value of employing a worker at a type-c firms, i.e. $J^c(E^c)$, is independent of the fraction of type-c (or type-m) firms in the market. If the minimum wage is binding, i.e. $\underline{w} \geq \underline{W}'$, unemployed and employed workers search actively. This decreases not only the value of employing a worker at a type-c firms but also the pool of workers (unemployed workers and workers employed at type-c firms) that a type-n firm can recruit from. Thus, an increase in the fraction of type-m firms decrease expected profits of type-c firms.

The expected profit of a type-m firm, i.e.

$$\Pi^m = [\lambda + s(U)]\, uJ^m(U) + \int_{-T_u^m}^{0} [\lambda + s(E^c)]\, J^m(E^c)\, dL^c(E^c)\,, \tag{34}$$

is similarly affected by a increase in the fraction of type-m firms. An increase in the fraction of type-m firms decreases the hiring rate of type-m firms by the same amount as the hiring rate of type-c firms. The value of employing a worker at a type-m firms, i.e. $J^m(E^m)$, does not depend on the fraction of type-m firms, since all firms offer workers employed at a type-m firms their marginal product, if they contact them.

Both types of wage policies coexist in equilibrium, if and only if

$$\Pi^c = \Pi^m > 0. \tag{35}$$

5.4 Optimal wage policy with homogenous firms

Given that both types of wage policies lead to the same hiring rate, we only need to compare the value of employing a worker hired at a given value of employment $E \in [U, E(p))$ for type-c and type-m wage policy in order to determine which wage policy is optimal. A type-c compared to a type-m wage policy has the advantage that workers employed at type-c firms never search actively for a job. Thus, type-c firms do not face any moral hazard problem. Furthermore, since firms are homogenous, they do not gain from matching an outside offer, since commiting to counter outside offers induces outside firms to offer a wage equal to the marginal product. Thus, a type-m wage policy despite being able to retain a worker does not leave any positive profit after the worker has meet another firm. Hence, the type-c wage policy.is optimal as shown in the following Proposition.

Proposition 3: *The type-c wage policy is always optimal, if all firms are equally productive.*

Proof: See Appendix.

The idea behind Proposition 3 is simple. Firms want to reduce the quitting probability of their workers in order to maximize the value of employment. The best way to reduce the quitting probability is to take away the workers incentive to search for another job. Since firms that do not counter outside offers provide no (or a lower) incentive to search actively, the type-c wage policy dominates type-m wage policy.

5.5 Optimal wage policy with heterogenous firms

The dominance of the type-c wage policy might no longer hold, if firms are heterogenous. If firms are heterogenous, high productivity firms with a type-c wage policy loose workers to low productivity firms although matching the outside offers of low productivity firms would still generate a positive profit for high productivity firms. Thus, more productive firms might benefit from a type-m wage policy. However, Proposition 3 clearly implies that committing to counter outside offers from equally or more productive firms is not optimal. Thus, only matching outside offers of less productive firms could be optimal. In the following analysis for high productivity firms, I will compare the profit of a type-m wage policy with a type-c wage policy for values of employment at which workers would benefit to changing to a low productivity firm that pays the marginal product, i.e. $E < E(p_l)$. Such values of employment occur at the beginning of an employment spell due to the nature of the optimal step contract.

Suppose firms differ in productivity with the fraction π_l of firms having a low productivity p_l and the fraction π_h a high productivity p_h, where $p_l < p_h$. Proposition 3 clearly implies that a type-c wage policy is optimal for all low productivity firms, since they cannot gain from matching an outside offer. The values of employing a worker at a high productivity firms at a type-c and type-m wage policy is therefore given by,

$$\begin{aligned} \delta J_h^c(E) &= p_h - \underline{w} + \dot{J}_h^c(E) - \lambda \pi_h J_h^c(E) - \lambda \pi_l [1 - I(E)] J_h^c(E), && (36) \\ \delta J_h^m(E) &= p_h - \underline{w} + \dot{J}_h^m(E) - \lambda \pi_h J_h^m(E) && (37) \\ &\quad + [\lambda + s(E)] \pi_l [1 - I(E)] [J_h^m(E(p_l)) - J_h^m(E)], \end{aligned}$$

where the index variable $I(E)$ indicates whether the value of employment E at which a worker is hired is higher than the maximum value of employment at a low productivity firm, i.e.

$$I(E) = \begin{cases} 0 & \text{if } E < E(p_l), \\ 1 & \text{if } E \geq E(p_l). \end{cases}$$

Since all high productivity firms do not match outside offers of high productivity firms, workers at both types of firms do not search, if their value of employment is at least as high as being employed at the marginal product at a low productivity firm, i.e. if $E \geq E(p_l)$. Applying $I(E) = 1$ to equations (36) and (37) shows that both wage policies are equally profitable.

If a worker is hired or employed at a value of employment that is below the value of being employed at the marginal product at a low productivity firm, i.e. if $E < E(p_l)$. High productivity firms with a type-c wage policy risk loosing a worker to a low productivity firm. On the other side, this prevents workers employed at a type-c wage policy from searching actively. Thus, high productivity firms with a type-m wage policy might be able to retain a worker, if he is contacted by a low productivity firm. However, the gains from meeting a low productivity firm increase the search intensity of workers employed at a type-m wage policy and thus the risk that the worker meets a high productivity firm and leaves.

Proposition 4 shows that, if a firms is allowed to change its wage policy over an employment spell, high productivity firms match the outside offers of low productivity firms, if they are not constrained by the minimum wage.

Proposition 4: *(i) If the minimum wage is not binding, i.e.* $\underline{w} < \underline{W}'$*, high productivity firms always match outside offers of low productiviy firms, but never match outside offers of high productivity firms, i.e.* $J_h^m(E) > J_h^c(E)$ *for any* $E \in [U, E(p_l))$*, and* $J_h^m(E) \leq J_h^c(E)$ *for any* $E \in [E(p_l), E(p_h)]$.

(ii) If the minimum wage is binding, i.e. $\underline{w} \geq \underline{W}'$*, high productivity firms do not match outside offers of low productivity firms at any* $E \in [U, E(\underline{w})]$*, they match ouside offers of low productivity firms at any* $E \in (E(\underline{w}), E(p_l))$*, and they never match outside offers of high productivity firms.*

Proof: See Appendix.

The idea behind the first part of Proposition 4 is that only a wage policy that counters outside offers that leave a positive profit for the firm are profitable. The fact that a type-c wage policy is never optimal for values of employment below the value of being employed at the marginal product at a low productivity firm, i.e. if $E < E(p_l)$, is due to a firms ability to change its wage policy over a worker's employment spell. If the value of employing a worker is close to value of being employed at the marginal product at a low productivity firm, then matching an outside offer does not cost much but ensures a profitable future employment relationship. Thus, for E close to $E(p_l)$ it is always optimal for a firm to match an outside offer. But given that it is optimal to keep a worker at E it cannot be optimal to let the worker leave the period before his value of employment reaches E, because the value of employing a worker at $E - \varepsilon$ with $\varepsilon > 0$ is higher than at E. This argument that underpins the Proof of the first part of Proposition 4 also explains why a high productivity firm does not counter an outside offer, if the minimum wage is binding.

6 Conclusions

In on-the-job search models based on Burdett and Mortensen (1998) firms commit not to match outside offers and to pay the same wage irrespective of how much workers' value their current job. Postel-Vinay and Robin (2002a,b) in contrary assume that firms condition their wage offers on the workers' current job value and react to outside offers.

Given these two wage policy options a firm has to trade off the effect of a wage policy on the recruitment and retention probability and on the search effort of its workers. Matching outside offers ensures that a firm is successful in retaining a worker as long as it is more productive than the competing firm. The prospect of an expected promotion, if a worker contacts another firm, however, increases workers' incentive to search for another job. This moral hazard problem of rent seeking on-the-job search makes it attractive for firms to commit not to counter outside offers. However, in the Burdett-Mortensen model firms that commit not to counter outside offers also commit not to condition their offers on workers' value of employment. Unconditional wage offers, however, limit the chances that a worker will accept an offer. Thus, a firm has to trade off the higher recruitment and retention probability, if it is willing to counter outside offers, with the higher on-the-job search effort that induces such a wage policy.

Besides these two classes of models two additional combinations are generally possible: (a) Firms that commit not to counter outside offers but condition on the workers' characteristics and (b) firms that counter outside offers but commit not to condition their wages on the workers' characteristics. Firms of type (b) that make unconditional wage offers but counter outside offers are not able to recruit as many workers as PVR firms. At the same time they can only make the same profits per matched worker as PVR firms. Thus, such a wage policy must be suboptimal. Type (a) firms make the same profit per matched worker as BM firms. In addition, however, they recruit workers employed at BM firms. Thus, they recruit with the same probability as PVR firms, but do not face the same moral hazard problem. This makes the wage policy where firms (i) make conditional outside offers and (ii) make not to counter offers to equally or more productive firms optimal.

Wage policies might also differ with respect to their time structure. Stevens (2004), Burdett and Coles (2003, 2007) and Carrillo-Tudela (2009b) have shown that a wage tenure contract is optimal in an on-the-job search environment where a firms can commit to offer the same wage contract to all workers and not to counter outside offers. Wage tenure contracts are optimal, if labor laws or credit constraints prohibit contracts where workers pay an entry fee on accepting the job and are paid the marginal product during employment. For risk neutral workers the optimal shape of a wage tenure contract is a step contract with a wage equal to the minimum wage in the beginning and the marginal product after a specified period (see Stevens, 2004). Optimality arises because wage tenure contracts reduce the quitting rate of workers. In the present paper I show that step contracts are also optimal for firms that counter outside offers and condition their wage contracts on how much workers' value their current job, because step contracts increase a worker's value of employment at the fastest possible rate and therefore reduce the rent seeking search effort to a minimum. Furthermore, step contracts offered by firms that counter outside offers have a longer time to promotion than step contracts offered by firms that do not match outside offers.

Having shown the optimal shape of the wage tenure contract for different types of wage policies, I first investigate under which conditions BM and PVR wage policies coexist. BM firms that don't counter outside offers and that don't condition their wage offers on how much employed workers' value their current job exist, if the minimum wage is high enough to reduce the rent that firms with a matching wage policy can extract from their workers. A high minimum wage reduces the rent that a firm with a matching wage policy can extract from its workers because matching outside offers gives workers a higher value of employment as not matching outside offers. This requires a lower minimum wage for firms that counter outside offers to be able to extract the whole rent from their employees. A high minimum wage, therefore, increases the return of a wage policy that does not counter outside offers relative to a firm that matches outside offers. However, PVR firms that counter outside offers and condition their wage contracts on how much workers' value their current job always exist, because they can recruit unemployed as well as employed workers while firms that do not match outside offers can only recruit unemployed workers.

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Appendix

Proof of Lemma 1

The proof of Lemma 1 follows Stevens (2004) proof of Proposition 5:

Step 1: I show that (i) the step contract $(\underline{w}, T^n)$ has a unique starting payoff $E^n(0|\underline{w}, T^n)$ and that (ii) $E^n(0|\underline{w}, T^n)$ is decreasing in T^n. Let the step contracts T^n and $\widetilde{T}^n$ be such that the time to promotion $\widetilde{T}^n = T^n + \varepsilon$ for any $\varepsilon > 0$.

(i) Suppose that $E^n(0|\underline{w}, T^n) = E^n\left(0|\underline{w}, \widetilde{T}^n\right)$. Using equation (2) it follows that $\dot{E}^n(0|\underline{w}, T^n) = \dot{E}^n\left(0|\underline{w}, \widetilde{T}^n\right)$. Continuity then implies $\dot{E}^n(t|\underline{w}, T^n) = \dot{E}^n\left(t|\underline{w}, \widetilde{T}^n\right)$ for any $t \in (0, T^n]$ and hence $E^n(T^n|\underline{w}, T^n) = E^n(p) = E^n\left(T^n|\underline{w}, \widetilde{T}^n\right)$. However, $\widetilde{T}^n = T^n + \varepsilon$ implies $E^n\left(T^n|\underline{w}, \widetilde{T}^n\right) < E(p)$ for all $t \in \left[0, \widetilde{T}^n\right)$. Hence, $E^n(0|\underline{w}, T^n) \neq E^n\left(0|\underline{w}, \widetilde{T}^n\right)$.

(ii) Now suppose that $E^n(0|\underline{w}, T^n) < E^n\left(0|\underline{w}, \widetilde{T}^n\right)$. Since $E^n(T^n|\underline{w}, T^n) > E^n\left(T^n|\underline{w}, \widetilde{T}^n\right)$ continuity implies that there exists a $t' \in [0, T^n)$ such that $E^n(t'|\underline{w}, T^n) = E^n\left(t'|\underline{w}, \widetilde{T}^n\right)$. Using the same argument as in (i) we have $E^n(t|\underline{w}, T^n) = E^n\left(t|\underline{w}, \widetilde{T}^n\right)$ for all $t \in (t', T^n]$. Hence, $E^n(0|\underline{w}, T^n) > E^n\left(0|\underline{w}, \widetilde{T}^n\right)$.

Note, that since $T^n = 0$ implies $E^n(0|\underline{w}, T^n) = E^n(p)$, any $T^n > 0$ must correspond to a $E^n(0|\underline{w}, T^n) < E^n(p)$. Using (i) and (ii), continuity then implies that for each $E_0^n \in [U, E^n(p))$ there exists a unique step contract with corresponding time to promotion $T^n > 0$.

Step 2: Let $\widetilde{w}^n(.)$ denote any other contract with $E^n(0|\widetilde{w}^n(.)) = E_0^n$. Subtracting the corresponding continuation payoffs implies

$$\begin{aligned}
& \dot{E}^n(t|\underline{w}, T^n) - \dot{E}^n(t|\widetilde{w}^n(.)) \\
= \; & \widetilde{w}^n(t) - \underline{w} + \delta\left[E^n(t|\underline{w}, T^n) - E^n(t|\widetilde{w}^n(.))\right] \\
& - \left[\lambda + s\left(E^n(t|\underline{w}, T^n)\right)\right]\gamma \int_{E^n(t|\underline{w}, T^n)}^{\overline{E}^n} \left[E_0^n - E^n(t|\underline{w}, T^n)\right] dF\left(E_0^n\right) \\
& + \left[\lambda + s\left(E^n(t|\widetilde{w}^n(.))\right)\right]\gamma \int_{E^n(t|\widetilde{w}^n(.))}^{\overline{E}^n} \left[E_0^n - E^n(t|\widetilde{w}^n(.))\right] dF\left(E_0^n\right) \\
& + c\left(s\left(E^n(t|\underline{w}, T^n)\right)\right) - c\left(s\left(E^n(t|\widetilde{w}^n(.))\right)\right)
\end{aligned}$$

for all $t < T^n$. Since $E^n(0|\widetilde{w}^n(.)) = E^n(0|\underline{w}, T^n) = E_0^n$ by assumption, it follows from $\widetilde{w}^n(0) \geq \underline{w}$ that $\dot{E}^n(0|\underline{w}, T^n) \geq \dot{E}^n(0|\widetilde{w}^n(.))$. Continuity of E^n then implies that $\dot{E}^n(t|\underline{w}, T^n) \geq \dot{E}^n(t|\widetilde{w}^n(.))$ and $E^n(t|\underline{w}, T^n) \geq E^n(t|\widetilde{w}^n(.))$ for all $t < T^n$. Furthermore, since $E^n(t|\underline{w}, T^n) = E^n(p)$ for all $t \geq T^n$, it follows that $E^n(t|\underline{w}, T^n) \geq E^n(t|\widetilde{w}^n(.))$ for all $t > 0$.

Step 3: Let $S^n(0|w^n(.)) = E^n(0|w^n(.)) + J^n(0|w^n(.))$ describe the total expected surplus of employment with a wage contract $w^n(.)$. Note that $E^n(p) \geq S^n(0|w^n(.))$ as $E^n(p) = p/\delta$ denotes the maximium value of a match. Since the objective of a firm is to chose a $w^n(.)$ to maximize $J^n(0|w^n(.))$ subject to $E^n(0|w^n(.)) = E_0^n$, the problem is equivalent to chose a $w^n(.)$ that maximizes $S^n(0|w^n(.))$.

Consider a step contract $(\underline{w}, T^n)$ and a contract $\widetilde{w}^n(.)$. It follows that $S^n(t|\underline{w}, T^n) = E^n(p) \geq S^n(t|\widetilde{w}^n(.))$ for $t \geq T^n$, where equation (2) and (7) characterise $\dot{S}^n(t|w^n(.))$, i.e.

$$\begin{aligned}
\dot{S}^n\left(t|w^n(.)\right) &= \left[\delta+\left[\lambda+s\left(E^n\left(t|w^n(.)\right)\right)\right]\left[1-\gamma F\left(E^n\left(t|w^n(.)\right)\right)\right]\right]S^n\left(t|w^n(.)\right) \\
&\quad -p+c\left(s\left(E^n\left(t|w^n(.)\right)\right)\right) \\
&\quad -\left[\lambda+s\left(E^n\left(t|w^n(.)\right)\right)\right]\gamma\int_{E^n(t|w^n(.))}^{\overline{E}^n} E_0^n dF\left(E_0^n\right) \\
&\quad -\left[\lambda+s\left(E^n\left(t|w^n(.)\right)\right)\right](1-\gamma)E^m\left(w^m\left(.,E^n\left(t|w^n(.)\right)\right)\right).
\end{aligned}$$

Subtracting and using the result of Step 2, i.e. $E^n\left(t|\underline{w},T^n\right)\geq E^n\left(t|\widetilde{w}^n(.)\right)$ for all $t>0$, yields

$$\begin{aligned}
&\dot{S}^n\left(t|\widetilde{w}^n(.)\right)-\dot{S}^n\left(t|\underline{w},T^n\right) \\
\geq\ &\left[\delta+\left[\lambda+s\left(E^n\left(t|\underline{w},T^n\right)\right)\right]\left[1-\gamma F\left(E^n\left(t|\underline{w},T^n\right)\right)\right]\right]\left[S^n\left(t|\widetilde{w}^n(.)\right)-S^n\left(t|\underline{w},T^n\right)\right]
\end{aligned}$$

for all $t\geq 0$. Since $S^n\left(t|\underline{w},T^n\right)\geq S^n\left(t|\widetilde{w}^n(.)\right)$ at $t=T^n$, continuity implies $S^n\left(0|\underline{w},T^n\right)\geq S^n\left(0|\widetilde{w}^n(.)\right)$. Hence, conditional on $E^n\left(0|\widetilde{w}^n(.)\right)=E^n\left(0|\underline{w},T^n\right)=E_0^n\in\left[U,E^n(p)\right)$, it follows that $J^n\left(0|\underline{w},T^n\right)\geq J^n\left(0|\widetilde{w}^n(.)\right)$.

Proof of Proposition 1

To prove that all type-n firms offer the same wage-tenure contract I follow Stevens (2004) proof of Proposition 6.

Differentiating the payoff function $\Pi^n(\underline{w},\tau)=h^n(\underline{w},\tau)J^n(\underline{w},\tau)$ using (9) and (7) implies for the first derivative,

$$\begin{aligned}
\dot{\Pi}^n(\underline{w},\tau) &= \frac{dh^n(\underline{w},\tau)}{d\tau}J^n(\underline{w},\tau)+h^n(\underline{w},\tau)\dot{J}^n(\underline{w},\tau) \qquad (38)\\
&= \left[\lambda+s\left(E^n(\tau)\right)\right]\int_\tau^0\left[\delta+\left[\lambda+s\left(E^n(\tau')\right)\right][1-\gamma]\right]d\overline{L}^n(\tau')\,J^n(\underline{w},\tau) \\
&\quad +h^n(\underline{w},\tau)\left[\delta+\left[\lambda+s\left(E^n(\tau)\right)\right][1-\gamma]\right]J^n(\underline{w},\tau) \\
&\quad -\left[\lambda+s\left(E^n(\tau)\right)\right]\int_{-T_u^n}^{\tau}\left[\lambda+s\left(E^n(\tau')\right)\right]d\overline{L}^n(\tau')\left[1-\overline{F}(\tau)\right]\gamma J^n(\underline{w},\tau) \\
&\quad +h^n(\underline{w},\tau)\left[\lambda+s\left(E^n(\tau)\right)\right]\left[1-\overline{F}(\tau)\right]\gamma J^n(\underline{w},\tau) \\
&\quad -h^n(\underline{w},\tau)\left[p-\underline{w}\right]
\end{aligned}$$

The second derivative is given by

$$\begin{aligned}
\ddot{\Pi}^n(\underline{w},\tau) = {} & [\lambda + s(E^n(\tau))] \int_\tau^0 [\delta + [\lambda + s(E^n(\tau'))][1-\gamma]]\, d\overline{L}^n(\tau')\, \dot{J}^n(\underline{w},\tau) \\
& - [\lambda + s(E^n(\tau))][\delta + [\lambda + s(E^n(\tau))][1-\gamma]] \frac{d\overline{L}^n(\tau)}{d\tau} J^n(\underline{w},\tau) \\
& + \frac{ds(E^n(\tau))}{d\tau} \int_\tau^0 [\delta + [\lambda + s(E^n(\tau'))][1-\gamma]]\, d\overline{L}^n(\tau')\, J^n(\underline{w},\tau) \\
& + h^n(\underline{w},\tau)[\delta + [\lambda + s(E^n(\tau))][1-\gamma]]\, \dot{J}^n(\tau) \\
& + [\lambda + s(E^n(\tau))] \frac{d\overline{L}^n(\tau)}{d\tau} [\delta + [\lambda + s(E^n(\tau))][1-\gamma]]\, J^n(\underline{w},\tau) \\
& + h^n(\underline{w},\tau) \frac{ds(E^n(\tau))}{d\tau} [1-\gamma]\, J^n(\underline{w},\tau) \\
& - [\lambda + s(E^n(\tau))] \int_{-T_u^n}^\tau [\lambda + s(E^n(\tau'))]\, d\overline{L}^n(\tau') \left[1 - \overline{F}(\tau)\right] \gamma \dot{J}^n(\underline{w},\tau) \\
& - [\lambda + s(E^n(\tau))][\lambda + s(E^n(\tau))] \frac{d\overline{L}^n(\tau)}{d\tau} \left[1 - \overline{F}(\tau)\right] \gamma J^n(\underline{w},\tau) \\
& - \frac{ds(E^n(\tau))}{d\tau} \int_{-T_u^n}^\tau [\lambda + s(E^n(\tau'))]\, d\overline{L}^n(\tau') \left[1 - \overline{F}(\tau)\right] \gamma J^n(\underline{w},\tau) \\
& + h^n(\underline{w},\tau)[\lambda + s(E^n(\tau))] \left[1 - \overline{F}(\tau)\right] \gamma \dot{J}^n(\underline{w},\tau) \\
& + [\lambda + s(E^n(\tau))] \frac{d\overline{L}^n(\tau)}{d\tau} [\lambda + s(E^n(\tau))] \left[1 - \overline{F}(\tau)\right] \gamma J^n(\underline{w},\tau) \\
& + h^n(\underline{w},\tau) \frac{ds(E^n(\tau))}{d\tau} \left[1 - \overline{F}(\tau)\right] \gamma J^n(\underline{w},\tau) \\
& - [\lambda + s(E^n(\tau))] \frac{d\overline{L}^n(\tau)}{d\tau} [p - \underline{w}]
\end{aligned}$$

Rearranging gives

$$
\begin{aligned}
\ddot{\Pi}^n(\underline{w},\tau) &= [\lambda+s(E^n(\tau))]\int_\tau^0 [\delta+[\lambda+s(E^n(\tau'))][1-\gamma]]\,d\overline{L}^n(\tau')\,\dot{J}^n(\underline{w},\tau) && (39)\\
&\quad +h^n(\underline{w},\tau)[\delta+[\lambda+s(E^n(\tau))][1-\gamma]]\,\dot{J}^n(\underline{w},\tau)\\
&\quad -[\lambda+s(E^n(\tau))]\int_{-T_u^n}^{\tau}[\lambda+s(E^n(\tau'))]\,d\overline{L}^n(\tau')\left[1-\overline{F}(\tau)\right]\gamma\dot{J}^n(\underline{w},\tau)\\
&\quad +h^n(\underline{w},\tau)[\lambda+s(E^n(\tau))]\left[1-\overline{F}(\tau)\right]\gamma\dot{J}^n(\underline{w},\tau)\\
&\quad +\frac{ds(E^n(\tau))}{d\tau}\left[\frac{d\overline{L}^n(\tau)}{d\tau}+h^n(\underline{w},\tau)\left[1-\gamma\overline{F}(\tau)\right]\right]J^n(\underline{w},\tau)\\
&\quad -[\lambda+s(E^n(\tau))]\frac{d\overline{L}^n(\tau)}{d\tau}[p-\underline{w}]\\
&= \left[\delta+[\lambda+s(E^n(\tau))]\left[1-\gamma\overline{F}(\tau)\right]\right]\dot{\Pi}^n(\underline{w},\tau) && (40)\\
&\quad +\frac{ds(E^n(\tau))}{d\tau}\left[\frac{d\overline{L}^n(\tau)}{d\tau}+h^n(\underline{w},\tau)\left[1-\gamma\overline{F}(\tau)\right]\right]J^n(\underline{w},\tau)\\
&\quad -2[\lambda+s(E^n(\tau))]\frac{d\overline{L}^n(\tau)}{d\tau}[p-\underline{w}]
\end{aligned}
$$

To prove that all type-n firms offer the same wage-tenure contract, start with supposing that $\overline{F}(\tau)$ is continuous and increasing over an interval $[\underline{\tau},\overline{\tau}]\subseteq[-T_u^n,0]$. Since the equilibrium condition (13) requires equal profits for all wage-tenure contracts posted over the interval, $\dot{\Pi}^n(\underline{w},\tau)=0$ and $\ddot{\Pi}^n(\underline{w},\tau)=0$. Equation (**??**) then implies $d\overline{L}^n(\tau)/d\tau\le 0$, since $ds(E^n(\tau))/d\tau<0$ for $s(E^n(\tau))>0$ and $ds(E^n(\tau))/d\tau=0$ for $s(E^n(\tau))=0$. However, if $d\overline{L}^n(\tau)/d\tau<0$ in the interval $[\underline{\tau},\overline{\tau}]$, firms recruit less workers, i.e. $dh^n(\underline{w},\tau)/d\tau<0$, and make less profit by offering $\tau>\underline{\tau}$ which contradicts the equal profit condition on the support of $\overline{F}(\tau)$. Furthermore, if $d\overline{L}^n(\tau)/d\tau=0$ and $s(E^n(\underline{w},\tau))=0$ in the interval $[\underline{\tau},\overline{\tau}]$, then equation (9) holds only for a constant $\overline{F}(\tau)$, a contradiction.

Suppose τ_1 is a mass point. If $\tau_1=-T_u^n$, equation (9) implies $d\overline{L}^n(-T_u^n)/d(-T_u^n)>0$. If $\tau_1>-T_u^n$ and $d\overline{L}^n(\tau_1)/d\tau_1=0$, then since $\overline{L}^n$ is continuous at τ_1 but $\overline{F}(\tau)$ is not, the equation (9) implies $d\overline{L}^n\left(\tau_1^-\right)/d\tau_1^-<0$, which contradicts $\tau_1>-T_u^n$ being a mass point, since it implies that a firm offering a lower position $\tau<\tau_1$ on the baseline salary scale will hire more workers, i.e. $dh^n(\underline{w},\tau)/d\tau>0$, and make more profits. Hence $d\overline{L}^n(-T_u^n)/d(-T_u^n)>0$. For $\tau_1=-T_u^n$ to be optimal it has to be the case that $\dot{\Pi}^n(\underline{w},-T_u^n)<0$. Equation (38) then implies $\dot{J}^n(\underline{w},-T_u^n)<0$, since $d\overline{L}^n(-T_u^n)/d(-T_u^n)>0\Longrightarrow dh^n(\underline{w},-T_u^n)/d(-T_u^n)>0$.

Now suppose that $\tau_2 > -T_u^n$ is also a mass point, and no firm offers $\tau \in (-T_u^n, \tau_2)$. For τ_2 to be optimal, it has to be that $\dot{\Pi}^n\left(\underline{w}, \tau_2^-\right) > 0$. Over $[-T_u^n, \tau_2)$, $\overline{F}(\tau) = \overline{F}(-T_u^n)$ is constant, so from equation (7) $\dot{J}^n(\underline{w}, \tau)$ is differentiable, i.e.

$$\begin{aligned}\ddot{J}^n(\underline{w}, \tau) &= \left[\delta + [\lambda + s(E^n(\tau))]\left[1 - \gamma\overline{F}(-T_u)\right]\right]\dot{J}^n(\underline{w}, \tau) \\ &\quad + \frac{ds(E^n(\tau))}{d\tau}\left[1 - \gamma\overline{F}(-T_u)\right]J^n(\underline{w}, \tau).\end{aligned} \tag{41}$$

Thus, since $ds(E^n(\tau))/d\tau \leq 0$ it follows from equation (41) that $\dot{J}^n(\underline{w}, \tau) < 0$ for all $\tau \in [-T_u^n, \tau_2)$. Hence equation (39) implies along with $\dot{\Pi}^n(\underline{w}, -T_u^n) < 0$ that $\dot{\Pi}^n(\underline{w}, \tau) < 0$ for all $\tau \in [-T_u^n, \tau_2)$, which contradicts $\dot{\Pi}^n\left(\underline{w}, \tau_2^-\right) > 0$. This completes the proof that all type-n firms offer a position $\tau = -T_u^n$ on the baseline salary scale.

Proof of Lemma 2

Step 1: Along the lines of the proof of Lemma 1, it follows that (i) the step contract $(\underline{w}, T^m)$ has a unique starting payoff $E^m(0|\underline{w}, T^m) \in (E^m(\underline{w}), E^m(p))$ and that (ii) $E^m(0|\underline{w}, T^m)$ is decreasing in T^m. Let the step contracts T^m and $\widetilde{T}^m$ be such that the time to promotion $\widetilde{T}^m = T^m + \varepsilon$ for any $\varepsilon > 0$.

Step 2: Let $\widetilde{w}^m(., E)$ denote any other contract with $E^m(0|\widetilde{w}^m(., E)) = E \in (E^m(\underline{w}), E^m(p))$. Following the same steps as in the proof of Lemma 1, it follows that $E^m(t|\underline{w}, T^m(E)) \geq E^m(t|\widetilde{w}^m(., E))$ for all $t > 0$ and for all $E \in (E^m(\underline{w}), E^m(p))$.

Step 3: Following the same steps as in the proof of Lemma 1, it follows that $J^m(0|\underline{w}, T^m(E)) \geq J^m(0|\widetilde{w}^m(., E))$ for all $E \in (E^m(\underline{w}), E^m(p))$.

Step 4: I next show that $J^m(0|\underline{w}, T^m(E))$ is increasing in $T^m(E)$ and converges to $J^m(0|\underline{w})$ as $T^m(E) \to \infty$. Let the step contracts $T^m(E)$ and $\widetilde{T}^m(E)$ be such that the time to promotion $\widetilde{T}^n(E) = T^m(E) + \varepsilon$ for any $\varepsilon > 0$. Now suppose that $J^m(0|\underline{w}, T^m(E)) > J^m\left(0|\underline{w}, \widetilde{T}^m(E)\right)$. Since $J^m(T^m(E)|\underline{w}, T^m(E)) = 0 < J^m\left(T^m(E)|\underline{w}, \widetilde{T}^m(E)\right)$ continuity implies that implies that there exists a $t' \in [0, T^m(E))$ such that $J^m(t'|\underline{w}, T^m(E)) = J^m\left(t'|\underline{w}, \widetilde{T}^m(E)\right)$. However, $J^m(t'|\underline{w}, T^m(E)) = J^m\left(t'|\underline{w}, \widetilde{T}^m(E)\right)$ implies using equation (10) that $\dot{J}^m(t'|\underline{w}, T^m(E)) = \dot{J}^m\left(t'|\underline{w}, \widetilde{T}^m(E)\right)$ for any $t \in [t', T^m(E)]$ and hence $J^m(t|\underline{w}, T^m(E)) = J^m\left(t|\underline{w}, \widetilde{T}^m(E)\right)$. This contradiction implies $J^m(0|\underline{w}, T^m(E)) < J^m\left(0|\underline{w}, \widetilde{T}^m(E)\right)$. Thus, $\lim_{T^m(E)\to\infty} J^m(0|\underline{w}, T^m(E)) = J^m(0|\underline{w})$.

Step 5: Given that $E^m(0|\underline{w}, T^m(E))$ decreases and $J^m(0|\underline{w}, T^m(E))$ increases with $T^m(E)$, it follows that for any $E \in [U, E^m(\underline{w})]$, it is optimal for firms to offer the minimum wage for ever.

Proof of Lemma 3

Let $(\underline{w}, T^m(E))$ and $(\underline{w}, T^n(E))$ denote the step contract at a type-m and type-n firm, respectively. Subtracting the corresponding continuation payoffs implies

$$\begin{aligned}
& \dot{E}^n(t|\underline{w}, T^n(E)) - \dot{E}^m(t|\underline{w}, T^m(E)) \\
= \; & \delta\left[E^n(t|\underline{w}, T^n(E)) - E^m(t|\underline{w}, T^m(E))\right] \\
& -\left[\lambda + s\left(E^n(t|\underline{w}, T^n(E))\right)\right]\gamma \int_{E^n(t|\underline{w}, T^n(E))}^{\overline{E}^n} \left[E_0^n - E^n(t|\underline{w}, T^n(E))\right] dF\left(E_0^n\right) \\
& +\left[\lambda + s\left(E^m(t|\underline{w}, T^m(E))\right)\right]\gamma \int_{E^m(t|\underline{w}, T^m(E))}^{\overline{E}^n} \left[E_0^n - E^m(t|\underline{w}, T^m(E))\right] dF\left(E_0^n\right) \\
& +\left[\lambda + s\left(E^m(t|\underline{w}, T^m(E))\right)\right](1-\gamma)\left[E^m(p) - E^m(t|\underline{w}, T^m(E))\right] \\
& +c\left(s\left(E^n(t|\underline{w}, T^n(E))\right)\right) - c\left(s\left(E^m(t|\underline{w}, T^m(E))\right)\right)
\end{aligned}$$

for all $t < \max[T^n(E), T^m(E)]$. Since $E^m(0|\underline{w}, T^m(E)) = E^n(0|\underline{w}, T^n(E)) = E$ by assumption, it follows that $\dot{E}^n(0|\underline{w}, T^n(E)) > \dot{E}^m(0|\underline{w}, T^m(E))$, since $s(E^m(0|\underline{w}, T^m(E))) > s(E^n(0|\underline{w}, T^n(E)))$. Continuity of E^m and E^n implies $\dot{E}^n(t|\underline{w}, T^n(E)) > \dot{E}^m(t|\underline{w}, T^m(E))$ for $t < T^n(E)$ and $E^n(t|\underline{w}, T^n(E)) > E^m(t|\underline{w}, T^m(E))$ for $t < T^m(E)$. Thus, $T^m(E) > T^n(E)$.

Proof of Proposition 2

Part (i): Let us first prove that type-m firms always exist. Suppose the contrary, i.e. $\gamma = 1$. Then no worker has an incentive to search actively, i.e. $s(E^m(\tau)) = 0$, Thus, equation (**??**) and (**??**) imply that $J^m(-T_u^m) = J^n(-T_u^n)$ and $T_u^m = T_u^n$. Hence type-m firms make the same profit from hiring unemployed workers. Since type-m firms hire in addition workers employed at type-n firms, they make higher profits. Thus, $\gamma < 1$.

Suppose $\gamma = 0$, i.e. no workers are employed at type-n firms. Thus, type-m firms also hire only unemployed workers.

The optimality conditions (14), (16) and (17) for the search intensity imply $s(E) \to 0$ as $\alpha \to 0$. For $\alpha \to 0$ and a minimum wage $\underline{w} \leq \underline{W}$ the difference in expected profits of type-m and type-n firms in equations (25) and (22) from hiring unemployed workers is positive due to Lemma 3, i.e. $T_u^m > T_u^n$ implies

$$\begin{aligned} \Pi^m - \Pi^n &= \lambda u \left[J^m \left(-T_u^m \right) - J^n \left(-T_u^n \right) \right] \\ &= \lambda u \frac{[p - \underline{w}]}{\delta + \lambda} \left[e^{-[\delta+\lambda]T_u^n} - e^{-[\delta+\lambda]T_u^m} \right] > 0. \end{aligned}$$

For $\alpha \to 0$ and a minimum wage $\underline{w} > \underline{W}$ the difference in expected profits of type-m and type-n firms in equations (25) and (22) from hiring unemployed workers is also positive, i.e.

$$\Pi^m - \Pi^n = \lambda u \left[\frac{p - \underline{w}}{\delta + \lambda} - \frac{p - \underline{w}}{\delta + \lambda} \left[1 - e^{-[\delta+\lambda]T_u^n} \right] \right] > 0$$

Thus, there exists a α^* such that for $\alpha \in (0, \alpha^*)$ only type-m firms exist, i.e. $\gamma = 0$ is an equilibrium.

As $\alpha \to \infty$ the search intensity increases such that the optimality condition (16) implies $E^m(\underline{w}) \to E(p)$. Thus, the equilibrium converges to a competitive equilibrium.

Part (ii): Suppose $\gamma = 0$, i.e. no workers are employed at type-n firms. Thus, type-m firms also hire only unemployed workers. Suppose also $\underline{w} > \underline{W}$. Expected profits of type-n firms are given by Π^n according to equation (22). Lemma 1 implies that paying the level of unemployment benefits b to unemployed workers is less profitable, i.e.

$$\Pi^n > [\lambda + s(U)] u \frac{p - b}{\delta + \lambda + s(E^n(b))}.$$

The difference in expected profits of type-m and type-n firms in equations (25) and (22) from hiring unemployed workers is

$$\Pi^n - \Pi^m > [\lambda + s(U)] u \left[\frac{p - b}{\delta + \lambda + s(E^n(b))} - \frac{p - \underline{w}}{\delta + \lambda + s(E^m(\underline{w}))} \right].$$

The optimality conditions for the search intensity imply

$$\begin{aligned}
c'\left(s\left(E^m\left(\underline{w}\right)\right)\right) &= \left[E^m\left(p\right) - E^m\left(\underline{w}\right)\right] = \frac{p-\underline{w}}{\delta+\lambda+\dfrac{s\left(E^m\left(\underline{w}\right)\right)}{1+\alpha}} \\
c'\left(s\left(E^n\left(b\right)\right)\right) &= \left[E^m\left(\underline{w}\right) - E^n\left(b\right)\right] \\
&= \frac{1}{\delta}\left[\delta E^m\left(\underline{w}\right) - \frac{\delta b + \left[\lambda + \dfrac{s\left(E^n\left(b\right)\right)}{1+\alpha}\right]\delta E^m\left(\underline{w}\right)}{\delta+\lambda+\dfrac{s\left(E^n\left(b\right)\right)}{1+\alpha}}\right] \\
\text{where } E^m\left(\underline{w}\right) &= \frac{\delta \underline{w} + \left[\lambda + \dfrac{s\left(E^m\left(\underline{w}\right)\right)}{1+\alpha}\right]p}{\delta+\lambda+\dfrac{s\left(E^m\left(\underline{w}\right)\right)}{1+\alpha}}
\end{aligned}$$

Rearranging and substituting implies

$$\begin{aligned}
\frac{c'\left(s\left(E^n\left(b\right)\right)\right)}{c'\left(s\left(E^m\left(\underline{w}\right)\right)\right)} &= \frac{\delta\left[\underline{w}-b\right] + \left[p-b\right]\left[\lambda+\dfrac{s\left(E^m\left(\underline{w}\right)\right)}{1+\alpha}\right]}{\left[p-\underline{w}\right]\left[\delta+\lambda+\dfrac{s\left(E^n\left(b\right)\right)}{1+\alpha}\right]} \\
&= \frac{\lambda+\dfrac{s\left(E^m\left(\underline{w}\right)\right)}{1+\alpha}}{\delta+\lambda+\dfrac{s\left(E^n\left(b\right)\right)}{1+\alpha}}\left[1-\frac{\left[b-\underline{w}\right]\left[\delta+\lambda+\dfrac{s\left(E^m\left(\underline{w}\right)\right)}{1+\alpha}\right]}{\left[p-\underline{w}\right]\left[\lambda+\dfrac{s\left(E^m\left(\underline{w}\right)\right)}{1+\alpha}\right]}\right]
\end{aligned}$$

Since the convexity of the search cost function implies that $s\left(E^n\left(b\right)\right) \leq s\left(E^m\left(\underline{w}\right)\right)$ cannot satisfy the above equation, $s\left(E^n\left(b\right)\right) < s\left(E^m\left(\underline{w}\right)\right)$ holds for all $\alpha > 0$ and $\underline{w} \geq \underline{W}$. For $\underline{w} = b$ it follows that

$$\frac{p-b}{\delta+\lambda+s\left(E^n\left(b\right)\right)} > \frac{p-b}{\delta+\lambda+s\left(E^m\left(b\right)\right)}.$$

Thus, there exists a $\underline{w}^* \in (\underline{W}, b)$ such that for $\underline{w} \in [\underline{w}^*, b]$ type-n and type-m firms coexist.

7 Proof of Proposition 3

Suppose the minimum wage is not binding, i.e., $\underline{w} < \underline{W}'$. In this case type-$m$ firms offer workers employed at type-c firms only epsilon more. Thus, workers employed at type-c firms never search actively, i.e., $s\left(E^c\left(t|\underline{w}, T^c\left(E\right)\right)\right) = 0$. Using this insight and equations

(31) and (32) and take the value of employment $E \in [U, E(p)]$ as the support for the value of employing a worker, i.e.

$$\begin{aligned} \delta J^c(E) &= p - \underline{w} + \dot{J}^c(E) - \lambda J^c(E) \\ \delta J^m(E) &= p - \underline{w} + \dot{J}^m(E) - [\lambda + s(E)] J^m(E) \end{aligned}$$

Using the value of employment E as the exogenous variable allows us to determine which wage policy is more profitable at a given value of employment $E \in [U, E(p)]$. Taking the difference gives

$$\dot{J}^c(E) - \dot{J}^m(E) = [\delta + \lambda] [J^c(E) - J^m(E)] - s(E) J^m(E).$$

At $E = E(p)$, $J^c(E(p)) = J^m(E(p)) = 0$. Suppose at some $E^* \in [U, E(p))$ that $J^c(E^*) \leq J^m(E^*)$. Since firms offer step contracts we know that $J^c(E^*) > 0$ and $s(E^*) > 0$. The above differential equation then implies $\dot{J}^c(E^*) < \dot{J}^m(E^*)$. Continuity then implies $J^c(E) < J^m(E)$ for all $E \in (E^*, E(p)]$ according to the above differential equation. This contradicts, however, $J^c(E(p)) = J^m(E(p))$. Thus, $J^c(E) > J^m(E)$ for all $E \in [U, E(p))$.

If the minimum wage is binding, i.e. $\underline{w} \geq \underline{W}'$, $J^c(E) > J^m(E)$ for all $E \in [E(\underline{w}), E(p))$ as shown above. Since $\dot{J}^m(E) = 0$ for $E \in [U, E(\underline{w}))$ while $\dot{J}^c(E) < 0$, i.e. $J^c(E)$ increases as E decreases, it follows that $J^c(E) > J^m(E)$ for all $E \in [U, E(p))$.

8 Proof of Proposition 4

Suppose the minimum wage is not binding, i.e., $\underline{w} < \underline{W}'$. By Proposition 3, low productivity firms choose type-c wage policy. Proposition 3 also implies that high productivity firms never match outside offers from other high productivity firms. This then implies according to equations (36) and (37) for the value of employing a worker,

$$\begin{aligned} \delta J_h^c(E) &= p_h - \underline{w} + \dot{J}_h^c(E) - \lambda \pi_h J_h^c(E) - \lambda \pi_l [1 - I(E)] J_h^c(E), \\ \delta J_h^m(E) &= p_h - \underline{w} + \dot{J}_h^m(E) - \lambda \pi_h J_h^m(E) \\ &\quad + [\lambda + s(E)] \pi_l [1 - I(E)] [J_h^m(E(p_l)) - J_h^m(E)], \end{aligned}$$

where π_l (π_h) denotes the fraction of low (high) productivity firms. The index variable $I(E)$ indicates whether the value of employment E is higher than the maximum value of employment at a low productivity firm, i.e.

$$I(E) = \begin{cases} 0 & \text{if } E < E(p_l) \\ 1 & \text{if } E \geq E(p_l) \end{cases}$$

Differentiating the two equations implies

$$\begin{aligned} [\delta + \lambda [1 - \pi_l I(E)]] [J_h^c(E) - J_h^m(E)] &= \dot{J}_h^c(E) - \dot{J}_h^m(E) + s(E) \pi_l [1 - I(E)] J_h^m(E) \\ &\quad - [\lambda + s(E)] \pi_l [1 - I(E)] J_h^m(E(p_l)). \end{aligned}$$

Thus, for all $E \geq E(p_l)$ it follows that $J_h^c(E) = J_h^m(E)$ since all firms do not match outside offers of firms that are equally or more productive. For $E < E(p_l)$ the differential equation reduces to

$$\begin{aligned} \dot{J}_h^c(E) - \dot{J}_h^m(E) &= [\delta + \lambda] [J_h^c(E) - J_h^m(E)] - s(E) \pi_l J_h^m(E) \\ &\quad + [\lambda + s(E)] \pi_l J_h^m(E(p_l)). \end{aligned}$$

First note that at a given E firms choose the wage policy that increases the value of employing a worker $J_h^t(E)$ at the fastest rate as E decreases, i.e. $\min\left[\dot{J}_h^c(E), \dot{J}_h^m(E)\right]$, since $\dot{J}_h^t(E) < 0$. At $E = E(p_l)$ the differential equation implies $\dot{J}_h^c(E(p_l)) > \dot{J}_h^m(E(p_l))$, since $J_h^c(E(p_l)) = J_h^m(E(p_l))$. Thus, continuity implies $J_h^c(E) < J_h^m(E)$ for some $E = E(p_l) - \varepsilon$, with $\varepsilon > 0$ small enough. Hence, it is optimal for high productivity firms to choose type-m wage policy at E.

Now suppose that a type-c wage policy is strictly profitable over some interval $[E^*, E^{**}) \in [U, E(p_l)]$. Optimality of type-$c$ wage policy requires $\dot{J}_h^c(E) < \dot{J}_h^m(E)$ for all $E \in [E^*, E^{**})$. Furthermore, since firms do not match outside offers at $E \in [E^*, E^{**})$ workers do not search, i.e. $s(E) = 0$ at all $E \in [E^*, E^{**})$. The differential equation then implies

$$-[\delta + \lambda] [J_h^c(E) - J_h^m(E)] = \lambda \pi_l J_h^m(E(p_l)),$$

which is only satisfied, if $J_h^c(E) < J_h^m(E)$ for all $E \in [E^*, E^{**})$. However, since all firms have chosen the same type-m wage policy over the interval $[E^{**}, E(p_l)]$ it must be true that $J_h^c(E^{**}) = J_h^m(E^{**})$. Since continuity of $J_h^c(E)$ and $J_h^m(E)$ rule out a jump at E^{**}, a type-c wage policy cannot be optimal at any $E \in [U, E(p_l))$.

If the minimum wage is binding, then $J_h^m(E)$ cannot increase beyond $J_h^m(E(\underline{w}))$. Thus, for $E \in [U, E(\underline{w})]$ a type-c wage policy is optimal.