Endogenous On-the-job Search and Frictional Wage Dispersion*

Matthias S. Hertweck†
University of Konstanz & IfW

This Version: January 5, 2011

Abstract

This paper addresses the large degree of frictional wage dispersion in US data. The standard job matching model without on-the-job search cannot replicate this pattern. With on-the-job search, however, unemployed job searchers are more willing to accept low wage offers since they can continue to seek for better employment opportunities. This explains why observably identical workers may be paid very differently. Therefore, we examine the quantitative implications of on-the-job search in a stochastic job matching model. Our key result is that the inclusion of *variable* on-the-job search increases the degree of frictional wage dispersion by an order of a magnitude.

JEL Codes: E24, J31, J64
Keywords: Matching, On-the-job Search, Wage Dispersion

*This paper is based on Chapter 2 of my doctoral dissertation written at the European University Institute. I am indebted to Morten Ravn and Salvador Ortigueira for their help and supervision. I am also very grateful to Antonella Trigari, Jean-Olivier Hairault, and Leo Kaas for extensive comments and suggestions. Part of this research project was conducted while the author was affiliated with the University of Basel, supported by the Swiss National Science Foundation (Project No. 118306).

†Contact Address: University of Konstanz, Department of Economics, Box 145, 78457 Konstanz, Germany. Email: `matthias.hertweck@uni-konstanz.de`, Homepage: `http://www.hertweck.org/`

1 Introduction

This paper addresses the large degree of frictional wage dispersion found in US data (Hornstein et al. 2007). The standard job matching model without on-the-job search cannot replicate this pattern. With on-the-job search, however, unemployed job searchers are more willing to accept low wage offers since they can continue to seek for better employment opportunities. This explains why observably identical workers may be paid very different wages. Therefore, we examine the quantitative implications of on-the-job search (Nagypál 2006) in a stochastic job matching model (Mortensen & Pissarides 1994). Our key result is that the inclusion of *variable* on-the-job search increases the degree of frictional wage dispersion by an order of a magnitude.

Hornstein et al. (2007) study frictional wage dispersion, i.e. wage differentials among observably identical workers, using a "mean-min-ratio" which relates the average wage paid to the lowest wage in the sample. They find that the mean-min-ratio takes values of 1.7 and above in US data. Adopting the framework of Mortensen & Pissarides (1994), the authors then examine the decision problem of an unemployed job searcher assuming that there is no aggregate uncertainty. Each firm-worker pair is characterized by an idiosyncratic productivity level. The worker and the firm form a match if and only if it yields a positive surplus and the wage rate of the worker is determined by Nash bargaining. Hornstein et al. (2007) demonstrate that unemployed job searchers are only willing to accept low wage offers if either (*i*) the job offer arrival rate is very low or (*ii*) the expected wage contract is very short. The former case implies that unemployed workers desperately accept any wage offer. Shimer (2007) estimates these values, where, consistent with the model presented, the possibility of direct flows from one employer to another is not taken into account. He finds that 45% of all unemployed workers find a new job within one month, while the average job duration is estimated to be about 2 1/2 years. Using these estimates, Hornstein et al. (2007) find that the model generated mean-min-ratio is not significantly different from unity.

This paper considers frictional wage dispersion in a model with on-the-job search. This seems to be a natural choice, given that the number of employed workers who change employer each month is about twice as large as the flow of unemployed job searchers into employment (Fallick & Fleischman 2004). The introduction of direct employment-to-employment[1] transitions has the potential to increase the degree of frictional wage dispersion for two reasons. First, unemployed job searchers who accept (temporarily) a low-wage job offer do not lose the option of labor market search. Hence, they may accept low wage offers for the moment, but continue to seek for better employment opportunities. This implies that a high degree of frictional wage dispersion and a high value of the average

[1]Our paper addresses only transitions of workers from one employer to another. We do not consider promotions within a given firm.

job finding rate may coexist at the same time. Second, Nagypál (2008) finds that average wage contract duration is overestimated when employment-to-employment transitions are not taken into account.

In particular, we modify the Mortensen & Pissarides (1994) set up as follows. Each firm-worker pair is characterized by its idiosyncratic productivity level, which is constant throughout the whole duration of the match. Search effort of both employed and unemployed job searchers is endogenous. All firm-worker matches are subject to exogenous and endogenous job separation hazard. Variations in the endogenous job separation margin are driven by aggregate productivity shocks. On-the-job search is motivated by the chance of finding a better job opportunity that promises (*i*) a higher real wage rate and (*ii*) a lower hazard of endogenous separation. An unemployed job searcher accepts any job offer, while an employed job searcher accepts a job offer (and quits the old job) only if it includes a higher surplus share.

We calibrate the model in order to match the empirical evidence presented in Nagypál (2008). Accordingly, conditional on not leaving the labor force, about 2/3 of all workers who separate from their employer are matched with a new one in the following month. In addition to that, the assumption of variable on-the-job search *and* endogenous job separation shocks helps us to replicate the observation that on-the-job search is most intense among workers in low-wage matches close to the separation margin (Fallick & Fleischman 2004, Christensen et al. 2005). Thus, our model is consistent with the empirical observation that (*i*) employment-to-employment transition rates are highest in matches that pay low wages and (*ii*) the aggregate employment-to-employment transition rate is slightly below the value of 3%.

High employment-to-employment transition rates imply that low-wage matches exhibit a high option value of labor market search. This stimulates unemployed job searchers to accept such offers and to search on-the-job for better employment opportunities. Moreover, we note that the expected duration of low-wage job matches is far below the average. Consequently, our calibrated model is able to generate a mean-min-ratio equal to 1.3. Compared to the value provided by Hornstein et al. (2007), the percentage difference between the average and the lowest wage paid rises by an order of a magnitude. Therefore, we conclude that the introduction of variable on-the-job search into a model with endogenous job separations is an effective means to generate frictional wage dispersion.

In addition, we examine the dynamic behavior of the model at the business cycle frequencies. Our analysis focuses on the cyclical movements of vacancies, unemployment, and the real wage rate. Interest in this issue has been sparked by the influential paper of Shimer (2005), which states that the job matching model with exogenous separations (Pissarides 1985) is not able to replicate the high degree of labor market volatility. Furthermore, Mortensen & Nagypál (2008) point out that counter-cyclical fluctuations in the endogenous separation margin are able to amplify the variations in the number of unemployed job searchers on the one hand. On the other hand, these strong counter-cyclical movements provide firms incentives to open more vacancies during economic downturns. Hence, the model generated Beveridge curve may be counter-factually positively sloped. In our model, on the contrary, variable on-the-job search uncouples aggregate search effort from the number of unemployed job searchers. Since on-the-job search is pro-cyclical, we note that total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, is relatively stable over the business cycle. This stimulates firms to open more vacancies when aggregate productivity is high. As a consequence, our model is able to replicate a negatively sloped Beveridge curve.

The remainder of this paper is organized as follows. Section 2 presents the model environment. Section 3 calibrates the model and evaluates its quantitative performance against US data. Section 4 concludes.

2 The Model Environment

2.1 Employment Relationships

There is a continuum of ex-ante identical workers in the economy, having unit mass and a continuum of potential firms, having infinite mass. Both firms and workers are risk-neutral. Production takes place in one-firm-one-worker matches. Each active match produces output according to a linear technology: $y(a, t) = az_t$. We assume that match-specific idiosyncratic productivity level a is constant throughout the whole duration of the match. The exogenous distribution of a is described by the cumulative distribution function $P(a)$ with support $[0, \infty)$. Aggregate productivity z_t, instead, is subject to an exogenous shock specified by following autoregressive process:

$$\ln(z_t) = (1 - \varrho)\ln(\bar{z}) + \varrho \ln(z_{t-1}) + \epsilon_t \text{ with } \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2) \text{ and } iid. \tag{1}$$

2.2 The Labor Market

In the beginning of period t, there are N_t matched firm-worker pairs. The endogenous distribution of firm-worker pairs over idiosyncratic productivity levels is described by a cumulative distribution function $G(a, t)$ with support $[0, \infty)$. All firm-worker pairs face exogenous separation with probability ρ^x. In addition, a match may be separated if its idiosyncratic productivity level is below the current reservation productivity $a_{r,t}$. Workers who lose their job, whether for exogenous or endogenous reasons, immediately enter the period t matching pool. Hence, the unemployment rate U_t is given by the share of workers who are not engaged in *active* employment relationships:[2]

$$U_t = 1 - N_t(1-\rho^x)\int_{a_{r,t}}^{\infty} g(a,t)da.$$

Labor market search takes place parallel to production. The pool of job searchers comprises all unemployed and all employed workers who search on-the-job for better employment opportunities. Search effort of both, unemployed and employed job searchers, is endogenous. Unemployed job searchers are identical and, hence, search all with the same intensity $e_{u,t}$ on the labor market. The search effort of an employed worker $e_w(a,t)$ depends on her idiosyncratic productivity level a. Thus, aggregate search effort E_t is given by:

$$E_t = U_t e_{u,t} + N_t(1-\rho^x)\int_{a_{r,t}}^{\infty} e_w(a,t)g(a,t)da.$$

Search effort of unemployed and employed job searchers incurs a cost. In particular, we assume that the respective cost functions are given as:

$$c\left[e_{u,t}\right] = \zeta_u \left[e_{u,t}\right]^{\phi_u}, \qquad c\left[e_w(a,t)\right] = \zeta_w \left[e_w(a,t)\right]^{\phi_w},$$

where the parameter ϕ_i, $i = u, w$ captures the fact that the level and the curvature of the search cost function may depend on the employment status.

Besides the pool of job searchers, the period t matching market consists of the aggregate number of vacancies V_t. Firms with unfilled positions may decide whether or not to post a vacancy, where posting a vacancy entails a cost κ per period. Free entry into the matching market determines the aggregate number of posted vacancies.

New matches are formed at the end of period t. The number of newly formed firm-worker pairs is given by a Cobb-Douglas matching function with constant returns to scale. This function relates aggregate job matches M_t to aggregate vacancies V_t and aggregate search effort E_t:

$$M(V_t, E_t) = \min\left[\chi V_t^{\gamma} E_t^{1-\gamma}, V_t, 1\right],$$

where χ is a constant scaling factor.

The ratio between aggregate vacancies and aggregate search effort measures the tightness of the labor market. By linear homogeneity of the matching function, the matching

[2] Our model abstracts from movements into and out of the labor force.

probability per unit search effort $f(\theta_t)$, and the matching rate per vacancy $q(\theta_t)$, respectively, depend solely on the value of market tightness θ_t:

$$f(\theta_t) = \frac{M(V_t, E_t)}{E_t} = \chi \left(\frac{V_t}{E_t}\right)^{\gamma} = \chi \theta_t^{\gamma}, \tag{2}$$

$$q(\theta_t) = \frac{M(V_t, E_t)}{V_t} = \chi \left(\frac{V_t}{E_t}\right)^{\gamma-1} = \chi \theta^{\gamma-1}. \tag{3}$$

The tighter the labor market, the longer the expected time to fill a vacancy, but the shorter the expected search for a job (and vice versa). The fact that firms and households do not internalize these adverse effects on the aggregate return rates gives rise to congestion externalities.

Labor market search and match formation entails that the employment distribution in the beginning of period $t+1$ is given by:

$$\begin{aligned} N_{t+1} \int_0^a g(a, t+1) da = & \int_0^a \Bigg\{ U_t f(\theta_t) e_{u,t} p(a) + \\ & I_{a > a_{r,t}} (1-\rho^x)(1-\tau(a,t)) N_t g(a,t) + \\ & (1-\rho^x) N_t f(\theta_t) \left(\int_{a_{r,t}}^a e_w(a', t) g(a', t) da' \right) p(a) \Bigg\} da, \end{aligned} \tag{4}$$

where I is an indicator function equal to zero if a is below the current reservation productivity $a_{r,t}$, and otherwise equal to one. The right hand side of equation (4) is made up of (*i*) the mass of unemployed job searchers who succeed in meeting an employer, (*ii*) the distribution of workers in existing job matches that experience neither job separation nor a transition (with probability $1 - \tau(a,t)$) to a new employer and, (*iii*) the distribution of new job matches established by successful employment-to-employment transitions. Evaluating equation (4) at $a \to \infty$ yields a more familiar law of motion:

$$N_{t+1} = (1-\rho^x) N_t [1 - G(a_{r,t})] + f(\theta_t) e_{u,t} U_t.$$

2.3 The Joint Surplus

Let the value of unemployment to a worker be $\mathcal{U}_t$, the value of a vacancy to a firm be $\mathcal{V}_t$, the value of a match to a worker be $\mathcal{W}(a,t)$, and the value of a match to a firm be $\mathcal{J}(a,t)$. Hence, the joint surplus of an active match $\mathcal{S}(a,t)$ is given by the value of output $y(a,t)$, net of expenses for search on-the-job $c\,[e_w(a,t)]$ and the joint outside alternative $(\mathcal{U}_t + \mathcal{V}_t)$, plus the joint continuation value of the current match $\mathcal{C}(a,t)$ in the future:

$$\mathcal{S}(a,t) = y(a,t) - c\,[e_w(a,t)] - (\mathcal{U}_t + \mathcal{V}_t) + \mathcal{C}(a,t). \tag{5}$$

The joint surplus is divided by Nash (1953) bargaining. Consequently, the firm gains period-by-period the fixed portion ξ, while the share $1-\xi$ is allocated to the worker. We assume that wage bargaining takes place in the beginning of period t, but *after* the employed worker has made her search decision. This timing assumption has two important implications. First, employed workers who have just made a transition to a new employer are not able to resume their old position. Therefore, the outside alternative of all workers is equal to the value of unemployment. Second, the bargaining outcome does not influence the search decision of the worker. Thus, the surplus shares are given as:[3]

$$\xi\mathcal{S}(a,t) = \mathcal{J}(a,t) - \mathcal{V}_t, \qquad (1-\xi)\mathcal{S}(a,t) = \mathcal{W}(a,t) - \mathcal{U}_t. \tag{6}$$

Equation (6) shows that the surplus share of the firm equals the value of an active match to the firm $\mathcal{J}(a,t)$ net of the value of an unfilled vacancy $\mathcal{V}_t$. The surplus share of the worker equals the value of an active match to the worker $\mathcal{W}(a,t)$ net of the value of unemployment $\mathcal{U}_t$. Furthermore, the Nash sharing rule implies that any endogenous separation decision is made by mutual consent. Consequently, the current reservation productivity $a_{r,t}$ has to satisfy the following job separation condition:

$$\mathcal{S}(a_{r,t},t) = 0 \;\Leftrightarrow\; y(a_{r,t},t) - c[e_w(a_{r,t},t)] + \mathcal{C}(a_{r,t},t) = \mathcal{U}_t + \mathcal{V}_t.$$

2.4 The Problem of an Unemployed Job Searcher

We now examine the decision problem of an unemployed job searcher. There is a continuum of ex-ante unemployed job searchers in the economy, having mass U_t. All unemployed job searchers take the aggregate matching rate per unit search effort f_t as given and search $e_{u,t}$ units on the labor market. Hence, an unemployed job searcher can expect to meet a firm at the rate:

$$\tilde{f}(e_{u,t},\theta_t) = f_t e_{u,t}. \tag{7}$$

When an unemployed job searcher meets a firm at the end of period t, the pair draws its idiosyncratic productivity level a, which is constant throughout the whole duration of the match. Given that next period's reservation productivity $a_{r,t+1}$ is still unknown, unemployed job searchers accept every match for the moment. If the match survives

[3] The present timing assumption is clearly a simplification. However, to our knowledge, there is no study investigating all aspects of a multi-player bargaining game under variable on-the-job search. Cahuc et al. (2006) examine the implications of a multi-player bargaining game, where the search intensity of employed job searchers is constant (Burdett & Mortensen 1998). They demonstrate that between-firm competition increases the average wage rate if employed job searchers can resume their old positions. Papp (2009) examines the implications of this approach in a general equilibrium model. On the other hand, Shimer (2006) analyzes a strategic bargaining game between an employed job searcher and a single firm, where the firm is willing to offer a higher wage rate in order to reduce the extent of transitions to other employers. Indeed, the examination of these questions in a stochastic environment is beyond the scope of this paper.

exogenous destruction at the beginning of period $t+1$, the firm and the worker observe the realization of aggregate productivity z_{t+1} and decide upon endogenous separation. Only if the idiosyncratic productivity of the firm-worker match is greater than the reservation productivity $a > a_{r,t+1}$, the match will become active. Thereby, the reservation strategy of the worker implies that she gains at least the value of unemployment $\mathcal{U}_{t+1}$. This setup implies that the expected value of a match to the worker is given by:

$$\overline{\mathcal{W}}_t = \int_0^{\infty} \mathcal{W}_t(a)p(a)da, \tag{8}$$

where $\mathcal{W}(a,t) = \mathcal{U}_t$ below the reservation productivity $a_{r,t}$. Hence, $\mathcal{W}(a,t)$ captures not only the value of active matches, but also the value of matches that are endogenously terminated. For this reason, the value of unemployment can be written as:

$$\mathcal{U}_t = \max_{e_{u,t} \geq 0} \left\{ b - c\left[e_{u,t}\right] + \beta E_t \left[(1-\rho^x)\tilde{f}_t \Big(\overline{\mathcal{W}}_{t+1} - \mathcal{U}_{t+1} \Big) + \mathcal{U}_{t+1} \right] \right\}. \tag{9}$$

Each worker who is not engaged in an active employment relationship receives the flow income b, net of $c[e_{u,t}]$ units search costs. Future utility is discounted at the rate β. The worker expects to find an average job in period $t+1$ with probability $(1-\rho^x)\ \tilde{f}_t$. In this case, the worker gains the expected surplus share $(\overline{\mathcal{W}}_{t+1} - \mathcal{U}_{t+1})$. In addition, the unemployed worker will receive at least the value of unemployment $\mathcal{U}_{t+1}$ – independent of whether or not she succeeds in finding an active employment relationship.

Consequently, the search effort choice $e_{u,t}$ of an unemployed worker has to satisfy following first order condition:

$$c'\left[e_{u,t}\right] = (1-\rho^x) f_t \beta E_t \left\{ \overline{\mathcal{W}}_{t+1} - \mathcal{U}_{t+1} \right\}. \tag{10}$$

The first order condition of an unemployed worker states that the marginal costs of labor market search (the left hand side) must be equal to the marginal benefits (the right hand side). The latter is given by the matching rate per unit labor market search f_t, the exogenous destruction rate ρ_x, and the expected present value of the worker's surplus share.

2.5 The Problem of an Employed Job Searcher

Employed workers who search on-the-job for better employment opportunities face the same meeting rate per unit search effort f_t as unemployed job searchers. The search effort of an employed worker, however, depends on her idiosyncratic productivity level a. Thus, the expected meeting rate of an employed job searcher is given as: $f_t e_w(a,t)$. Moreover, the reservation strategy of an employed worker implies that she accepts matches at the end of period t only if the idiosyncratic productivity of the new match $\hat{a}$ is strictly larger than the one of the old match a. Otherwise, she remains on her old job. Given that the idiosyncratic productivity level of new matches is drawn from the exogenous distribution $P(a)$, the current setting implies that an employed job searcher of type a rejects new job offers at the rate $P(a)$. Hence, the expected employment-to-employment transition rate of an employed worker with idiosyncratic productivity a reads as:

$$\tau(a,t) = f_t e_w(a,t)[1 - P(a)]. \tag{11}$$

Consequently, the worker's value of a match with idiosyncratic productivity a can be written as:

$$\begin{aligned} \mathcal{W}(a,t) \quad = \quad & \max_{e_w(a,t)\geq 0} \Big\{ w(a,t) - c[e_w(a,t)] + \beta E_t \Big\{ \\ & \Big(\Big\langle (1-\tau(a,t))(1-\rho^x) \Big(\mathcal{W}(a,t+1) - \mathcal{U}_{t+1} \Big) \Big\rangle + \\ & \Big\langle f_t e_w(a,t)(1-\rho^x) \int_a^\infty \Big(\mathcal{W}(\hat{a},t+1) - \mathcal{U}_{t+1} \Big) p(\hat{a}) d\hat{a} \Big\rangle + \mathcal{U}_{t+1} \Big) \Big\} \Big\}. \end{aligned} \tag{12}$$

The worker's value of a match $\mathcal{W}(a,t)$ is given by the real wage rate $w_{a,t}$, net of expenses for search on-the-job $c[e_w(a,t)]$, and the continuation value of the match to the worker. The worker's continuation value is given by (i) the expected surplus share of the *current* employment relationship, (ii) the expected surplus share of a prospective job opportunity with higher ($\hat{a} > a$) idiosyncratic productivity, and (iii) the expected present value of unemployment (the minimum gain of the worker). The first term (i) refers to the worker who does not find a better employment opportunity, but suffers neither exogenous nor endogenous job destruction. The second term (ii) characterizes the worker who succeeds in finding a better job opportunity which survives exogenous and endogenous job destruction. The third term (iii) applies if the *current* or the *prospective* employment relationship is terminated either for *exogenous* or *endogenous* reasons.

When employed workers make their search effort decision, they anticipate the Nash wage rate $w(a,t)$ correctly. Wage bargaining, however, does not start until the search effort decision has been made.[4] Hence, the optimal search intensity of an employed worker with idiosyncratic productivity a is given by following first order condition:

$$c'[e_w(a,t)] = f_t(1-\rho^x)\beta E_t \Big\{ \int_a^\infty \Big(\mathcal{W}(\hat{a},t+1) - \mathcal{W}(a,t+1) \Big) p(\hat{a}) d\hat{a} \Big\}. \tag{13}$$

Equation (13) shows that the search intensity of employed workers decreases in the idiosyncratic productivity level a. The expected upgrading value $\mathcal{W}(\hat{a},t+1) - \mathcal{W}(a,t+1)$, which is determined by the expected surplus from a prospective job opportunity $\hat{a}$ given

[4] See Footnote (3).

the value of the current job a, is highest if the idiosyncratic productivity level a is slightly above the current reservation productivity level $a_{r,t}$. Workers engaged in these matches fear to loose their job in the case of a negative shock to aggregate productivity.

2.6 Optimal Vacancy Posting

There is an infinite mass of firms with unfilled positions. Each firm with an unfilled position may decide whether or not to post a vacancy. Equation (3) shows that a firm with an open vacancy expects to meet a job searcher at rate $q(\theta_t)$. However, the firm anticipates the possibility of meeting an employed job searcher who might reject the job offer in favor of her old job. Hence, the probability of *filling* a vacancy $\tilde{q}_t$ is given by the probability of *meeting* a worker $q(\theta_t)$, the job offer acceptance rate $(1-\rho_t^j)$, and the exogenous separation rate ρ^x:

$$\tilde{q}_t = q(\theta_t)\left(1-\rho_t^j\right)(1-\rho^x), \tag{14}$$

where the share of rejected job offers ρ_t^j is given by the number of rejected job offers divided by the number of aggregate matches:

$$\rho_t^j = \left(N_t(1-\rho^x)f_t\int_{a_{r,t}}^{\infty} e_w(a,t)P(a)g(a,t)da\right)\Big/ M_t. \tag{15}$$

Since next period's reservation productivity is still unknown, the firm accepts matches of any idiosyncratic productivity level a for the moment. When the worker and the firm decide whether to engage in production in period $t+1$, the reservation strategy of the firm ensures that the value of the match is greater or equal to the value of an unfilled vacancy $\mathcal{V}_t$. Hence, the expected value of a filled vacancy is given by:

$$\overline{\mathcal{J}}_t = \int_0^{\infty}\mathcal{J}(a,t)p(a)da, \tag{16}$$

where the value of a match to the firm $\mathcal{J}(a,t)$ can be written as:

$$\mathcal{J}(a,t) = y(a,t) - w(a,t) + \beta E_t\left\{(1-\tau(a,t))(1-\rho^x)\mathcal{J}(a,t+1)\right\}. \tag{17}$$

The firm enjoys the value of production, net of labor costs $w(a,t)$. The firm's continuation value is determined by the current employment-to-employment transition rate $\tau(a,t)$, the exogenous job destruction rate ρ^x, and the expected present value of the current match in the next period. Furthermore, we note that $\mathcal{J}(a,t) = \mathcal{V}_t$ below the reservation productivity $a_{r,t}$. Hence, $\mathcal{J}(a,t)$ measures not only the value of active matches, but also the value of matches that are consensually terminated. Hence, the value of an unfilled vacancy is given by:

$$\mathcal{V}_t = -\kappa + \beta E_t \left[\tilde{q}_t \overline{\mathcal{J}}_{t+1} + (1-\tilde{q}_t)\mathcal{V}_{t+1} \right]. \tag{18}$$

Recall that posting a vacancy entails a cost κ per period. Therefore, the firm expects to gain the value of a filled vacancy $\overline{\mathcal{J}}_t$ with probability $\tilde{q}_t$. Otherwise, the vacancy remains unfilled. Free entry into the matching market ensures that the firm's outside option, i.e. the value of an unfilled vacancy, is zero in every period: $\mathcal{V}_t = 0$. Thus, the number of posted vacancies has to satisfy following condition:

$$\frac{\kappa}{\tilde{q}_t} = \beta E_t \overline{\mathcal{J}}_{t+1}. \tag{19}$$

2.7 The Wage Function

Using the Nash sharing rule (6), the value of a match to the worker (12), and the value of a match to the firm (17), the joint surplus of the match (5) can be rewritten as follows:

$$\begin{aligned} \mathcal{S}(a,t) \;&=\; y(a,t) - c[e_w^*(a,t)] - \mathcal{U}_t - \mathcal{V}_t + \\ &\quad \beta E_t \Bigg\{ \Bigg\langle (1-\tau(a,t))(1-\rho^x)\mathcal{S}(a,t+1) \Bigg\rangle + \\ &\quad \Bigg\langle f_t e_w(a,t)(1-\rho^x) \int_a^\infty (1-\xi)\mathcal{S}(\hat{a},t+1)p(\hat{a})d\hat{a} \Bigg\rangle + \mathcal{U}_{t+1} \Bigg\}. \end{aligned} \tag{20}$$

where $e_w^*(a,t)$ satisfies equation (13).

Equation (20) shows that the joint continuation value of the match consists of (*i*) the option value of the current match, (*ii*) the option value of on-the-job search, and (*iii*) the option value of unemployment. Thereby, only the benefits of on-the-job search going to the worker $(1-\xi)\mathcal{S}(\hat{a},t+1)$ enter the mutual surplus, while the benefits going to the new employer $\xi\mathcal{S}(\hat{a},t+1)$ are not taken into account. Furthermore, the Nash bargaining solution entails following real wage function, depending on the idiosyncratic productivity level a:

$$\begin{aligned} w(a,t) \;&=\; (1-\xi)y(a,t) + \xi\left(c[e_w^*(a,t)] + \mathcal{U}_t\right) - \\ &\quad \xi\beta E_t \Bigg\{ \Bigg\langle f_t e_w(a,t)(1-\rho^x) \int_a^\infty (1-\xi)\mathcal{S}(\hat{a},t+1)p(\hat{a})d\hat{a} \Bigg\rangle + \mathcal{U}_{t+1} \Bigg\}. \end{aligned} \tag{21}$$

The real wage rate is given by the weighted average of (*i*) the value of production and (*ii*) the value of unemployment, plus compensation for search on-the-job, minus the option value of on-the-job search to the worker. The positive option value to the worker reduces her reservation wage. Therefore, we observe that the Nash wage in our model with on-the-job search is lower than in the baseline matching model.

2.8 Equilibrium

The competitive equilibrium is given by the unemployment rate U_t, the aggregate number of vacancies V_t, the search effort of unemployed workers $e_{u,t}$, the function of employed workers' search effort $e_w(a,t)$, the distribution of firm-worker matches $G(a,t)$, and a wage function $w(a,t)$, such that:

- $\mathcal{U}_t$ and $\mathcal{W}(a,t)$ are the value of unemployment and of a match, respectively, for workers making optimal search effort decisions, given U_t, V_t, E_t, $w(a,t)$, and $G(a,t)$. $e_w(a,t)$ and $e_{u,t}$ are the corresponding optimal search effort policies.
- $\mathcal{V}_t$ and $\mathcal{J}(a,t)$ are the value of a vacancy and of a match for firms making optimal vacancy creation decisions, given U_t, V_t, E_t, $w(a,t)$, $e_w(a,t)$, $e_{u,t}$, and $G(a,t)$.
- Total factor productivity z_t follows the exogenous stochastic process (1).
- There is free entry into the matching pool of vacancies.
- New firm-worker matches draw their idiosyncratic productivity a from an exogenously given distribution $P(a)$.
- Wages are set by sharing the surplus of an active firm-worker match in fractions ξ and $1-\xi$, respectively, given the wage function $w(a,t)$.
- The distribution $G(a,t)$, the unemployment rate U_t, the aggregate number of vacancies V_t, and the total search effort E_t are consistent with the decisions of the agents in the economy.
- An initial condition for the share of matched firm worker pairs N_0 is given.

3 Model Evaluation

3.1 Computational Issues

We analyze the cyclical properties of the model economy by value function iteration on a discrete state space. Thereby, the treatment of the endogenous productivity distribution as an endogenous state variable establishes a computational challenge. Particularly, as in the case of endogenous separations the endogenous productivity distribution exhibits a discontinuity at the reservation productivity $a_{r,t}$. This might be the reason why only a small number of authors so far have addressed the issue of on-the-job search in a job matching model. Among others, Nagypál (2006) examines the stationary equilibrium of a job matching model with variable on-the-job search, where idiosyncratic productivity shocks lead to endogenous separations. She concludes that workers close to the endogenous separation margin show the highest intensity of on-the-job search. In a companion paper, Nagypál (2007) considers a log-linear approximation around the non-stochastic steady state. However, in order to ensure differentiability of the endogenous distribution, the model drops the assumption of endogenous separations. On the contrary, Fahr (2007) argues that firm-worker pairs are subject to noisy signals about the idiosyncratic productivity level. This implies that some profitable matches separate endogenously by mistake, while some non-profitable matches are continued. Thus, the discontinuity at the reservation productivity may be smoothed out by a logistic distribution. For this reason, the model with on-the-job search and endogenous separations can be solved by a linear approximation. Tasci (2007) extends the model by Pries & Rogerson (2005) in order to allow for *exogenous* on-the-job search. In this setting, it suffices to approximate the worker's acceptance probability. For this purpose, the algorithm by Krusell & Smith (1998) is utilized. Krause & Lubik (2007) assume segregated markets for good and bad jobs, where separations are exogenous and constant in both market segments. Finally, Menzio & Shi (2010) introduce on-the-job search into a model of directed search akin to Moen (1997). They demonstrate that, in this setting, the agents' decisions are independent of the endogenous productivity distribution.

Instead, our approach is based on the observation that, in the US, the half life deviation of the actual unemployment rate from its stationary value is close to one month (Elsby et al. 2009). Consequently, the correlation between the stationary rate and the actual unemployment rate in the following month is very close to unity (Shimer 2007). Since the lag is so short, Hall (2009) suggests neglecting the fact that the unemployment rate is governed by a backward-looking law of motion. In addition, as described above, we note that the flow of workers from one employer to another is more than twice as large as the flow from unemployment to employment. For this reason, we treat neither the unemployment rate nor the corresponding endogenous productivity distribution as endogenous state variables. The only state variable in our model is aggregate productivity.

3.2 Calibration

In order to capture the high transition rates in the US economy, we calibrate the model so that one period corresponds to one month. When simulating the model, we time-aggregate the artificial series to quarterly data and evaluate it against the quarterly US time series. Table 1 summarizes the parameter values of our model.

Preferences Workers are risk-neutral and supply labor inelastically. The discount factor β is chosen to match an annual real interest rate of 4 percent (Kydland & Prescott 1982). Furthermore, estimates by Christensen et al. (2005) and Yashiv (2000) indicate that both types of job searchers face a quadratic cost of search effort ($\phi_e = \phi_u = 2$).

Matching and the Labor Market We target an average unemployment rate $U = 10\%$ and a workers' meeting rate $\tilde{f}_t = 27\%$ (Hall 2006). These figures refer to the officially unemployed job searchers plus the pool of marginally attached non-participants (Jones & Riddell 1999). Our calibrated value of unemployment benefits $b = 0.85$, together with the implied average value of search disutility ($c[\bar{e}^*_w] = 0.14$) and average output per worker $\bar{y} = 1.25$, yields a replacement rate equal to 0.56. This value lies within the range found in the literature.[5] Targets for the transition rate out of employment are provided by Hall (2006). Therefore, we set the exogenous separation rate ρ^x to 0.0275. Furthermore, our calibration implies that the endogenous separation rate ρ^n is close to 0.01, and the average employment-to-employment transition rate $\overline{\tau}$ is equal to 0.04. Beside that, we choose the firm's bargaining power $\xi = 0.25$ and the per-period vacancy posting cost $\kappa = 0.06$, such that the unemployment rate and the percentage of vacancy posting costs in aggregate output $(\kappa V)/Y = 1.9\%$ are in line with our targets. The latter value implies that the steady-state asset value of a match to the firm is equal to 19% of average output (Yashiv 2000). Finally, we calibrate the two parameters of the matching function. First, we set the matching function constant χ equal to 0.45.[6] Second, we calibrate the matching elasticity of vacancies η equal to 0.5, which is within the plausible range $[0.3 - 0.5]$ proposed by Petrongolo & Pissarides (2001).

Productivity We approximate the stochastic process (Equation 1) with a discrete valued first-order Markov process using the method by Tauchen (1986). Thereby, we use the values ($\varrho = 0.97, \sigma_\epsilon = 0.007$) suggested by Hagedorn & Manovskii (2008).[7] The number of grid points in the state space is set equal to 29. The exogenous productivity distribution is assumed to be log-normal with mean $\mu = 1$ and a standard deviation equal to $\sigma_p = 0.1$. We compute the exogenous and the endogenous productivity distribution, $P(a)$ and $G(a)$ respectively, on a very fine grid with 500 points between 0.7 and 1.5.

[5] The value used by Shimer (2005), $b = 0.4$, comprises only pecuniary benefits and, thus, is considered to be a lower bound. Values beyond are usually justified by the reference to "leisure gain from unemployment". Angerhausen et al. (2010) provide a microfoundation for this claim, while the results of Costain & Reiter (2008) indicate that this "gain" is quantitatively important. An upper bound is provided by the estimate $b = 0.95$ of Hagedorn & Manovskii (2008), who attempt to match the elasticity of the real wage rate.

[6] As argued by Shimer (2005), the model allows the normalization of this value.

[7] The published version of the model is calibrated so that one period corresponds to one week. Please refer to the working paper version (available at: `http://www.econ.umn.edu/macro/2005/hagedorn.pdf`) for the monthly calibration.

3.3 Steady State Analysis

The Endogenous Productivity Distribution Figure (1a) displays the probability density function of the exogenous productivity distribution $P(a)$, represented by a blue line, as well as the *endogenous* productivity distribution at 5 different levels of aggregate productivity (at grid points 1, 8, 15, 22, and 29), *after* endogenous separation has taken place. Grid point 15 (turquoise line) represents the stochastic steady state. The graph clearly shows that the positive selection of employed workers towards matches with higher idiosyncratic productivity shifts the endogenous distribution to the right. In the steady state, average match quality is about 25% higher than the mean of the exogenous productivity distribution. Moreover, we note that the impact of aggregate productivity on average match quality follows a non-linear pattern (Figure 1b). The slope of the right tail is much steeper than the slope of the left tail. This suggests that the channel from aggregate productivity to average match quality is shaped by two opposing effects. On the one hand, recessions destroy low-quality matches which are likely to be profitable during economic upswings. This is referred to as the “cleansing” effect of recessions (Caballero & Hammour 1994). On the other hand, we observe that search intensity of on-the-job searchers is pro-cyclical (Figure 1d). Consequently, the number of employment-to-employment transitions is pro-cyclical as well (Figure 1f). This implies that recessions are times when workers tend to stay in low quality matches. Barlevy (2002) refers to this as the “sullying” effect of recessions. In our calibrated model, the “sullying” effect is dominant. Therefore, average match quality is pro-cyclical. Nevertheless, the flat shape of the left tail shows that also the “cleansing” effect is present.

The Average Real Wage Rate The evolution of average match quality over the business cycle has important implications for the dynamic behavior of the real wage rate. In the presence of period-by-period Nash bargaining, both variables are linked very closely. This is manifested by Figure 1b. Individual wages, instead, increase monotonously in aggregate productivity (Figure 1g). This finding indicates that our model may be able to match an important empirical observation. Several authors, among others Solon et al. (1994), Bowlus et al. (2002) and Hart (2006), report that individual wage profiles are strongly pro-cyclical. Aggregate data, however, show (at most) a weakly pro-cyclical pattern. The literature argues that the apparent discrepancy is due to the fact that aggregate data often neglects the cyclical variations in the average match quality. Our model seems to be consistent with this claim.

Frictional Wage Dispersion Furthermore, we observe that the real wage rate paid in the least productive active match is essentially independent of aggregate productivity (Figure 1g). Consequently, the ratio between the average and the lowest wage paid in the economy, the so-called “mean-min-ratio”, follows a similar pattern as the average real wage rate (Figure 1h). The mean-min-ratio measures the degree of frictional wage dispersion, i.e. wage differentials among ex-ante identical workers. Hornstein et al. (2007) demonstrate that the job matching model without on-the-job search is unable to match the high degree of frictional wage dispersion found in the data. In the US, the average wage paid to similar workers is about 70% higher than the lowest wage within the set. In contrast, the mean-min ratio generated by a reasonably calibrated version of the job matching model without on-the-job search is not significantly different from unity. In our calibrated model, the steady state value of the mean-min-ratio is equal to 1.4. Thus, our model is able to explain about one-half of the empirical degree of frictional wage dispersion.

Hornstein et al. (2007) argue that the inclusion of on-the-job search is essential for generating these results. In the presence of high job finding rates, unemployed job searchers have no incentive to accept a long-term job that pays a low wage rate. With on-the-job search, employed workers are more likely to accept low wage offers for the moment and continue to seek for better employment opportunities. However, a model with exogenous on-the-job search (Burdett & Mortensen 1998) requires far too high EE-transition rates in order to replicate the degree of frictional wage dispersion found in the data. For this reason, we introduce variable on-the-job search and endogenous separations. These two modifications allow us to concentrate on-the-job search among workers in low quality matches. These workers have (*i*) a large probability of finding a better employment opportunity and (*ii*) a high probability of entering unemployment. Therefore, our model is able to generate significant frictional wage dispersion. At the same time, the share of employed workers who change employer each month is very close to the empirical value of 3% (Figure 1f).

On-the-Job Search Figure (1c) illustrates the search effort decision of employed and unemployed job searchers, respectively. The horizontal lines represent the effort per unemployed job searcher, which depends only on the level of aggregate productivity. The search effort choice of employed job searchers, instead, is a function of aggregate and idiosyncratic productivity. As expected, search effort of any type of worker rises with aggregate productivity, while search effort of employed job searchers falls with idiosyncratic productivity. This happens for two reasons. First, the probability of finding a better employment opportunity is smaller the higher the quality of the current match. Second, the hazard of endogenous job separation decreases with the distance to the current reservation productivity. We observe that the search behavior of an employed worker at the reservation productivity (the "marginal" worker) is almost identical to the search behavior of an unemployed. Due to the low reservation productivity, these workers are (*i*) likely to find a better employment opportunity and (*ii*) face a huge endogenous separation hazard in the case of a negative productivity shock. Moreover, we note that the bulk of employed workers, i.e. workers in matches with idiosyncratic productivity $a > 1.1$, do not make any significant effort to find a new employer. This observation is consistent with the findings by Fallick & Fleischman (2004) and Christensen et al. (2005).

Total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, seems to be relatively stable (Figure 1d). The composition, however, changes considerably over the business cycle. Search effort of employed job searchers rises with aggregate productivity, since there are more on-the-job searchers (extensive margin) who search more than the employed workers already in place (intensive margin). In particular, note that the search effort of a given employed worker increases only by little when economic conditions improve. This indicates that escaping from imminent unemployment is probably the main motive for on-the-job search. In addition to that, we observe that aggregate search effort of unemployed job searchers is counter-cyclical. Even though the search intensity per unemployed job searcher rises (intensive margin, see Figure 1c), the number of unemployed job searchers declines sharply during economic upswings (extensive margin, see Figure 2b). This implies that a firm is much more likely to meet an employed job searcher when aggregate productivity is high.

Employment-to-Employment Transitions Figure (1e) presents the rates at which employed workers find new employers. Given that the meeting rate per unit search effort is not very elastic, EE-transition rates follow the same pattern as search intensity of employed workers. Consequently, marginal workers enjoy almost the same transition probability (well above 30%) as unemployed job searchers during economic booms (grid point 29). During recessions (grid point 1), on the contrary, the EE-transition rate of marginal workers is only slightly above 10%. The job finding rate of unemployed job searchers, on the other hand, never falls below a value of 25%. The huge differences in the transition probabilities of marginal workers may help to explain the shape of the endogenous productivity distribution at the reservation productivity (Figure 1a). When aggregate productivity is low, the endogenous productivity distribution exhibits a clear cut-off point. Yet, when aggregate productivity is high, we observe that the endogenous productivity distribution is very smooth.

Aggregate Labor Market Dynamics In addition, our model allows analyzing the cyclical behavior of the labor market. Interest in this issue has been sparked by the influential paper of Shimer (2005). The survey paper by Yashiv (2008) provides a general picture of US labor market dynamics over the business cycle. Accordingly, gross worker flows between employment and unemployment are counter-cyclical and volatile. On the one hand, counter-cyclical flows from employment to unemployment are driven by counter-cyclical movements in the job separation margin (Fujita 2009). On the other hand, counter-cyclical flows from unemployment to employment are due to the fact that the percentage fall in the job finding rate is smaller than the percentage rise in the unemployment rate in the aftermath of a negative productivity shock (Fallick & Fleischman 2004). This implies that the number of unemployed job searchers is strongly counter-cyclical. As pointed out by Mortensen & Nagypál (2008), such a scenario might give firms incentives to open more vacancies during economic downturns. Hence, the model generated Beveridge curve may be counter-factually positively sloped. In the case of variable on-the-job search, however, total search effort is uncoupled from the number of unemployed job searchers. For this reason, strong counter-cyclical movements in the aggregate unemployment rate do not necessarily induce counter-cyclical variations in the number of posted vacancies.

Figure (2a) shows that gross worker flows between employment and unemployment are, indeed, counter-cyclical.[8] The flow of employed workers into unemployment is determined by exogenous and endogenous separation (Figure 2d).[9] The graph depicting the aggregate unemployment rate (Figure 2b) clearly demonstrates that the number of unemployed job searchers rises during economic downturns.

The flow of workers from unemployment into employment, as displayed in Figure (2a), represents all unemployed job searchers that succeed in *meeting* an employer at the end of period $t-1$. Since the reservation productivity at time t is still unknown, unemployed job searchers accept matches of any quality for the moment. Whether meeting an employer actually results in finding an active match in period t, depends on the realization of aggregate productivity. Therefore, Figure (2c) gives the worker's expected *meeting* rate, represented by a blue line, and the job *finding* rate, represented by a pink line. We observe that both transition rates are pro-cyclical. This is due to the fact that more unemployed job searchers compete for fewer vacancies during economic downturns. Moreover, the counter-cyclical flow in Figure (2a) suggests that the percentage rise in the number of unemployed job searchers is larger than the percentage fall in the worker's meeting rate.

Turning to the set of unmatched firms that have posted a vacancy (Figure 2e), we note that there are three different transition rates. The firms' *meeting* rate, represented by a blue line, gives the rate at which a firm can expect to meet a worker at the end of period $t-1$. As some employed job searchers will immediately *reject* the offer (Figure 2f), the pink line gives the rate at which a firm can expect to be matched with a worker at the beginning of period t. In addition to that, some matches will be separated in the beginning of period t, due to exogenous and endogenous job *separation*. Hence, the yellow line shows at which rate a firm can actually expect to find an active job match in period t.

[8] All graphs in Figure (2) represent stationary values, given different values of aggregate productivity. Consequently, the flow from employment to unemployment is always identical to the flow into the opposite direction.

[9] As the graphs depict stationary values, all endogenous separations at time t are due to newly formed matches in period $t-1$ that do not become active in period t. In addition to that, we observe a spike in the endogenous separation rate every time the level of aggregate productivity decreases.

Finally, note that the introduction of variable on-the-job search uncouples aggregate search effort from the number of unemployed job searchers (Figure 1d). For this reason, the ratio of vacancies to the number of unemployed job searchers is no longer identical to "labor market tightness", i.e. the ratio of vacancies to total search effort. Since on-the-job search is pro-cyclical, we observe that total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, is relatively stable over the business cycle. This stimulates firms to open more vacancies when aggregate productivity is high (Figure 2h). Consequently, we observe that the ratio of vacancies to unemployed job searchers is clearly pro-cyclical (Figure 2g). Labor market tightness, instead, is much less elastic. This indicates that the negative feedback effect of labor market tightness on vacancy creation, which prevails in the standard job matching model, is much weaker in a model with variable on-the-job search.

In summary, we notice that our model replicates salient features of the US labor market. Gross worker flows between employment and unemployment are counter-cyclical. Vacancies are pro-cyclical, while the number of unemployed job searchers is counter-cyclical. Hence, the model generated Beveridge curve is positively sloped.

3.4 Business Cycle Analysis

This section examines the quantitative performance of the job matching model with variable on-the-job search. We evaluate the model against business cycle moments of the US labor market from 1955:1 to 2008:4 (Table 2a). All data are logged and de-trended using a Hodrick & Prescott (1997) filter with smoothing parameter 1600. As a measure of aggregate activity we choose output per worker. We observe that both vacancies and unemployment are very volatile and very persistent. Vacancies are pro-cyclical, but the unemployment rate is counter-cyclical. The average real wage rate is less volatile than output per worker. In addition, we analyze whether the model is able to replicate the pattern of a relative stable *total* separation rate (Hall 2006), which is due to the offsetting behavior of its components (Nagypál 2008). In particular, the employment-to-unemployment (EU) transition rate is counter-cyclical, the employment-to-employment (EE) transition rate is pro-cyclical, and the employment to out-of-the labor force (EO) transition rate is essentially a-cyclical. Furthermore, Nagypál (2008) estimates that 50 to 60% of the volatility in the unemployment rate is due to composition changes in the total separation rate. The remaining share is caused by variations in the job finding rate.

As demonstrated by Shimer (2005), the baseline job matching model without on-the-job search is not able to replicate this pattern. The average real wage moves almost one-to-one with output per worker, providing firms not enough incentives to amplify the supply of vacancies over the business cycle. Hence, the model is not able to match the high degree of labor market volatility found in the data (Table 2b). Moreover, we note that all model generated time series closely follow the exogenous stochastic process. This result suggests that the internal propagation mechanism of the matching model without physical capital is very weak. Besides, due to the exogenous separation margin, we observe that the model generated flow from employment to unemployment is, by construction, pro-cyclical (Davis 2006). This implies that the baseline job matching model is not able to account for the composition changes in the total separation rate. Instead, all variation in the aggregate unemployment rate is attributed to the job creation margin.

Table 2c presents the second moments of our calibrated model. We observe that the inclusion of variable on-the-job search increases the volatility of vacancies and unemployment significantly. Compared to the baseline model, the volatility of unemployment rises by more than an order of a magnitude. This enormous rise is due to the counter-cyclical variations in the endogenous separation margin. Figure (3) illustrates that recessions involve a spike in the number of job separations. The unemployment rate tracks these movements almost exactly. On the other hand, the volatility of vacancies rises only modestly when variable on-the-job search is introduced. This result suggests that our model generates most of the volatility in unemployment along the job separation margin, and only little along the job creation margin.

This is a well-known problem in job matching models with endogenous separations (Mortensen & Nagypál 2008), where counter-cyclical fluctuations in the job separation margin may induce strong counter-cyclical movements in the number of unemployed job searchers. Without on-the-job search, when aggregate search effort is directly linked to the number of unemployed job searchers, these movements stimulate firms to open more vacancies when aggregate productivity is low. In other words, the model generated Beveridge curve is positively sloped. This is in stark contrast to the data. Our model, however, uncouples total search effort from the number of unemployed job searchers. Since on-the-job search is pro-cyclical, aggregate search effort is relatively stable over the business cycle. Hence, consistent with the data, we observe that firms post more vacancies when aggregate productivity is high. Yet, the degree of amplification remains below the empirical estimate.

The main reason for the low volatility of vacancies is the fact that on-the-job search involves job offer rejections (Figure 2f). Firms suffer from job offer rejections by employed job searchers who prefer to stay in their old jobs. Each job offer rejection implies that the sunk cost of labor market search is lost. This effect discourages firms to open vacancies. Nevertheless, our model is able to generate a pro-cyclical time path of vacancies. This partial success is due to the impact of two effects. First, we observe that only employed job searchers in low productivity matches make great efforts to find a new employment opportunity. These workers try to "escape" imminent unemployment and, therefore, tend to accept most of the job offers. Second, conditional on job offer acceptance, firms enjoy a higher expected payoff from an employed job searcher than from an unemployed. This is due to the fact that employed job searchers accept only attractive job offers and, therefore, are unlikely to quit later on. Hence, their expected match duration is longer (Nagypál 2007).

Figure (4) presents the cyclical behavior of the *total* job separation rate and its components. The total job separation rate, represented by a yellow line, is made up of (i) the sum of the exogenous and the endogenous separation rate, represented by a pink line, and (ii) the employment-to-employment transition rate, represented by a blue line. The graph shows that the two main components of the total job separation rate are negatively correlated. The employment-to-employment transition rate is pro-cyclical, while the sum of the exogenous and the endogenous separation rate is counter-cyclical. The total job separation rate clearly exhibits less variability than its two main components.

Finally, we recall that our model is able to distinguish between the average real wage rate the real wage rate of an individual worker.[10] The average real wage rate is clearly less volatile than the real wage rate of an individual worker. The co-movement with output, however, is almost the same between both variables.

4 Conclusion

This paper addresses the large degree of frictional wage dispersion found in US data. Hornstein et al. (2007) demonstrate that the average wage paid to observably identical workers is about 70% higher than the lowest wage in the sample. The standard job matching model without on-the-job search cannot replicate this pattern. With on-the-job search, however, unemployed job searchers are more willing to accept low wage offers since they can continue to seek for better employment opportunities. This explains why observably identical workers may be paid very different wages. Therefore, we examine the quantitative implications of variable on-the-job search (Nagypál 2006) in a stochastic job matching model (Mortensen & Pissarides 1994).

Our key result is that the inclusion of *variable* on-the-job search increases the degree of frictional wage dispersion by an order of a magnitude (from about 3% to 40%). Variable on-the-job search allows us to replicate the fact that search effort is most intense among workers in low-paid matches close to the separation margin (Fallick & Fleischman 2004, Christensen et al. 2005). These "marginal" workers try to escape imminent unemployment and tend to accept most of the job offers (Nagypál 2007). Hence, marginal workers enjoy very high employment-to-employment transition rates. These high career expectations stimulate unemployed job searchers to accept such low-paid matches. For this reason, we observe that the average wage paid to ex-ante identical workers in our model is about 40% higher than the lowest wage in the sample.

[10]The analyzed individual real wage corresponds to an employed worker with idiosyncratic productivity level $a = 1.1$ who stays permanently on the same job.

Furthermore, we evaluate our modified job matching model at the business cycle frequencies. We observe that counter-cyclical variations in the endogenous job separation rate amplify the cyclical variations in the aggregate unemployment rate. Indeed, our model uncouples aggregate search effort from the number of unemployed job searchers. Since on-the-job search is pro-cyclical, we note that total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, is relatively stable over the business cycle. This stimulates firms to open more vacancies when aggregate productivity is high. As a consequence, we are able to replicate a negatively sloped Beveridge curve. This is an interesting result, given that models with endogenous separations, but without on-the-job search, imply that aggregate search effort is strongly counter-cyclical (Mortensen & Nagypál 2008). Thus, in stark contrast to the data, firms are likely to post more vacancies when the number of unemployed job searchers is high.

It would be interesting to extend our analysis to a more general wage bargaining set up. To our knowledge, there is no study investigating all aspects of a multi-player bargaining game under variable on-the-job search. Cahuc et al. (2006) examine the implications of a multi-player bargaining game, where the search intensity of employed job searchers is constant (Burdett & Mortensen 1998). They demonstrate that between-firm competition increases the average wage rate if employed job searchers can resume their old positions. Papp (2009) examines the implications of this approach in a general equilibrium model. On the other hand, Shimer (2006) analyzes a strategic bargaining game between an employed job searcher and a single firm, where the firm is willing to offer a higher wage rate in order to reduce the extent of transitions to other employers.

References

Angerhausen, J., Bayer, C. & Hehenkamp, B. (2010), 'Strategic unemployment', *Journal of Institutional and Theoretical Economics* . Forthcoming.

Barlevy, G. (2002), 'The sullying effect of recessions', *Review of Economic Studies* **69**(1), 65–96.

Bowlus, A., Liu, H. & Robinson, C. (2002), 'Business cycle models, aggregation, and real wage cyclicality', *Journal of Labor Economics* **20**(2), 308–335.

Burdett, K. & Mortensen, D. T. (1998), 'Wage differentials, employer size, and unemployment', *International Economic Review* **39**(2), 257–73.

Caballero, R. J. & Hammour, M. L. (1994), 'The cleansing effect of recessions', *American Economic Review* **84**(5), 1350–1368.

Cahuc, P., Postel-Vinay, F. & Robin, J.-M. (2006), 'Wage bargaining with on-the-job search: Theory and evidence', *Econometrica* **74**(2), 323–364.

Christensen, B. J., Lentz, R., Mortensen, D. T., Neumann, G. R. & Werwatz, A. (2005), 'On-the-job search and the wage distribution', *Journal of Labor Economics* **23**(1), 31–58.

Costain, J. S. & Reiter, M. (2008), 'Business cycles, unemployment insurance, and the calibration of matching models', *Journal of Economic Dynamics and Control* **32**(4), 1120–1155.

Davis, S. J. (2006), Comments on 'Job loss, job finding, and unemployment in the U.S. economy over the past fifty years' by Robert E. Hall, *in* M. Gertler & K. Rogoff, eds, 'NBER Macroeconmics Annual 2005', MIT Press, Cambridge, Massachusetts, pp. 139–157.

Elsby, M. W. L., Michaels, R. & Solon, G. (2009), 'The ins and outs of cyclical unemployment', *American Economic Journal: Macroeconomics* **1**(1), 84–110.

Fahr, S. (2007), Essays on innovations, technology, and the labor market, PhD thesis, European University Institute. Chapter 1: On-the-job search and v-u volatility.

Fallick, B. & Fleischman, C. A. (2004), Employer-to-employer flows in the U.S. labor market: The complete picture of gross worker flows, Finance and Economics Discussion Series No. 2004-34, Board of Governors of the Federal Reserve System (U.S.).

Fujita, S. (2009), 'Dynamics of worker flows and vacancies: Evidence from the sign restriction approach', *Journal of Applied Econometrics* . Forthcoming.

Hagedorn, M. & Manovskii, I. (2008), 'The cyclical behavior of equilibrium unemployment and vacancies revisited', *American Economic Review* **98**(4), 1692–1706.

Hall, R. E. (2006), Job loss, job finding, and unemployment in the U.S. economy over the past fifty years, *in* M. Gertler & K. Rogoff, eds, 'NBER Macroeconmics Annual 2005', Vol. 20, MIT Press, Cambridge, Massachusetts, pp. 101–137.

Hall, R. E. (2009), 'Reconciling cyclical movements in the marginal value of time and the marginal product of labor', *Journal of Political Economy* **117**(2), 281–323.

Hart, R. A. (2006), 'Worker-job matches, job mobility and real wage cyclicality', *Economica* **73**(290), 287–298.

Hodrick, R. J. & Prescott, E. C. (1997), 'Postwar U.S. business cycles: An empirical investigation', *Journal of Money, Credit and Banking* **29**(1), 1–16.

Hornstein, A., Krusell, P. & Violante, G. L. (2007), Frictional wage dispersion in search models: A quantitative assessment, NBER Working Paper Series No. 13674, National Bureau of Economic Research.

Jones, S. R. G. & Riddell, W. C. (1999), 'The measurement of unemployment: An empirical approach', *Econometrica* **67**(1), 147–162.

Krause, M. U. & Lubik, T. A. (2007), On-the-job search and the cyclical dynamics of the labor market, Working Paper Series No. 779, European Central Bank.

Krusell, P. & Smith, A. A. (1998), 'Income and wealth heterogeneity in the macroeconomy', *Journal of Political Economy* **106**(5), 867–896.

Kydland, F. E. & Prescott, E. C. (1982), 'Time to build and aggregate fluctuations', *Econometrica* **50**(6), 1345–1370.

Menzio, G. & Shi, S. (2010), 'Block recursive equilibria for stochastic models of search on the job', *Journal of Economic Theory* **145**(4), 1453–1494.

Moen, E. R. (1997), 'Competitive search equilibrium', *Journal of Political Economy* **105**(2), 385–411.

Mortensen, D. & Nagypál, É. (2008), 'Labor-market volatility in matching models with endogenous separations', *Scandinavian Journal of Economics* **109**(4), 645–665.

Mortensen, D. T. & Pissarides, C. A. (1994), 'Job creation and job destruction in the theory of unemployment', *Review of Economic Studies* **61**(3), 397–415.

Nagypál, É. (2006), On the extent of job-to-job transitions, 2006 Meeting Papers 10, Society for Economic Dynamics.

Nagypál, É. (2007), Labor-market fluctuations and on-the-job search, Unpublished manuscript, Northwestern University.

Nagypál, É. (2008), Worker reallocation over the business cycle: The importance ofemployer-to employer transitions, Unpublished manuscript, Northwestern University.

Nash, J. (1953), 'Two-person cooperative games', *Econometrica* **21**, 128–140.

Papp, T. K. (2009), Explaining frictional wage dispersion, Unpublished manuscript, Princeton University, Department of Economics.

Petrongolo, B. & Pissarides, C. A. (2001), 'Looking into the black box: A survey of the matching function', *Journal of Economic Literature* **39**(2), 390–431.

Pissarides, C. A. (1985), 'Short-run equilibrium dynamics of unemployment, vacancies and real wages', *American Economic Review* **75**(4), 676–690.

Pries, M. & Rogerson, R. (2005), 'Hiring policies, labor market institutions, and labor market flows', *Journal of Political Economy* **113**(4), 811–839.

Shimer, R. (2005), 'The cyclical behavior of equilibrium unemployment and vacancies', *American Economic Review* **95**(1), 25–49.

Shimer, R. (2006), 'On-the-job search and strategic bargaining', *European Economic Review* **50**(4), 811–830.

Shimer, R. (2007), Reassessing the ins and outs of unemployment, NBER Working Paper Series No. 13421, National Bureau of Economic Research.

Solon, G., Barsky, R. & Parker, J. A. (1994), 'Measuring the cyclicality of real wages: How important is composition bias?', *The Quarterly Journal of Economics* **109**(1), 1–25.

Tasci, M. (2007), On-the-job search and labor market reallocation, Working Paper No. 0725, Federal Reserve Bank of Cleveland.

Tauchen, G. (1986), 'Finite state Markov-Chain approximations to univariate and vector autoregressions', *Economics Letters* **20**(2), 177–181.

Yashiv, E. (2000), 'The determinants of equilibrium unemployment', *American Economic Review* **90**(5), 1297–1322.

Yashiv, E. (2008), 'U.S. labor market dynamics revisited', *Scandinavian Journal of Economics* **109**(4), 779–806.

A Tables

Parameter	Description	Value	Source
U	unemployment rate	0.10	Jones & Riddell (1999)
$\tilde{f}$	job finding rate	0.27	Hall (2006)
ρ^x	*EU* transition rate	0.01	Hall (2006)
ρ^n	*EO* transition rate	0.03	Hall (2006)
τ	*EE* transition rate	0.04	Hall (2006)
b	unemployment benefits	0.85	Costain & Reiter (2008)
ξ	firm's bargaining power	0.25	Yashiv (2000)
κ	vacancy posting cost	0.06	Yashiv (2000)
χ	matching function constant	0.45	normalization
γ	matching elasticity of vacancies	0.50	Petrongolo & Pissarides (2001)
Z	number of productivity states	29	
m	grid width	3	Tauchen (1986)
ϱ	1st order autocorrelation	0.97	Hagedorn & Manovskii (2008)[11]
σ_ϵ	standard deviation	0.007	Hagedorn & Manovskii (2008)

Table 1: Calibrated Parameter Values

	Y	V	U	V/U	W
$\sigma(X)$	0.01	0.13	0.11	0.24	0.01
$\sigma(X)/\sigma(Y/N)$	1	14.1	12.0	25.5	0.72
$\rho(X, Y/N)$	1	0.54	-0.42	0.49	0.38
$\rho(X_t, X_{t-1})$	0.71	0.90	0.89	0.90	0.70

(a) US Business Cycle Facts

	Y	V	U	V/U	W
$\sigma(X)$	0.02	0.02	0.01	0.03	0.02
$\sigma(X)/\sigma(Y/N)$	1	1.2	0.4	1.6	1
$\rho(X, Y/N)$	1	1	-0.95	1	0.84
$\rho(X_t, X_{t-1})$	0.78	0.75	0.80	0.78	0.70

(b) Second Moments without On-the-job Search

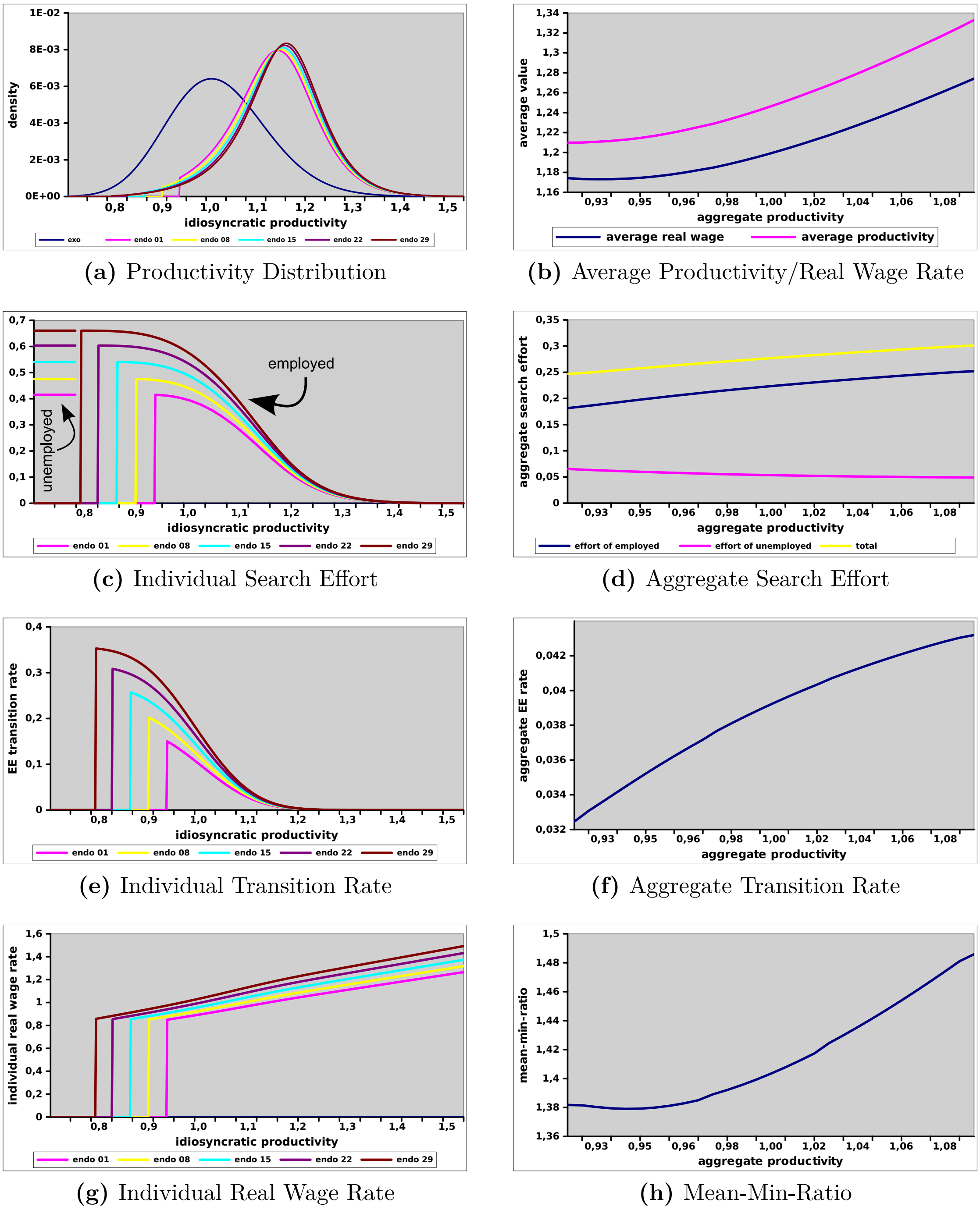
	Y	V	U	V/U	$\overline{W}$	W_i
$\sigma(X)$	0.01	0.04	0.06	0.11	0.01	0.01
$\sigma(X)/\sigma(Y/N)$	1	2.9	4.4	7.3	0.54	0.92
$\rho(X, Y/N)$	1	1	-0.99	0.99	0.97	1
$\rho(X_t, X_{t-1})$	0.77	0.77	0.76	0.77	0.76	0.77

(c) Second Moments with On-the-job Search

Table 2: US Business Cycle Facts and the Corresponding Moments of our Model Economy. All Data are logged and de-trended with an HP-Filter 1600. All US data (but the real wage) are taken directly from OECD.Stat. The real wage is constructed using the time series "Compensation of Employees: Wages and Salary Accruals" and "Total Nonfarm Payrolls: All Employees".

[11]See Footnote (7)

B Figures



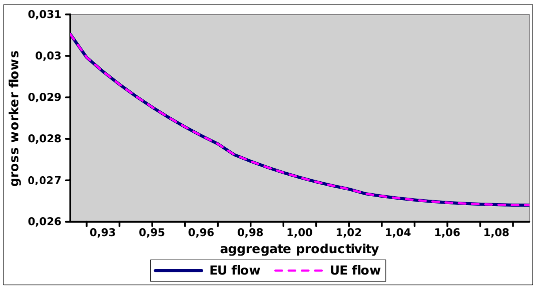
(a) Productivity Distribution **(b)** Average Productivity/Real Wage Rate

(c) Individual Search Effort **(d)** Aggregate Search Effort

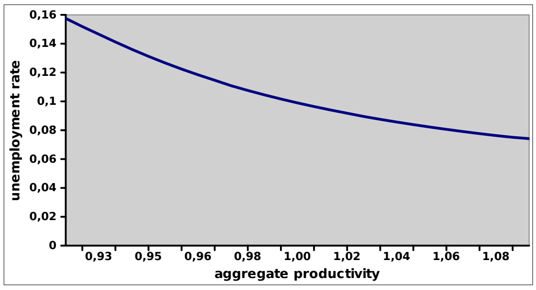
(e) Individual Transition Rate **(f)** Aggregate Transition Rate

(g) Individual Real Wage Rate **(h)** Mean-Min-Ratio

Figure 1: Endogenous Distribution, the Real Wage Rate, and On-the-job Search

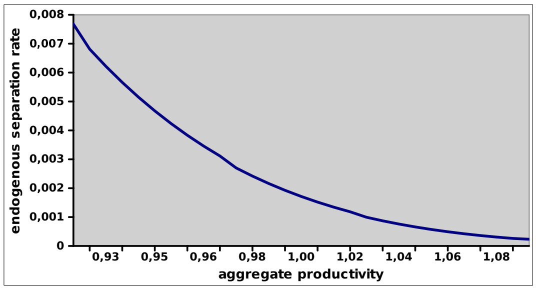


(a) Gross Flows

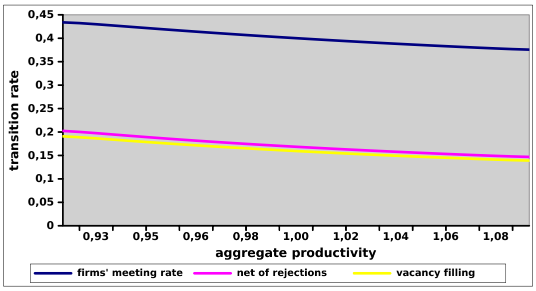


(b) Unemployment

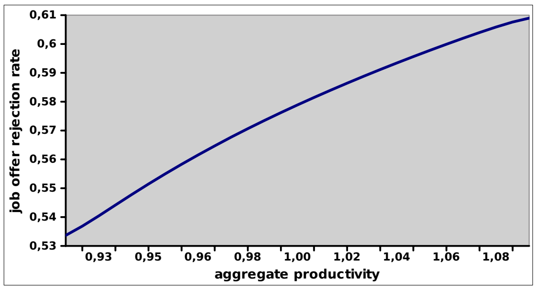
(c) Job Finding Rate



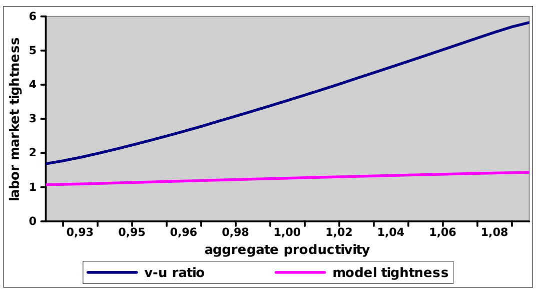
(d) Endogenous Separation Rate



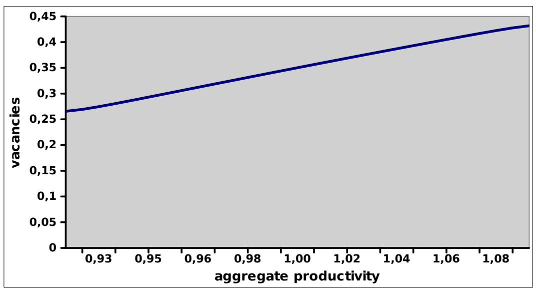
(e) Vacancy Filling Rate



(f) Aggregate Rejection Rate



(g) Labor Market Tightness



(h) Vacancies

Figure 2: Aggregate Labor Market Dynamics

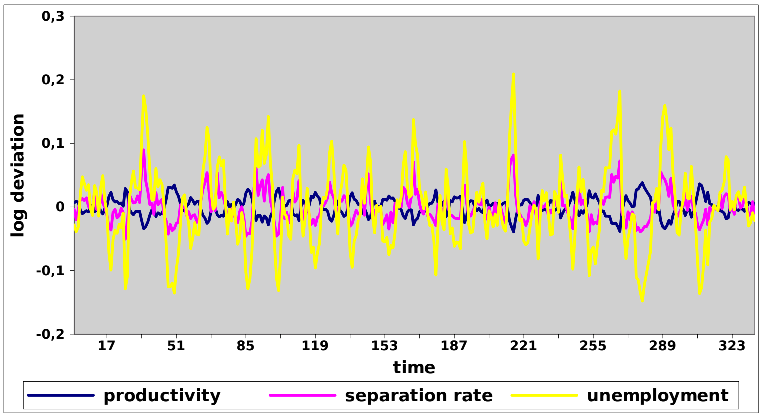


Figure 3: Counter-cyclical Spikes in the Separation Rate

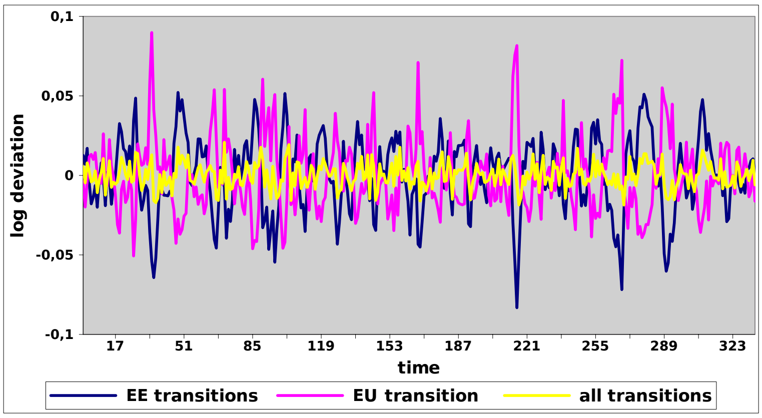


Figure 4: Composition Changes in the Total Separation Rate