Part I Modelling Money in General Equilibrium: a Primer Lecture 3 Welfare Cost of Inflation in the Basic MIU model

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- \rightarrow Some features of fiat money regimes:
 - private opportunity cost of holding real balances (m) depends on the level of the short-term nominal interest rate (i)
 - social opportunity cost of providing money is essentially zero, ie the central bank can make the amount of money in circulation arbitrarily large (and thereby depress *i*)
 - the **wedge** between the private and social cost at positive interest rates creates an **inefficiency**
 - moreover, in equilibrium, the **nominal interest rate**, the **inflation rate** and the **quantity of money** in circulation are linked to each other

 \rightarrow These features give rise to **two** types of (closely linked) **policy questions** which have been addressed in monetary economics for centuries:

From a positive perspective: How large is the welfare cost of inflation?
 From a normative perspective: What is the optimal rate of inflation?

 \rightarrow The **MIU-model** offers a transparent and tractable **general equilibrium perspective**, ie it is a widely used starting point to address these questions, qualitatively and quantitatively (in particular, see *Lucas, 2000*):

- \rightarrow Why is the MIU model a good model to analyze these questions? Because:
 - The costs of inflation (via the interest rate) are clearly captured by the expression

$$u_m(c_t, m_t) = \frac{i_t}{1 + i_t} u_c(c_t, m_t)$$
(1)

- Steady-state superneutrality implies that we don't have to worry about the real side of the economy when comparing steady states with different inflation and interest rates
- The flow objective u(c, m) offers a simple and direct metric for welfare comparisons

Question 2: What is the optimal rate of inflation?

 \rightarrow In the basic MIU model the answer to this question is rather straightforward, since monetary policy instruments (*i* or θ) do not affect the steady-state value of *c*, only of *m*

 \rightarrow Money growth rule (via θ) : Optimality requires $\frac{\partial u(c,m)}{\partial \theta} = \frac{\partial u(c,m)}{\partial m} \frac{\partial m}{\partial \theta} = 0$, or $\frac{\partial u(c,m)}{\partial m} = 0$, implying in eqn (1)

 \rightarrow This is the **Friedman rule** which implies (using $1 + i \approx 1 + r + \pi$)

$$\pi \approx -r$$
,

i.e. the (long-run) optimal inflation rate is a rate of deflation approximately equal to the return on capital and government bonds (*Friedman, 1969*)

Friedman rule: preliminary comments

Comment 1: Satiation vs. approximate validity of Friedman rule

- Depending on the assumed utility function the strict equality $\frac{\partial u(c,m)}{\partial m} = 0$ with i = 0 obtains if there exists a finite satiation level (as in (A 1))
- In other cases (like the discussed case of log-utility) *m* should be made arbitrarily large such that *i* → 0 (approximate validity of the Friedman rule)

Comment 2: Implementation

• The result is equally valid if monetary policy is directly implemented via an interest rate rule (rather than a money growth rule)

Friedman rule: an important qualification

- This reasoning is not yet a basis for policy advice, since the interest rate *i* (and indirectly π) is the only distortionary policy instrument
- Recall from the model set-up: government has access to lump-sum transfers/taxes τ, making distortionary seigniorage revenues 'costly'
- The discussion about the optimality of the Friedman rule becomes only meaningful when we introduce **distortionary taxes**
- Then, as argued by *Phelps (1973)*, the Friedman rule may well break down

 \rightarrow We will return to **Question** 2 in detail in Part II which covers the Friedman vs. Phelps debate, assuming distortionary taxation

Let us return to **Question 1:** How large is the welfare cost of inflation?

 \rightarrow Lucas (2000) uses the MIU model to provide quantitative guidance for the likely range of welfare costs of inflation

 \rightarrow The methodology proposed by Lucas has been influential since it exploits the general equilibrium dimension of the MIU model, as opposed to partial equilibrium estimates that were traditionally used (*Bailey 1956*)

To reproduce the findings from Lucas, we proceed in 2 steps:

Step I): Two types of money demand specifications and discussion of the empirical evidenceStep II): Partial vs. general equilibrium measures of welfare costs of inflation

 \rightarrow Any welfare measure of the costs of inflation depends critically on the form of the money demand equation and the sensitivity of money demand to the opportunity cost of holding real balances

 \to In the basic MIU model these issues are directly related to the specification of the utility function which drives the shape of

$$u_m(c_t, m_t) = \frac{i_t}{1+i_t} u_c(c_t, m_t)$$

 \rightarrow The **empirical literature** commonly distinguishes between **log-log** specifications as opposed to **semi-log specifications** of **money demand**

CES utility function between c and m:

 \rightarrow Consider the following **CES utility function** which displays a constant elasticity of substitution between consumption and real balances:

$$u(c_t, m_t) = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}} \text{ with: } 0 < a < 1, b > 0 \text{ (and } b \neq 1)$$
(2)

 \rightarrow This function satisfies

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = (\frac{1-a}{a})(\frac{c_t}{m_t})^b$$

and, when combined with the first-order optimality condition

$$u_m(c_t, m_t) = \frac{i_t}{1+i_t} u_c(c_t, m_t),$$

it gives rise to the money-demand function

$$m_t = \left(\frac{1-a}{a}\right)^{\frac{1}{b}} \cdot \left(\frac{i_t}{1+i_t}\right)^{-\frac{1}{b}} \cdot c_t \tag{3}$$

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Log-log specification:

 \rightarrow In the empirical literature it is common to rewrite equations of type (3), ie

$$m_t = (\frac{1-a}{a})^{\frac{1}{b}} \cdot (\frac{i_t}{1+i_t})^{-\frac{1}{b}} \cdot c_t$$

by taking logs, leading to

$$\log(m) = \frac{1}{b}\log(\frac{1-a}{a}) + \log(c) - \frac{1}{b}\log(\frac{i}{1+i})$$
(4)

Features of eqn (4):

- Demand for *m* depends positively on *c* and negatively on *i*
- Elasticity of money demand w.r.t. consumption (η_{m,c} = dm/dc) is 1 → often c is replaced by y, yielding an income elasticity of 1
- Elasticity of money demand w.r.t. the opportunity cost variable $\frac{i}{1+i}$ (ie:

$$\eta_{m, \frac{i}{1+i}} = -\frac{dm}{m} / \frac{d\frac{i}{1+i}}{\frac{i}{1+i}}$$
 is $\frac{1}{b}$

 \rightarrow for simplicity $\eta_{m, i/1+i}$ is often referred to as the **interest elasticity of money demand** (and, in any case, for small $i : \frac{i}{1+i} \approx i$)

 \rightarrow since on the RHS of eqn (4) we consider the log of $\frac{i}{1+i}$ (and *not* the level of $\frac{i}{1+i}$) it is called a **log-log specification of money demand** \rightarrow

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Log-log specification:

 \rightarrow **Special case of** $b \rightarrow 1$ in eqn (2):

$$u(c_t, m_t) = c_t^a \cdot m_t^{1-a}$$
(5)

Features of (5):

• The money demand equation (3) becomes

$$m_t = (\frac{1-a}{a}) \cdot (\frac{i_t}{1+i_t})^{-1} \cdot c_t$$

• The consumption and the interest elasticity of money demand are both equal to 1

Semi-log specification:

 \rightarrow In the empirical literature commonly studied alternatives to (4) are semi-log specifications of money demand

 \rightarrow Semi-log money demand equations regress the log of m_t on the level of i_t (or some alternative measure of the opportunity cost of real balances like $\frac{i_t}{1+i_t}$)

Illustration:

• With a semi-log specification, eqn (4), ceteris paribus, turns into

$$\log(m) = \tilde{a} + \log(c) - \xi \frac{i}{1+i},$$
(6)

- The coefficient ξ in front of $\frac{i}{1+i}$ denotes the **semielasticity of money** demand w.r.t. $\frac{i}{1+i}$, ie $\xi = -\frac{dm}{m}/d\frac{i}{1+i}$
- Hence, the elasticity and the semi-elasticity are linked via the relationship:

$$\xi = -\frac{dm}{m} \frac{\frac{i}{1+i}}{d\frac{i}{1+i}} \frac{1}{\frac{i}{1+i}} \Leftrightarrow \eta_{m, \frac{i}{1+i}} = \xi \cdot \frac{i}{1+i}$$

Empirical evidence:

Lucas (2000):

 \rightarrow How to account for US annual time series of short-term nominal interest rates and the 'money-income ratio' (ie the ratio of M1 to nominal GDP: $\frac{M}{Py}$) over the 95 year period 1900-1994?

 \rightarrow Can we explain the relationship between the two series by log-log or semi-log specifications of money demand?

 \rightarrow Figures 1-4 in Lucas (2000) entail 3 stylized empirical findings:

Finding 1 (see Figure 1 from Lucas, 2000):

- Over the 95 year period, US real GDP grew at an average rate of 3%, M1 at 5.6% and the GDP deflator at 3.2%
- This makes the 'money-income ratio' (^M/_{Py}) essentially trendless over the entire period (although there has been a significant decline since World War II)
- A value of the income elasticity of money demand larger than unity (ie $\eta_{m,\;y}>1)$ would have produced an upward trend
- \rightarrow Stationarity of the money-income ratio not to be rejected

 \rightarrow Assumed money demand functions $m^d(i,y)$ are of type $m^d=m(i)\cdot y$, satisfying a **unit income elasticity of money demand** $(\eta_{m,v}=1)$

Finding 2 (see Figures 2 and 3 from Lucas, 2000):

- Figures 2 and 3 plot observations for the money-income ratio m/y and the short-term nominal interest rate *i*
- To account for this relationship compare predictions from the log-log specification (using the η-values 0.3, 0.5, 0.7)

$$\frac{m}{y} = A \cdot i^{-\eta} \iff \log(\frac{m}{y}) = \log(A) - \eta \log(i)$$

and the **semi-log specification** (using the ξ -values 5, 7, 9)

$$\frac{m}{y} = \widetilde{A} \cdot e^{-\xi i} \quad \Leftrightarrow \quad \log(\frac{m}{y}) = \log(\widetilde{A}) - \xi i,$$

where A and \widetilde{A} , respectively, are fitted such that the curves pass through the geometric means of the data pairs

- \rightarrow Log-log curves give a better fit than semi-log curves
- ightarrow Within the class of log-log curves $\eta=0.5$ gives the best fit

Finding 3 (see Figures 1 and 4 from Lucas, 2000):

 Figure 4 plots observed levels of real balances (ie m) against the real balances predicted by the estimated log-log demand curve (with η = 0.5)

$$m_t = A \cdot i_t^{-0.5} \cdot y_t$$

 \rightarrow success:

- Fitted values successfully track secular increase in *m/y* prior to World War II (in a period characterized by declining nominal interest rates)
- Fitted values track well the decline in *m*/*y* after World War II until 1980s (in a period characterized by a secular rise in *i*)
- \rightarrow but:
 - Fitted values become poor since the mid 1980s (a period of low *i* and significant financial deregulation)
 - Elasticities needed to fit long-run trends do *not* permit a good fit on a year-to-year or even quarterly basis

Related empirical literature:

- Lucas' preferred estimate from a log-log specification of η = 0.5, implying a value b = 2 in eqn (4), is broadly in line with related studies:
 → Chari, Kehoe and McGrattan (2000) report a US value of b = 2.6
 → Hoffmann, Rasche and Tieslau (1995) report cross-country evidence and find similar estimates for the US and Canada, a somewhat higher value for the UK, and lower values for Germany and Japan
- The failure to obtain a good short-run fit via estimated long-run elasticities fitting secular trends has led to a vast research agenda on money demand specifications, with the aim to reconcile evidence at different frequencies

 \rightarrow For example, see the **distributed lag specifications** summarized by Walsh (Table 2.1), allowing, by construction, for **differences between short-run** and **long-run elasticities**

 \rightarrow Estimations of more **flexible money demand systems**, allowing for additional variables (various assets and interest rates etc.)

Related empirical literature:

- The mentioned problems to account for stable money demand relationships since the 1980s have been explained through various channels, like
 - \rightarrow Changes in financial regulation (which have increased the range of money substitutes available for transactions at low cost) and associated portfolio shifts
 - \rightarrow Non-linearities in money demand equations (suggesting that in environments of low i and π interest elasticities of money demand tend to be lower)
- Ireland (2009) argues that for post-1980 US-data the fit of semi-log specifications improves and outperforms log-log specifications:
 → Findings of this type have implications for estimates of the welfare cost of inflation, as to be discussed next

Partial equilibrium estimates of welfare costs of inflation:

Bailey (1956):

- inflation taxes real balances
- focus: fully anticipated inflation
- welfare effects of inflation to be assessed similar to the effects of any other tax

 \rightarrow when comparing welfare implications of two different levels of i, the natural welfare measure is the area under the (inverse) money demand curve, ie the consumer surplus

 \rightarrow ceteris paribus, this amounts to a **partial equilibrium** perspective

Lucas (2000) reproduces this approach, but suggests to consider instead of the demand for real balances (ie $m = \frac{M}{P}$) the 'money income ratio' (ie $\frac{m}{y} \equiv \tilde{m}$) \rightarrow idea: express area under the money demand curve as a fraction of income \rightarrow since his estimates use $m^d = m(i) \cdot y$ this transformation is legitimate \rightarrow in terms of dimension, this is a more satisfactory measure of how to compensate people to be indifferent between different steady states.

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III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

- \rightarrow Welfare costs of inflation in steady states comparison
- \rightarrow Consider two pairs of observations s.t. 0 < i_0 < i_1 with $\widetilde{m}_0 > \widetilde{m}_1$

Here: Figure I (Welfare costs of inflation from a partial equilibrium perspective)

Initial steady state: i_0 , \widetilde{m}_0

- Area *Ai*₀*C*: consumer surplus
- Area $A\tilde{m}_0 0i_0$: surplus extracted by gov't (since it saves $\approx i_0 \tilde{m}_0$ resources relative to raising them via issuance of gov't bonds at nominal rate i_0)

New steady state: i_1 , \widetilde{m}_1

- Area *Bi*₁*C*: consumer surplus
- Area $B\tilde{m}_10i_1$: surplus extracted by gov't (since it saves $\approx i_1\tilde{m}_1$ resources)

Comparison between initial and new steady state:

• Area $A\widetilde{m}_0\widetilde{m}_1B$: Welfare loss (ie the difference between the total surpluses) is the shaded area under the inverted money demand curve (in terms of \widetilde{m}), ie the combined net loss of surpluses extracted by the gov't and consumers when moving from i_0 up to i_1

Partial equilibrium estimates of welfare costs of inflation:

Finding 4 (see Figures 5 and 6 from Lucas, 2000):

Lucas (2000) quantifies the partial equilibrium welfare gains from reducing the nominal interest rate from $i_1 > 0$ to $i_0 = 0$ by evaluating the consumer surplus expression

$$\int_0^{i_1} \widetilde{m}(x) dx - i_1 \widetilde{m}(i_1)$$

for his money demand estimates

 \rightarrow For the benchmark log-log specification with $\eta=0.5$ a permanent reduction of short-term nominal interest rates from 10% to 0% leads to a welfare gain of $\approx 1.6\%$ of annual US real GDP

 \rightarrow Assuming that for the US over the 95 year horizon a nominal interest rate of 3% on average leads approximately to price stability, a **permanent reduction** of short-term nominal interest rates from 3% to 0% still leads to a sizeable welfare gain of $\approx 0.9\%$ of annual US real GDP

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III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

Comments:

- For the log-log specification $\lim_{i \to 0} \widetilde{m}(i) \to \infty$
- This makes welfare gains at very low levels of *i* relatively 'large'

 \rightarrow Gains like 1.6% or 0.9% of US real GDP may seem small, but they would be available every year, ie the discounted **present value** of them is **substantial** \rightarrow However, these gains need to be compared with **costs of disinflation** (see Ball, 1993). Such costs typically arise because of (short-run) nominal rigidities and unexpected disinflation surprises

 \rightarrow Such features are not addressed in the flex-price rational expectations approach considered so far

Partial equilibrium estimates of welfare costs of inflation:

Comments:

- For the semi-log specification, $\tilde{m}(i = 0)$ is a finite number (ie \tilde{A})
- This makes welfare gains at very low levels of *i* relatively 'small'

 \rightarrow What is meant by **'large' vs. 'small'?** The numbers can be computed from the expressions (using $A=0.05,~\widetilde{A}=0.35)$:

 \rightarrow **Upshot:** Under the semi-log specification virtually all gains are realised if one stops at i = 3%, while under the log-log specification there remain sizeable gains if one goes all the way to i = 0%

Partial equilibrium estimates of welfare costs of inflation:

Lucas (2000): Criticism

 \rightarrow Estimated money demand specifications of type (4) or (6) can well be satisfactory in isolation, but they lack a general equilibrium perspective

 \rightarrow To use such estimates for welfare comparisons, in general, can be misleading since policy changes affect all equilibrium relationships (while the partial equilibrium approach going back to Bailey relies on the ceteris paribus assumption)

General equilibrium estimates of welfare costs of inflation:

Lucas (2000): To address this conceptual challenge, consider as an alternative the following general equilibrium strategy:

 \rightarrow Start out from the estimated money demand relationship

$$\widetilde{m} = \frac{m}{y} = A \cdot i^{-0.5} \quad \text{with:} A = 0.05 \tag{7}$$

 \rightarrow Find an appropriately adjusted version of the basic MIU-model such that eqn (7) can be recovered from a first-order optimality condition

 \rightarrow Solve for the steady state of this model

 \rightarrow Identify the (permanent) welfare cost associated with i > 0 as the percentage increase in annual steady-state consumption that would be needed to make the representative household indifferent between a steady state with i = 0 and i > 0

 $[\rightarrow$ For consistency we use from now on the exposition in Walsh, Section 2.3]

General equilibrium estimates of welfare costs of inflation:

Simplifications and adjustments:

1) Since the basic MIU model displays steady-state superneutrality, c and y are independent of i, π

 \rightarrow Simplify the set-up and consider an economy with an exogenous and constant endowment (period by period) y, yielding the resource constraint

y = c

2) For consistency with the general first-order condition of the MIU-model

$$\frac{u_m}{u_c} = \frac{i}{1+i} \tag{8}$$

replace i in eqn (7) by the alternative opportunity cost measure $\frac{i}{1+i}$

General equilibrium estimates of welfare costs of inflation:

• As a point of departure for finding a utility function u(c, m) which displays a first-order condition consistent with eqn (7) consider the general specification

$$u(c,m) = \frac{1}{1-\sigma} \{ [c \cdot \varphi(\frac{m}{c})]^{1-\sigma} - 1 \}, \tag{9}$$

with $\varphi(\frac{m}{c})$ to be determined below

- Define $x \equiv \frac{m}{c}$
- For eqn (9) the general first-order condition of the MIU-model (8) becomes

$$\frac{u_m}{u_c} = \frac{\varphi'(x)}{\varphi(x) - x\varphi'(x)} = \frac{i}{1+i}$$
(10)

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General equilibrium estimates of welfare costs of inflation:

- Choose the function $\varphi(x)$ such that eqn (10) will be consistent with the estimated money-demand function (7)
- Accordingly, let

$$\varphi(x) = \frac{1}{1 + A^2 \cdot x^{-1}} = \frac{1}{1 + A^2 \cdot \frac{c}{m}},$$
(11)

implying

$$\frac{\varphi'(x)}{\varphi(x) - x\varphi'(x)} = \frac{A^2}{(\frac{m}{c})^2}$$
(12)

• Combining eqns (8) and (12) yields

$$m = c \cdot A \cdot \left(\frac{i}{1+i}\right)^{-0.5},\tag{13}$$

which confirms eqn (7), since we assumed y = c and replaced *i* by $\frac{i}{1+i}$ $\langle \Box \rangle \langle \Box \rangle \langle$

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General equilibrium estimates of welfare costs of inflation:

Comment:

• Using (9) and (11), the analysis will be based on

$$u(c,m) = \frac{1}{1-\sigma} \{ [c \cdot \varphi(\frac{m}{c})]^{1-\sigma} - 1 \}, \quad \text{with:} \quad \varphi(\frac{m}{c}) = \frac{1}{1+A^2 \cdot \frac{c}{m}}$$

and not on the utility function (2), ie

$$u(c,m) = [ac^{1-b} + (1-a)m^{1-b}]^{rac{1}{1-b}}$$

• Does this matter? no

(since the utility functions are monotonic transformations of each other, leaving all welfare results established below unaffected)

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

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Initial steady state (with i \rightarrow 0):
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- Normalize the initial steady-state consumption s.t. $c^* = 1$
- Let m^* denote the level of m yielding the highest possible level of utility if $i \rightarrow 0$
- From eqn (13), ie $m = c \cdot A \cdot (\frac{i}{1+i})^{-0.5}$, it is clear that

$$m^* \to \infty$$

This implies

$$arphi(rac{m^*}{1})=1$$
 and $u(1,m^*)=0$

from the definition of φ in eqn (11) and the general utility specification (9), respectively

Appendix

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

New steady state (with i > 0):

• Define the welfare cost $w(\frac{i}{1+i})$ associated with i > 0 implicitly via

$$u\left(1+w(\frac{i}{1+i}), \ m(\frac{i}{1+i})\right) = u(1, m^*) = 0$$
 (14)

which, using the definition of m via (13), is equivalent to

$$u\left(1+w, (1+w)\cdot A\cdot (\frac{i}{1+i})^{-0.5}\right) = u(1, m^*) = 0$$

• Use eqn (9), ie $u(c, m) = \frac{1}{1-\sigma} \{ [c \cdot \varphi(\frac{m}{c})]^{1-\sigma} - 1 \}$, to see that this implies

$$(1+w)\cdot\varphi(\frac{m}{1+w})=1$$

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General equilibrium estimates of welfare costs of inflation:

New steady state (with i > 0):

• From the definition of φ the last step, ie $(1+w)\cdot \varphi(\frac{m}{1+w})=1,$ is equivalent to

$$(1+w) \cdot \frac{1}{1+A^2 \cdot \frac{1+w}{(1+w) \cdot A \cdot (\frac{i}{1+i})^{-0.5}}} = 1$$

which can be rearranged to obtain the desired welfare measure $w(\frac{i}{1+i})$ of the cost of inflation

$$w(\frac{i}{1+i}) = A \cdot \sqrt{\frac{i}{1+i}}$$
(15)

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General equilibrium estimates of welfare costs of inflation:

Finding 5 (Lucas, 2000):

• Use the estimated value of A = 0.05 to calculate the welfare gains from a permanent reduction of short-term nominal interest rates from 10% to 0% from the expression

$$w(\frac{i}{1+i}) = A \cdot \sqrt{\frac{i}{1+i}}$$

• The welfare gain is $0.05 \cdot \sqrt{\frac{0.1}{1.1}} \approx 1.5\%$ of annual steady-state consumption (which is identical to output because of the assumption c = y)

IV Comments

 \rightarrow Quantitatively, the derived welfare costs of inflation from a partial and general equilibrium perspective are very similar. This reflects the steady-state superneutrality of money in the basic MIU model.

 \rightarrow Conceptually, Lucas' point that one needs to distinguish between partial and general equilibrium welfare measures, of course, is not affected by this finding

 \rightarrow To the contrary, the simple MIU model had been chosen to show that under special and well understood assumptions the general and the partial equilibrium welfare measures can approximately coincide

 \rightarrow In larger models with richer interactions such coincidence is likely to disappear (but researchers are invited to motivate their findings from transparent benchmark models, to make sure that we understand the driving forces behind their findings)

IV Comments

 \rightarrow Recall from above that **Ireland (2009)** finds for **post-1980 US-data** a much improved fit of the **semi-log specification**, calling for some caution when using welfare costs from a log-log specification

 \rightarrow See Figures 1 and 2 from Ireland (2009) which extend the Figures from Lucas until 2006

 \rightarrow Moreover, for about the same period FED policy was arguably implemented via an **interest rate rule** (and not, for example, a monetary aggregates policy). This improves the fit of estimated money demand equations like (4) or (6) which use the interest rate as an instrument. This reinforces the caution expressed by Ireland

Appendix: Figures



Source: Lucas, R., Inflation and welfare, Econometrica, 68, 2, 247-274, 2000.

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Appendix: Figures



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FIGURE 1. US MONEY DEMAND, 1900–1994

Source: Ireland, P. On the welfare cost of inflation and the recent behavior of money, American Economic Review, 99, 3, 1040-1052, 2009



FIGURE 2. US MONEY DEMAND, 1900–2006

Source: Ireland, P. On the welfare cost of inflation and the recent behavior of money, American Economic Review, 99, 3, 1040-1052, 2009.

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