Part I
Modelling Money in General Equilibrium: a Primer
Lecture 5
Extensions of the MIU model

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Motivation

This lecture gives a *short* (and in many ways not exhaustive) *overview* over the remaining aspects covered in Chapter 2 by Walsh

- Notice that until now our discussion of the basic MIU model had a clear focus on steady-state properties
- In line with the DSGE-agenda we sketch now an extension of the basic MIU model that can be used to study the dynamic behaviour of the economy over the business cycle as it adjusts to exogenous disturbances
- Relative to the basic model this calls for 2 main extensions:

→ 1) the introduction of a labour-leisure trade-off since employment variations are an important characteristic of business cycles

→ 2) the introduction of uncertainty, ie we will allow for exogenous shocks that disturb the system from its steady-state equilibrium

Motivation

- Because of these complications, it will not any longer be possible to find at all stages explicit closed-form solutions
- Instead, the DSGE-agenda uses numerical methods to characterize intertemporal equilibrium responses of carefully constructed medium-scale economies to various types of shocks
- The results can be compared to actual data generated by real economies
- Moreover, since the parameters of the model can be varied numerical methods permit answering a broad range of ‘what if’ questions

→ Example: how does the dynamic response to a temporary change in the growth rate of the money supply depend on certain elasticities characterizing individual preferences or, alternatively, on the persistence of the disturbance that affects the money growth rate?

Motivation

Main advantages of the DSGE agenda compared with traditional macroeconomic analysis:

- Internal consistency of all policy evaluations because of the general equilibrium nature of the analysis
- Recognition of the forward-looking nature of decision-making in economics, ie the DSGE agenda has internalized that policy advice can well be misleading if it simply extrapolates reactions of agents from past observations ($\longrightarrow$ Lucas critique)

Upshot: (Calibrated or estimated) DSGE-models are equally useful for forecasting purposes and for the evaluation of policy options via scenarios

Motivation

Steps to be addressed:

1) Extensions of the basic model

2) Formulation of the decision problem and derivation of equilibrium conditions

3) Characterization of the deterministic steady state

4) Linear approximation of the equilibrium conditions around the steady state

5) Calibration of the model economy

6) Simulation results

Extensions

Model extensions:

1) Preferences allow for **labour-leisure trade-off**, ie

$$u = u(c_t, m_t, \underbrace{1 - n_t}_{=l_t}),$$

where n_t is the per capita labour supply (and $l_t = 1 - n_t$ denotes leisure)

Extensions

Model extensions:

2) Two types of (persistent) stochastic disturbances

i) Productivity shock (z_t)
$\longrightarrow$ Specify the production function as

$$y_t = f(k_{t-1}, n_t, z_t) = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}$$

with

$$z_t = \rho_z \cdot z_{t-1} + e_t \quad \text{with: } \rho_z \in (0,1), \quad e_t \sim \text{iid } (0, \sigma_e^2)$$

$\longrightarrow$ Notice: $f(k_{t-1}, n_t, z_t)$ has constant returns to scale in k_{t-1} and n_t

Extensions

Model extensions:

2) Two types of (persistent) stochastic disturbances

ii) Money growth shock (θ_t)

$\longrightarrow$ Consider the law of motion of the nominal stock of money

$$M_{t+1} = (1+\theta_t)M_t$$

$\longrightarrow$ Let

$$u_t = \theta_t - \theta^{ss}$$

denote the deviation of the money growth rate θ_t from its mean θ^{ss} and assume u_t evolves according to

$$u_t = \rho_u \cdot u_{t-1} + \phi \cdot z_{t-1} + \varphi_t \quad \text{with: } \rho_u \in (0,1), \quad \varphi_t \sim \text{iid } (0, \sigma_\varphi^2),$$

ie the shock to the growth rate displays persistence and reacts to the productivity shock (if $\phi \neq 0$)

Decision problem and derivation of equilibrium conditions

- Write the per capita budget constraint as

$$f(k_{t-1}, n_t, z_t) + (1-\delta)k_{t-1} + a_t = c_t + k_t + b_t + m_t$$

where

$$a_t \equiv \tau_t + \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{1+\pi_t}$$

defines the real financial wealth (plus transfers) at the beginning of period t

- Assume: the realization of z_t becomes known prior to the choice of n_t
- Notice: The entire LHS can no longer be summarized as a single state variable (ω_t), since income in period t is affected by the (optimal) choice of the control variable n_t
- Way out: Define a **value function** which depends on **two state variables**, ie a and k

Decision problem and derivation of equilibrium conditions

- Value function (depending on the state variables a and k):

$$V(a_t, k_{t-1}) = \max_{c_t, n_t, b_t, m_t} \{u(c_t, m_t, 1-n_t) + \beta E_t V(a_{t+1}, k_t)\}$$

with a_{t+1} and k_t defined as

$$\begin{aligned} a_{t+1} &= \tau_{t+1} + \frac{(1+i_t)b_t + m_t}{1+\pi_{t+1}} \\ k_t &= f(k_{t-1}, n_t, z_t) + (1-\delta)k_{t-1} + a_t - c_t - b_t - m_t \end{aligned}$$

and with a_t and k_{t-1} to be taken as given

- If one combines the optimality conditions with respect to the control variables c_t, n_t, b_t, m_t and the state variables a_t, k_{t-1} this leads to a set of consolidated intertemporal equilibrium conditions which are mostly known from the preceding analysis

Decision problem and derivation of equilibrium conditions

- Related to the **optimal choice of** n_t there emerges one **additional static optimality condition**

$$U_l(c_t, m_t, 1-n_t) = U_c(c_t, m_t, 1-n_t) \cdot f_n(k_{t-1}, n_t, z_t)$$

  which summarizes the trade-off on how to allocate time between labour and leisure

- **Interpretation:** at the margin the representative HH needs to be indifferent between allocating time to leisure which gives a marginal benefit of

$$U_l(c_t, m_t, 1-n_t)$$

  or, alternatively, to work which gives a marginal benefit of

$$U_c(c_t, m_t, 1-n_t) \cdot f_n(k_{t-1}, n_t, z_t)$$

  where we use the linear transformation between labour and leisure as given by

$$l_t = 1 - n_t$$

Decision problem and derivation of equilibrium conditions

Overview of intertemporal equilibrium conditions:

1) Consumption Euler equation

$$u_c(c_t, m_t, 1 - n_t) = \beta E_t (1 + r_t) u_c(c_{t+1}, m_{t+1}, 1 - n_{t+1}) \quad (1)$$

2) Definition of real interest rate

$$r_t = f_k(k_t, n_{t+1}, z_{t+1}) - \delta \quad (2)$$

3) Resource constraint

$$k_t = (1 - \delta) k_{t-1} + y_t - c_t \quad (3)$$

4) Labour-leisure choice

$$U_l(c_t, m_t, 1 - n_t) = U_c(c_t, m_t, 1 - n_t) \cdot f_n(k_{t-1}, n_t, z_t) \quad (4)$$

5) Choice of real balances

$$U_m(c_t, m_t, 1 - n_t) = U_c(c_t, m_t, 1 - n_t) \cdot \frac{i_t}{1 + i_t} \quad (5)$$

Decision problem and derivation of equilibrium conditions

Overview of intertemporal equilibrium conditions:

6) Production function

$$y_t = f(k_{t-1}, n_t, z_t) = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha} \tag{6}$$

7) Law of motion of real balances

$$m_t = \left(\frac{1+\theta_t}{1+\pi_t}\right) m_{t-1} \tag{7}$$

8) Fisher equation

$$1 + i_t = (1 + r_t) \cdot E_t(1 + \pi_{t+1}) \tag{8}$$

Summary: The equations (1)-(8) constitute a **non-linear system** of 8 **equations** in 8 **endogenous variables** $k_t, r_t, c_t, n_t, m_t, y_t, \pi_t, i_t$. The equilibrium values of these variables can be determined once the processes of the exogenous disturbances z_t and $\theta_t = \theta^{ss} + u_t$ are taken into account, ie

$$z_t = \rho_z \cdot z_{t-1} + e_t \quad \text{with: } \rho_z \in (0,1), \quad e_t \sim \text{iid } (0, \sigma_e^2) \tag{9}$$

$$u_t = \rho_u \cdot u_{t-1} + \phi \cdot z_{t-1} + \varphi_t \quad \text{with: } \rho_u \in (0,1), \quad \varphi_t \sim \text{iid } (0, \sigma_\varphi^2) \tag{10}$$

Decision problem and derivation of equilibrium conditions

What to do next?

To analyze the dynamic behaviour of the economy we will proceed as follows:

$\longrightarrow$ **First**, we will characterize the **deterministic steady state of the economy** at which **all variables are constant** and **all shocks are set to zero**. This steady state is the rest point of the system around which dynamic reactions of the economy to shocks (so-called impulse responses) and business cycle features will be analyzed

$\longrightarrow$ **Second**, to characterize such dynamic reactions we will consider a **(first-order) linearized version of all equilibrium conditions**

Notice: the linearized system will be saddlepath-stable such that the dynamic responses will be uniquely defined

Decision problem and derivation of equilibrium conditions

What is still missing?

→ To make the analysis operational we need to consider a particular utility function. Below this function will be given by

$$u(c_t, m_t, 1 - n_t) = \frac{\left[ac_t^{1-b} + (1-a)m_t^{1-b}\right]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta} \tag{11}$$

Steady state

- The characterization of steady states of the equation system (1)-(8) is relatively straightforward because of a certain recursive structure of the equations
- However, there is one important caveat to this: the inclusion of **leisure** as a third argument in the utility function $u(c, m, 1-n)$ **challenges the steady-state superneutrality of money**
- This feature needs to be addressed in some detail because it matters strongly for the overall dynamics of the system. And it will become relevant for the calibration of the utility function (11)

Steady state

Recursive elements of the system (1)-(8) in steady state:

- The Euler eqn (1) and the eqn (2) for the real interest rate imply

$$\begin{aligned} \beta &= \frac{1}{1+r} \\ \frac{1}{\beta} - 1 + \delta &= f_k(k, n, 0) = f_k(\frac{k}{n}, 1, 0), \end{aligned} \tag{12}$$

  where the last step uses that the production function has constant returns to scale in k and n

- Hence, equation (12) determines **uniquely** the **steady-state capital-labour ratio** as a function of β and δ only (ie independently of the money growth rate θ).
- Let's indicate this unique steady-state ratio of $\frac{k}{n}$ with a bar, ie $\overline{\frac{k}{n}}$

Steady state

Recursive elements of the system (1)-(8) in steady state:

Memo: Implications of constant returns to scale:

- Start out from Euler's theorem:

$$y = f(k,n,0) = f_k(k,n,0)\cdot k + f_n(k,n,0)\cdot n$$

- Use that the derivatives of $f(k,n,0)$ are homogenous of degree zero, ie

$$y = f_k(\frac{k}{n},1,0)\cdot k + f_n(\frac{k}{n},1,0)\cdot n$$

- Hence

$$\underbrace{\frac{y}{n}}_{\phi(\frac{k}{n})} = \underbrace{f_k(\frac{k}{n},1,0)}_{\phi'(\frac{k}{n})}\cdot\frac{k}{n} + \underbrace{f_n(\frac{k}{n},1,0)}_{w(\frac{k}{n})},$$

$\rightarrow$ The intensive form production function $\frac{y}{n} = \phi(\frac{k}{n})$ depends only on the capital-labour ratio

$\rightarrow$ Similarly, the real wage depends only on the capital-labour ratio

$$w = f_n(k,n,0) = f_n(\frac{k}{n},1,0) = \phi(\frac{k}{n}) - \phi'(\frac{k}{n})\cdot\frac{k}{n} = w(\frac{k}{n})$$

Steady state

Recursive elements of the system (1)-(8) in steady state:

- Hence, in steady-state $\frac{y}{n}$ is **uniquely determined** from $\frac{y}{n} = \phi(\overline{\frac{k}{n}})$
- Similarly, $\frac{c}{n}$ is **uniquely determined** from the resource constraint (3)

$$c = f(k, n, 0) - \delta k = \underbrace{[\phi(\overline{\frac{k}{n}}) - \delta \overline{\frac{k}{n}}]}_{\equiv \overline{\phi}} \cdot n \quad \Leftrightarrow \quad \frac{c}{n} = \overline{\phi}$$

- and the **real wage rate** $w = f_n(\frac{k}{n}, 1, 0)$ is **uniquely determined** from

$$w = \phi(\overline{\frac{k}{n}}) - \phi'(\overline{\frac{k}{n}}) \cdot \overline{\frac{k}{n}} = \overline{w}$$

Steady state

Recursive elements of the system (1)-(8) in steady state:
$\longrightarrow$ In other words: money is superneutral with respect to the **ratios** $\frac{k}{n}$, $\frac{y}{n}$, $\frac{c}{n}$ and the real wage (w)
$\longrightarrow$ But what about the **levels** of the variables k, y, c, and n? This can be assessed from the two static optimality conditions (4) and (5), ie
Labour-leisure choice

$$\frac{U_l(\overline{\phi}n, m, 1-n)}{U_c(\overline{\phi}n, m, 1-n)} = \overline{w} \tag{13}$$

Choice of real balances

$$\frac{U_m(\overline{\phi}n, m, 1-n)}{U_c(\overline{\phi}n, m, 1-n)} = \frac{i}{1+i} = \frac{1+\theta-\beta}{1+\theta}, \tag{14}$$

where the last line uses the Fisher equation (8), ie

$$1+i = (1+r)\cdot(1+\theta) = \frac{1}{\beta}\cdot(1+\theta)$$

$\longrightarrow$ **Notice:** Equations (13) and (14) are 2 equations in 2 unknowns n and m

Steady state

Recursive elements of the system (1)-(8) in steady state:

- Consider eqns (13) and (14), ie

$$\frac{U_l(\overline{\phi}n, m, 1-n)}{U_c(\overline{\phi}n, m, 1-n)} = \overline{w} \quad \text{and} \quad \frac{U_m(\overline{\phi}n, m, 1-n)}{U_c(\overline{\phi}n, m, 1-n)} = \frac{1+\theta-\beta}{1+\theta},$$

- **Money** can be **superneutral** or **not,** depending on the **structure of the utility function** $u(c, m, 1-n)$
- **Special case:** if u is separable in real balances (such that the marginal utilities of consumption and leisure become independent of m), then **superneutrality** prevails, ie not only the ratios, but also the levels of the real variables (except, of course, m itself) are independent of θ
- **General case** of **non-superneutrality** (used below for calibrating the utility function (11)): u is not separable in real balances, and instead we assume that **consumption and real balances are Edgeworth complements**, ie $u_{cm} > 0$

Steady state

Recursive elements of the system (1)-(8) in steady state:

- **General case: consumption and real balances are Edgeworth complements**, ie $u_{cm} > 0$
- Money will **not** be **superneutral**. In particular: an increase in θ reduces n and, hence, k, y, and c
- **Why?**

$\rightarrow$ The increase in θ leads to higher inflation π and lower real balances m... (this direct effect is unambiguous, ie higher inflation makes it more costly to hold real balances, so HHs want to hold less of them)
$\rightarrow$ ...Since m and c are complements, the marginal utility of consumption declines...
$\rightarrow$ ...but such decline matters now for the optimal trade-off between consumption and leisure in eq (4)...
$\rightarrow$ ...HH substitute away from labor to leisure...
$\rightarrow$ ...and through this channel output, capital, and consumption need to decline

Steady state

Comments: Superneutrality vs. non-superneutrality

Consider the particular utility function (11), ie

$$u(c_t, m_t, 1-n_t) = \frac{\left[ac_t^{1-b} + (1-a)m_t^{1-b}\right]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta}$$

1) Special case (superneutrality):

$$b = \Phi \quad \Leftrightarrow \quad u_{cm} = 0$$

2) General case (non-superneutrality)

$$b > \Phi \quad \Leftrightarrow \quad u_{cm} > 0$$

Steady state

Comments: Superneutrality vs. non-superneutrality

$\longrightarrow$ Conceptually, the MIU-model has not good enough microfoundations to settle this. There is some evidence that long-run inflation may hamper real activity (as indirectly captured by the assumption $u_{cm} > 0$), but to model the mechanisms responsible for such (controversial) effects of inflation are beyond the scope of the MIU-model

$\longrightarrow$ Feldstein (1978), for example, identifies the non-indexation of tax systems with respect to inflation as a major source of non-superneutrality

Linear approximation

- To understand the structure of the linearized system of equations used by Walsh notice that he adds to the original ten equations (1)-(10) listed above 2 further variables (and 2 further equations), namely

$$\lambda_t = u_c(c_t, m_t, 1 - n_t) \tag{15}$$
$$x_t = k_t - (1 - \delta)k_{t-1}, \tag{16}$$

  capturing the marginal utility of consumption and (gross) investment
- The linearized system (→ **see Annex**) expresses most **variables as percentage deviations from their steady-state values**, ie for some representative variable q we use the hat-notation

$$\widehat{q_t} = \frac{q_t - q^{SS}}{q^{SS}} \qquad q = k,\ y,\ c,\ n,\ m,\ \lambda,\ x$$

- **Exceptions:** There are 3 widely used exceptions, ie we use

$$\widehat{r_t} = r_t - r^{SS}, \quad \widehat{i_t} = i_t - i^{SS}, \quad \widehat{\pi_t} = \pi_t - \pi^{SS},$$

  to capture deviations of interest rates and inflation from their steady-state values in terms of percentage points

Linear approximation

Features of the (linearized) dynamics:

1) Superneutrality for the special case of $b = \Phi$:

- If $b = \Phi$ one can isolate within the system of intertemporal equilibrium conditions a sub-system of 3 dynamic equations in inflation $(\widehat{\pi}_t)$, the nominal interest rate $(\widehat{i}_t)$, and real balances $(\widehat{m}_t)$, ie

$$\begin{aligned}
\widehat{i}_t &= \widehat{r}_t + E_t \widehat{\pi}_{t+1} \\
\widehat{m}_t - \widehat{c}_t &= -\frac{1}{b} \cdot \frac{1}{i^{ss}} \cdot \widehat{i}_t \\
\widehat{m}_t &= \widehat{m}_{t-1} - \widehat{\pi}_t + u_t
\end{aligned}$$

- These 3 variables do not enter any other equation. Thus, these 3 equations, subject to the exogenous money supply process u_t and with $\widehat{r}_t$ and $\widehat{c}_t$ being independently determined within the core block of equations which summarize the real side of the economy, support superneutrality (at steady state and during transitional dynamics). They can be used to study 'purely nominal' aspects of the intertemporal equilibrium.
- Such analysis will be similar to the discussion of Examples 1 and 2 in Lecture 2, in the spirit of Obstfeld and Rogoff (1983).

Linear approximation

Features of the (linearized) dynamics:

2) $b \neq \Phi$: **Only anticipated changes in money growth matter**
$\longrightarrow$ Even if money is non-superneutral ($b \neq \Phi$), only **anticipated changes in money growth matter**

- **Experiment 1: Unanticipated money growth shock**
  Consider the process describing u_t,

  $$u_t = \rho_u \cdot u_{t-1} + \phi \cdot z_{t-1} + \varphi_t \quad \text{with: } \rho_u \in (0,1), \quad \varphi_t \sim \text{iid } (0, \sigma_\varphi^2),$$

  and assume $\rho_u = \phi = 0$ such that $u_t = \varphi_t$ describes an unanticipated one-off change in the money growth rate (inducing a **permanent and unanticipated one-off shock to the level of the money supply**)
  $\longrightarrow$ Future money growth and expected inflation are unaffected by this shock
  $\longrightarrow$ The shock leaves real balances $\widehat{m}_t$ in period t unaffected
  $\longrightarrow$ The shock leads to a one-period change in current inflation ($\widehat{\pi}_t = u_t = \varphi_t$), ie it induces a proportionate permanent change in the price level
  $\longrightarrow$ **Money does not matter**

Linear approximation

Features of the (linearized) dynamics:

2) $b \neq \Phi$: **Only anticipated changes in money growth matter**

- **Experiment 2: Anticipated money growth shock**
  Consider the process describing u_t,

$$u_t = \rho_u \cdot u_{t-1} + \phi \cdot z_{t-1} + \varphi_t \quad \text{with: } \rho_u \in (0,1), \quad \varphi_t \sim \text{iid } (0, \sigma_\varphi^2),$$

  and assume now $\rho_u > 0$ (but maintain $\phi = 0$) such that $\varphi_t > 0$ implies that **future money growth will be above average**
  $\rightarrow$ This leads to a rise in expected inflation and a higher nominal interest rate...
  $\rightarrow$ ...affecting real balances $\widehat{m}_t$ in period t ...
  $\rightarrow$ ...with implications for $\widehat{c}_t$ and the other variables forming the real side of the economy (channel for this: $b \neq \Phi$ in Eq (15 hat), see Annex)
  $\rightarrow$ **Money does matter**

Linear approximation

Features of the (linearized) dynamics:

2) $b \neq \Phi$: **Only anticipated changes in money growth matter**

Importantly, **money matters in a special** and **model-specific way:**

$\longrightarrow$ Higher money growth works on impact exclusively through the channel of higher expected inflation (and higher nominal interest rates)

$\longrightarrow$ In particular, because of the assumption of flexible prices, real balances decline on impact, ie the monetary shock generates a jump in the price level

$\longrightarrow$ *Notice:* this channel is different from the standard **liquidity effect of money** which leads on impact under rigid nominal prices to an increase in real balances and a decline in the nominal interest rate

Calibration and simulation results

Baseline Parameter Values

α	δ	β	Φ	η	a	b	$1+\theta^{ss}$	ρ_z	σ_e	ρ_u	σ_φ
0.36	0.019	0.989	2	1	0.99	3	1.01	0.95	0.34	0.75	0.9

Steady-State Values at Baseline Parameter Values

$1+r^{ss}$	$\frac{y^{ss}}{k^{ss}}$	$\frac{c^{ss}}{k^{ss}}$	$\frac{m^{ss}}{k^{ss}}$	$\frac{n^{ss}}{k^{ss}}$
1.011	0.084	0.065	0.051	0.021

Source: Walsh, C., Monetary Theory and Policy, 3rd edition, MIT Press, 2010, p. 72.

Calibration and simulation results

Comments on calibration (*for details, see Walsh: p.71*)

→ Parameter values are chosen to match key properties of quarterly detrended US-data (largely in line with standard RBC calibrations)

Selected features:

- US capital share: $\approx 36\%$; Annual real interest rate: $\approx 4\%$; Annual depreciation rate: $\approx 8\%$; Annual growth rate of $M1$: $\approx 4\%$; AR(1)-coefficient of $M1$: ≈ 0.75
- Intertemporal elasticity of substitution in consumption: $1/\Phi = 0.5$; Implied per capita labour supply: $n^{ss} = 1/3$; Intertemporal elasticity of substitution in labour: $(1 - n^{ss})/(n^{ss}\eta) = 2$
- Edgeworth complementarity between c and m (ie $u_{cm} > 0$), since: $b - \Phi = 1 > 0$
  Implications: $\frac{\partial m^{ss}}{\partial \theta} < 0 \Rightarrow \frac{\partial c^{ss}}{\partial \theta} < 0$, $\frac{\partial n^{ss}}{\partial \theta} < 0$, $\frac{\partial y^{ss}}{\partial \theta} < 0$

Calibration and simulation results

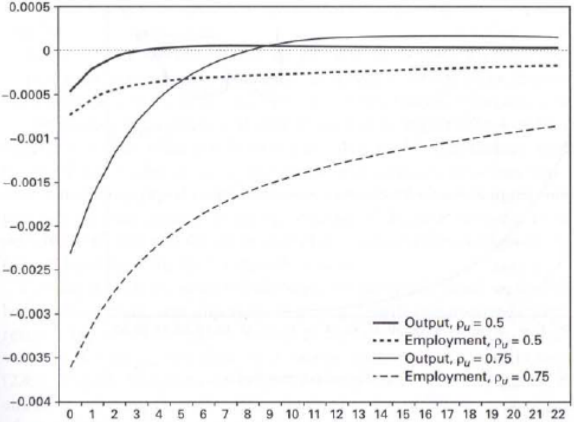

Figure 2.3
Response of output and employment to a positive money growth shock.

Source: Walsh, C., Monetary Theory and Policy, 3rd edition, MIT Press, 2010, p. 73.

Calibration and simulation results

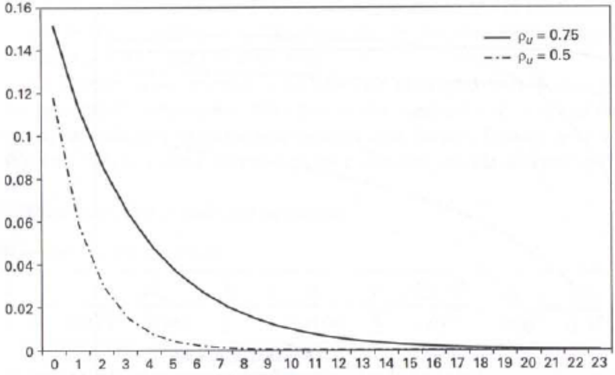

Figure 2.4
Response of the nominal interest rate to a positive money growth shock.

Source: Walsh, C., Monetary Theory and Policy, 3rd edition, MIT Press, 2010, p. 74.

Calibration and simulation results

Comments on simulation: *Responses of output, employment and nominal interest rate to money growth shock (Figures 2.3 and 2.4)*

Selected features:

- Figures show responses to one standard deviation money growth shock (φ_t) for $\rho_u = 0.5$ and $\rho_u = 0.75$
- $\rho_u = 0$: there would be no effect (case of unanticipated money growth shock as discussed above)
- $\rho_u > 0$: effects are in line with above discussion of anticipated money growth shock
  $\rightarrow$ negative effects on y and n because of Edgeworth complementarity between c and m (ie $u_{cm} > 0$)
  $\rightarrow$ strength of these effects increases in ρ_u
- Similarly: positive effect on i (channel: expected higher inflation)

Memo: Definition of *money growth shock*:

$$u_t = \rho_u \cdot u_{t-1} + \phi \cdot z_{t-1} + \varphi_t \quad \text{with:} \;\; \rho_u \in (0,1), \;\; \varphi_t \sim \text{iid } (0, \sigma_\varphi^2)$$

Calibration and simulation results

Table 2.2
Effects of the Money Process

	$\phi = -0.15$			$\phi = 0$			$\phi = 0.15$		
	s.d.	s.d. rel. to y	Corr. with y	s.d.	s.d. rel. to y	Corr. with y	s.d.	s.d. rel. to y	Corr. with y
y	1.51	1.00	1.00	1.49	1.00	1.00	1.48	1.00	1.00
n	0.58	0.38	0.02	0.58	0.39	−0.04	0.59	0.40	−0.10
c	0.91	0.60	0.88	0.88	0.59	0.87	0.86	0.58	0.86
x	4.22	2.79	0.94	4.25	2.84	0.94	4.28	2.90	0.94
r	0.04	0.02	0.19	0.04	0.02	0.17	0.04	0.02	0.14
m	7.86	5.20	0.90	3.59	2.40	0.24	6.35	4.30	−0.84
i	0.45	0.30	−0.88	0.22	0.15	−0.03	0.44	0.30	0.87
π	3.59	2.37	−0.11	3.23	2.16	−0.00	3.51	2.38	0.11

Source: Walsh, C., Monetary Theory and Policy, 3rd edition, MIT Press, 2010, p. 74.

Calibration and simulation results

Comments on simulation: *Effects of the money process (Table 2.2)*

Selected features:

- The Table summarizes reactions to a one standard deviation productivity shock (e_t), conditional on how money growth responds to productivity shocks (ie 3 different values of ϕ)
- Benchmark ($\phi = 0$): positive effect on y;
  moreover: positive effect on c, weakly negative effects on n and π
- If $\phi > 0$: monetary policy accommodates productivity shock...this leads to expected inflation...since $u_{cm} > 0$: this induces ceteris paribus negative effects on c and n...
  $\rightarrow$ output effect (marginally) weaker than in the benchmark
- If $\phi < 0$: converse pattern, ie output effect (marginally) stronger than in the benchmark

Memo: Definition of *productivity shock* and *money growth shock*:

$$z_t = \rho_z \cdot z_{t-1} + e_t \quad \text{with: } \rho_z \in (0,1), \quad e_t \sim \text{iid } (0, \sigma_e^2)$$

$$u_t = \rho_u \cdot u_{t-1} + \phi \cdot z_{t-1} + \varphi_t \quad \textit{with}: \ \rho_u \in (0,1), \quad \varphi_t \sim iid(0, \sigma_\varphi^2)$$

Annex: System of linearized equilibrium conditions

Marginal utility of consumption:

$$\widehat{\lambda}_t = \Omega_1 \widehat{c}_t + \Omega_2 \widehat{m}_t \qquad \text{with:} \tag{15 hat}$$

$$\Omega_1 = (b-\Phi)\gamma - b, \;\; \Omega_2 = (b-\Phi)(1-\gamma), \;\; \gamma = \frac{a\left(c^{ss}\right)^{1-b}}{a\left(c^{ss}\right)^{1-b} + (1-a)\left(m^{ss}\right)^{1-b}}$$

Gross investment:

$$\frac{x^{ss}}{k^{ss}} \widehat{x}_t = \widehat{k}_t - (1-\delta)\widehat{k}_{t-1} \tag{16 hat}$$

Consumption Euler equation:

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \widehat{r}_t \tag{1 hat}$$

Definition of real interest rate:

$$\widehat{r}_t = \alpha\left(\frac{y^{ss}}{k^{ss}}\right)\left(E_t \widehat{y}_{t+1} - \widehat{k}_t\right) \tag{2 hat}$$

Resource constraint:

$$\left(\frac{y^{ss}}{k^{ss}}\right)\widehat{y}_t = \left(\frac{c^{ss}}{k^{ss}}\right)\widehat{c}_t + \delta \widehat{x}_t \tag{3 hat}$$

Labour-leisure choice:

$$-\widehat{\lambda}_t + \eta\left(\frac{n^{ss}}{1-n^{ss}}\right)\widehat{n}_t = \widehat{y}_t - \widehat{n}_t \tag{4 hat}$$

Annex: System of linearized equilibrium conditions

Choice of real balances:

$$\widehat{m}_t - \widehat{c}_t = -(\frac{1}{b})\frac{1}{i^{ss}}\widehat{i}_t \tag{5 hat}$$

Production function:

$$\widehat{y}_t = \alpha \widehat{k}_{t-1} + (1-\alpha)\widehat{n}_t + z_t \tag{6 hat}$$

Law of motion of real balances:

$$\widehat{m}_t = \widehat{m}_{t-1} - \widehat{\pi}_t + u_t \tag{7 hat}$$

Fisher equation:

$$\widehat{i}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1} \tag{8 hat}$$

These equations constitute a **linear system** of 10 **equations** in 10 **endogenous variables** $\widehat{\lambda}_t, \widehat{x}_t, \widehat{k}_t, \widehat{r}_t, \widehat{c}_t, \widehat{n}_t, \widehat{m}_t, \widehat{y}_t, \widehat{\pi}_t, \widehat{i}_t$. The equilibrium values of these variables can be determined once the processes of the exogenous disturbances z_t and $\theta_t = \theta^{ss} + u_t$ are taken into account, ie

Productivity shock:

$$z_t = \rho_z \cdot z_{t-1} + e_t \quad \text{with: } \rho_z \in (0,1), \quad e_t \sim \text{iid } (0, \sigma_e^2) \tag{9}$$

Money growth shock:

$$u_t = \rho_u \cdot u_{t-1} + \phi \cdot z_{t-1} + \varphi_t \quad \text{with: } \rho_u \in (0,1), \quad \varphi_t \sim \text{iid } (0, \sigma_\varphi^2) \tag{10}$$