1 Projection Bias

Let utility of an individual today in t be given by $u(c_t, s_t)$, where c_t is consumption and s_t is her current state. The individual's prediction for her future utility $u(c_\tau, s_\tau)$ with $\tau > t$ will be denoted by $\tilde{u}(c_\tau, s_\tau | s_t)$, which indicates that the prediction for some future point τ is conditional on the current state s_t. Thus an accurate prediction is defined as one where $\tilde{u}(c_\tau, s_\tau | s_t) = u(c_\tau, s_\tau)$.

Now, we say that the individual's predicted utility exhibits projection bias if there exists an $\alpha \in]0, 1]$ such that

$$\tilde{u}(c_\tau, s_\tau | s_t) = (1 - \alpha) u(c_\tau, s_\tau) + \alpha u(c_\tau, s_t).$$

All individuals who exhibit such a projection bias understand the qualitative nature of changes in the state, but underestimate the magnitude of these changes. Clearly, predicted utility lies in between future utility given the true state and future utility given the current state.

1. Let an individual have the following intertemporal utility function

$$\tilde{U}_t(c_t, \ldots, c_T | s_t) = \sum_{\tau=t}^{T} \delta^{\tau - t} \tilde{u}(c_\tau, s_\tau | s_t). \tag{1}$$

   Give an interpretation.

2. Let the state evolve according to $s_{t+1} = (1 - \gamma) s_t + \gamma c_t$. Now, maximize the individual's utility function (1) subject to the evolution of the state.

3. Write down the relationship between marginal utility on day t and $t + 1$ for the following three cases:

   (a) An individual without projection bias

   (b) A person with projection bias, i.e. $\alpha \in (0, 1)$

   (c) Somebody with the strongest possible projection bias, that is an individual who perceives that her future tastes will be identical to her current tastes.

2 Optimal saving with uncertain labour income I

Let wealth a of a household evolve according to

$$da = \{ra + z - c\}\, dt.$$

Wealth increases per unit of time dt by the amount da which depends on current savings $ra + z - c$. The interest rate is deterministic. Labour income is denoted by z which includes income w when employed and unemployment benefits b when unemployed, $z = w, b$. Labour income follows a stochastic Poisson differential equation as there is job creation and job destruction,

$$dz = \Delta dq_b - \Delta dq_w,$$

where $\Delta \equiv w - b$. Job destruction takes place at an exogenous state-dependent arrival rate $s(z)$. The corresponding Poisson process counts how often our household moved from employment into unemployment is q_w. Job creation takes place at an exogenous rate $\lambda(z)$. The Poisson process related to the matching process is denoted by q_b. It counts how often a household leaves her "b-status", i.e. how often she found a job. As an individual cannot loose her job when she does not have one and as finding a job makes (in this setup) no sense for someone who has a job, both arrival rates are state dependent. As an example, when an individual is employed, $\lambda(w) = 0$, when she is unemployed, $s(b) = 0$.

z	w	b
$\lambda(z)$	0	λ
$s(z)$	s	0

Table State dependent arrival rates

Let the individual maximize expected utility $E_t \int_t^\infty e^{-\rho[\tau - t]} u(c(\tau))\, d\tau$, where instantaneous utility is of the CES type, $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$ subject to the budget constraint and the labour income constraint above. It is well-known that the Bellman equation for this problem looks like

$$\rho V(a, z) = \max_c \left\{ \begin{array}{c} u(c) + V_a [ra + z - c] + V_z g z \\ +s(z)[V(a, z - \Delta) - V(a, z)] + \lambda(z)[V(a, z + \Delta) - V(a, z)] \end{array} \right\}.$$

1. Compute the first-order condition for both states of z.
2. Compute the evolution of the shadow price of wealth.
3. Show that under optimal behaviour marginal utility in the state of employment, i.e. $z = w$, follows

$$\begin{aligned} du'(c(a_w, w)) = {} & \{(\rho - r) u'(c(a_w, w)) - s[u'(c(a_w, b)) - u'(c(a_w, w))]\}\, dt \\ & + [u'(c(a_w, b)) - u'(c(a_w, w))]\, dq_s. \end{aligned}$$

4. Compute the Keynes-Ramsey rule for optimal consumption. To this end, let $f(.)$ be the inverse function for u', i.e. $f(u') = c$.

   (a) Show that $f'(u'(c(a_w, w))) = \frac{1}{u''(c(a_w,w))}$.

   (b) Apply the appropriate change-of-variable-formula to $f(u'(c(a_w, w)))$.

3 Optimal saving with uncertain labour income II

Consider a household that maximizes utility. The objective function is given by

$$U_t = E_t \Sigma_{\tau=t}^{\infty} \beta^{\tau-t} u\left(c_\tau\right).$$

It is maximized subject to a budget constraint

$$a_{t+1} = \left(1 + r_t\right) a_t + w_t - p_t c_t.$$

Assume that the price p_t and the interest rate r_t are non-stochastic. Let the wage follow a stochastic process described by

$$w_{t+1} = \gamma w_t + \varepsilon_{t+1},$$

where $\gamma < 1$ is a positive constant and ε_{t+1} in normally distributed with mean w_0 and variance σ^2.

1. What is the expected labour income for the long run?
2. Compute the Euler equation for optimal consumption.