Part I
Modelling Money in General Equilibrium: a Primer
Lecture 3
Welfare Cost of Inflation in the Basic MIU model

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I Motivation

→ **Some features of fiat money regimes:**

- **private opportunity cost** of holding real balances (m) depends on the level of the short-term nominal interest rate (i)
- **social opportunity cost** of providing money is essentially zero, ie the central bank can make the amount of money in circulation arbitrarily large (and thereby depress i)
- the **wedge** between the private and social cost at positive interest rates creates an **inefficiency**
- moreover, in equilibrium, the **nominal interest rate**, the **inflation rate** and the **quantity of money** in circulation are linked to each other

→ These features give rise to **two** types of (closely linked) **policy questions** which have been addressed in monetary economics for centuries:

1) From a **positive** perspective: How large is the **welfare cost of inflation**?
2) From a **normative** perspective: What is the **optimal rate of inflation**?

I Motivation

→ The **MIU-model** offers a transparent and tractable **general equilibrium perspective,** ie it is a widely used starting point to address these questions, qualitatively and quantitatively
(in particular, see *Lucas, 2000*):

→ Why is the MIU model a good model to analyze these questions? Because:

- The costs of inflation (via the interest rate) are clearly captured by the expression
$$u_m(c_t, m_t) = \frac{i_t}{1+i_t} u_c(c_t, m_t) \tag{1}$$
- Steady-state superneutrality implies that we don't have to worry about the real side of the economy when comparing steady states with different inflation and interest rates
- The flow objective $u(c, m)$ offers a simple and direct metric for welfare comparisons

I Motivation

Question 2: What is the **optimal rate of inflation**?

$\rightarrow$ In the basic MIU model the answer to this question is rather straightforward, since monetary policy instruments (i or θ) do not affect the steady-state value of c, only of m

$\rightarrow$ Money growth rule (via θ) : Optimality requires $\frac{\partial u(c,m)}{\partial \theta} = \frac{\partial u(c,m)}{\partial m} \frac{\partial m}{\partial \theta} = 0$, or $\frac{\partial u(c,m)}{\partial m} = 0$, implying in eqn (1)

$$i = 0$$

$\rightarrow$ This is the **Friedman rule** which implies (using $1 + i \approx 1 + r + \pi$)

$$\pi \approx -r,$$

i.e. the (long-run) optimal inflation rate is a rate of deflation approximately equal to the return on capital and government bonds *(Friedman, 1969)*

I Motivation

Friedman rule: preliminary comments

Comment 1: *Satiation vs. approximate validity of Friedman rule*

- Depending on the assumed utility function the strict equality $\frac{\partial u(c,m)}{\partial m} = 0$ with $i = 0$ obtains if there exists a finite satiation level (as in *(A 1)*)
- In other cases (like the discussed case of log-utility) m should be made arbitrarily large such that $i \longrightarrow 0$ (approximate validity of the Friedman rule)

Comment 2: *Implementation*

- The result is equally valid if monetary policy is directly implemented via an interest rate rule (rather than a money growth rule)

I Motivation

Friedman rule: an important qualification

- **This reasoning is not yet a basis for policy advice**, since the interest rate i (and indirectly π) is the only distortionary policy instrument
- Recall from the model set-up: government has access to **lump-sum transfers/taxes** τ, making distortionary seigniorage revenues 'costly'
- The discussion about the optimality of the Friedman rule becomes only meaningful when we introduce **distortionary taxes**
- Then, as argued by *Phelps (1973)*, the Friedman rule may well break down

$\rightarrow$ We will return to **Question** 2 in detail **in Part II** which covers the **Friedman vs. Phelps debate,** assuming **distortionary taxation**

I Motivation

Let us return to
Question 1: How large is the **welfare cost of inflation**?

→ **Lucas (2000)** uses the MIU model to provide **quantitative guidance** for the likely range of **welfare costs of inflation**

→ The methodology proposed by Lucas has been influential since it exploits the **general equilibrium dimension of the MIU model,** as opposed to **partial equilibrium** estimates that were traditionally used *(Bailey 1956)*

To reproduce the findings from Lucas, we proceed in 2 steps:

Step I): Two types of money demand specifications and discussion of the empirical evidence
Step II): Partial vs. general equilibrium measures of welfare costs of inflation

II Money demand specifications and empirical evidence

→ Any welfare measure of the costs of inflation depends critically on the form of the money demand equation and the sensitivity of money demand to the opportunity cost of holding real balances

→ In the basic **MIU model** these issues are directly related to the specification of the utility function which drives the shape of

$$u_m(c_t, m_t) = \frac{i_t}{1+i_t} u_c(c_t, m_t)$$

→The **empirical literature** commonly distinguishes between **log-log** specifications as opposed to **semi-log specifications** of **money demand**

II Money demand specifications and empirical evidence

CES utility function between c and m :

$\longrightarrow$ Consider the following **CES utility function** which displays a constant elasticity of substitution between consumption and real balances:

$$u(c_t, m_t) = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}} \quad \text{with: } 0 < a < 1,\ b > 0 \text{ (and } b \neq 1) \tag{2}$$

$\longrightarrow$ This function satisfies

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = (\frac{1-a}{a})(\frac{c_t}{m_t})^b$$

and, when combined with the first-order optimality condition

$$u_m(c_t, m_t) = \frac{i_t}{1+i_t} u_c(c_t, m_t),$$

it gives rise to the **money-demand function**

$$m_t = (\frac{1-a}{a})^{\frac{1}{b}} \cdot (\frac{i_t}{1+i_t})^{-\frac{1}{b}} \cdot c_t \tag{3}$$

II Money demand specifications and empirical evidence

Log-log specification:

→ In the empirical literature it is common to rewrite equations of type (3), ie

$$m_t = \left(\frac{1-a}{a}\right)^{\frac{1}{b}} \cdot \left(\frac{i_t}{1+i_t}\right)^{-\frac{1}{b}} \cdot c_t$$

by taking logs, leading to

$$\log(m) = \frac{1}{b}\log\left(\frac{1-a}{a}\right) + \log(c) - \frac{1}{b}\log\left(\frac{i}{1+i}\right) \tag{4}$$

Features of eqn (4):

- Demand for m depends positively on c and negatively on i
- Elasticity of money demand w.r.t. consumption ($\eta_{m,c} = \frac{dm}{m} / \frac{dc}{c}$) is 1
  → often c is replaced by y, yielding an income elasticity of 1
- Elasticity of money demand w.r.t. the opportunity cost variable $\frac{i}{1+i}$ (ie: $\eta_{m,\frac{i}{1+i}} = -\frac{dm}{m} / \frac{d\frac{i}{1+i}}{\frac{i}{1+i}}$) is $\frac{1}{b}$
  → for simplicity $\eta_{m,\, i/1+i}$ is often referred to as the **interest elasticity of money demand** (and, in any case, for small $i: \frac{i}{1+i} \approx i$)
  → since on the RHS of eqn (4) we consider the log of $\frac{i}{1+i}$ (and *not* the level of $\frac{i}{1+i}$) it is called a **log-log specification of money demand**

II Money demand specifications and empirical evidence

Log-log specification:

$\longrightarrow$ **Special case of** $b \longrightarrow 1$ in eqn (2):

$$u(c_t, m_t) = c_t^a \cdot m_t^{1-a} \tag{5}$$

Features of (5):

- The money demand equation (3) becomes

$$m_t = \left(\frac{1-a}{a}\right) \cdot \left(\frac{i_t}{1+i_t}\right)^{-1} \cdot c_t$$

- The **consumption** and the **interest elasticity** of **money demand** are **both equal to** 1

II Money demand specifications and empirical evidence

Semi-log specification:

$\rightarrow$ In the empirical literature commonly studied alternatives to (4) are semi-log specifications of money demand
$\rightarrow$ Semi-log money demand equations regress the log of m_t on the level of i_t (or some alternative measure of the opportunity cost of real balances like $\frac{i_t}{1+i_t}$)

Illustration:

- With a semi-log specification, eqn (4), ceteris paribus, turns into

$$\log(m) = \widetilde{a} + \log(c) - \xi \frac{i}{1+i}, \tag{6}$$

- The coefficient ξ in front of $\frac{i}{1+i}$ denotes the **semielasticity of money demand** w.r.t. $\frac{i}{1+i}$, ie $\xi = -\frac{dm}{m} / d\frac{i}{1+i}$
- Hence, the elasticity and the semi-elasticity are linked via the relationship:

$$\xi = -\frac{dm}{m} \frac{\frac{i}{1+i}}{d\frac{i}{1+i}} \frac{1}{\frac{i}{1+i}} \quad \Leftrightarrow \quad \eta_{m,\frac{i}{1+i}} = \xi \cdot \frac{i}{1+i}$$

II Money demand specifications and empirical evidence

Empirical evidence:

Lucas (2000):

$\longrightarrow$ How to account for US annual time series of short-term nominal interest rates and the 'money-income ratio' (ie the ratio of M1 to nominal GDP: $\frac{M}{Py}$) over the 95 year period 1900-1994?

$\longrightarrow$ Can we explain the relationship between the two series by log-log or semi-log specifications of money demand?

$\longrightarrow$ *Figures 1-4* in Lucas (2000) entail 3 stylized **empirical findings**:

II Money demand specifications and empirical evidence

Finding 1 *(see Figure 1 from Lucas, 2000):*

- Over the 95 year period, US real GDP grew at an average rate of 3%, M1 at 5.6% and the GDP deflator at 3.2%
- This makes the 'money-income ratio' ($\frac{M}{Py}$) essentially trendless over the entire period (although there has been a significant decline since World War II)
- A value of the income elasticity of money demand larger than unity (ie $\eta_{m,\,y} > 1$) would have produced an upward trend

$\rightarrow$ **Stationarity of the money-income ratio not to be rejected**

$\rightarrow$ Assumed money demand functions $m^d(i, y)$ are of type $m^d = m(i) \cdot y$, satisfying a **unit income elasticity of money demand** ($\eta_{m,\,y} = 1$)

II Money demand specifications and empirical evidence

Finding 2 *(see Figures 2 and 3 from Lucas, 2000):*

- Figures 2 and 3 plot observations for the money-income ratio m/y and the short-term nominal interest rate i
- To account for this relationship compare predictions from the **log-log specification** (using the $\eta-$values 0.3, 0.5, 0.7)

$$\frac{m}{y} = A \cdot i^{-\eta} \quad \Leftrightarrow \quad \log(\frac{m}{y}) = \log(A) - \eta \log(i)$$

and the **semi-log specification** (using the $\xi-$values 5, 7, 9)

$$\frac{m}{y} = \widetilde{A} \cdot e^{-\xi i} \quad \Leftrightarrow \quad \log(\frac{m}{y}) = \log(\widetilde{A}) - \xi i,$$

where A and $\widetilde{A}$, respectively, are fitted such that the curves pass through the geometric means of the data pairs

$\longrightarrow$ **Log-log curves** give a **better fit than semi-log curves**

$\longrightarrow$ Within the class of log-log curves $\eta = 0.5$ **gives the best fit**

II Money demand specifications and empirical evidence

Finding 3 *(see Figures 1 and 4 from Lucas, 2000):*

- Figure 4 plots observed levels of real balances (ie m) against the real balances predicted by the estimated log-log demand curve (with $\eta = 0.5$)

$$m_t = A \cdot i_t^{-0.5} \cdot y_t$$

$\longrightarrow$ **success:**

- Fitted values successfully track secular increase in m/y prior to World War II (in a period characterized by declining nominal interest rates)
- Fitted values track well the decline in m/y after World War II until 1980s (in a period characterized by a secular rise in i)

$\longrightarrow$ **but:**

- Fitted values become poor since the mid 1980s (a period of low i and significant financial deregulation)
- Elasticities needed to fit long-run trends do *not* permit a good fit on a year-to-year or even quarterly basis

II Money demand specifications and empirical evidence

Related empirical literature:

- Lucas' preferred estimate from a log-log specification of $\eta = 0.5$, implying a value $b = 2$ in eqn (4), is broadly **in line with related studies**:
  $\longrightarrow$ Chari, Kehoe and McGrattan (2000) report a US value of $b = 2.6$
  $\longrightarrow$ Hoffmann, Rasche and Tieslau (1995) report cross-country evidence and find similar estimates for the US and Canada, a somewhat higher value for the UK, and lower values for Germany and Japan

- The failure to obtain a good short-run fit via estimated long-run elasticities fitting secular trends has led to a vast research agenda on money demand specifications, with the aim to reconcile evidence at different frequencies
  $\longrightarrow$ For example, see the **distributed lag specifications** summarized by Walsh (Table 2.1), allowing, by construction, for **differences between short-run** and **long-run elasticities**
  $\longrightarrow$ Estimations of more **flexible money demand systems**, allowing for additional variables (various assets and interest rates etc.)

II Money demand specifications and empirical evidence

Related empirical literature:

- The mentioned **problems to account for stable money demand relationships since the 1980s** have been explained through various channels, like
  $\longrightarrow$ **Changes in financial regulation** (which have increased the range of money substitutes available for transactions at low cost) and associated **portfolio shifts**
  $\longrightarrow$ **Non-linearities** in **money demand equations** (suggesting that in environments of low i and π interest elasticities of money demand tend to be lower)
- *Ireland (2009)* argues that for **post-1980 US-data** the fit of **semi-log specifications** improves and outperforms log-log specifications:
  $\longrightarrow$ Findings of this type have **implications** for estimates of the **welfare cost of inflation**, as to be discussed next

III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

Bailey (1956):

- inflation taxes real balances
- focus: **fully anticipated** inflation
- welfare effects of inflation to be assessed similar to the effects of any other tax

$\longrightarrow$ when comparing welfare implications of two different levels of i, the natural welfare measure is the area under the (inverse) money demand curve, ie the **consumer surplus**
$\longrightarrow$ ceteris paribus, this amounts to a **partial equilibrium** perspective

Lucas (2000) reproduces this approach, but suggests to consider instead of the demand for real balances (ie $m = \frac{M}{P}$) the 'money income ratio' (ie $\frac{m}{y} \equiv \widetilde{m}$)
$\longrightarrow$ **idea:** express area under the money demand curve as a **fraction of income**
$\longrightarrow$ since his estimates use $m^d = m(i) \cdot y$ this transformation is legitimate
$\longrightarrow$ in terms of dimension, this is a more satisfactory measure of how to compensate people to be indifferent between different steady states

III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

$\rightarrow$ Welfare costs of inflation in steady states comparison
$\rightarrow$ Consider two pairs of observations s.t. $0 < i_0 < i_1$ with $\widetilde{m}_0 > \widetilde{m}_1$

Here: *Figure I (Welfare costs of inflation from a partial equilibrium perspective)*

Initial steady state: $i_0, \widetilde{m}_0$

- Area Ai_0C: consumer surplus
- Area $A\widetilde{m}_0 0 i_0$: surplus extracted by gov't (since it saves $\approx i_0\widetilde{m}_0$ resources relative to raising them via issuance of gov't bonds at nominal rate i_0)

New steady state: $i_1, \widetilde{m}_1$

- Area Bi_1C: consumer surplus
- Area $B\widetilde{m}_1 0 i_1$: surplus extracted by gov't (since it saves $\approx i_1\widetilde{m}_1$ resources)

Comparison between initial and new steady state:

- Area $A\widetilde{m}_0\widetilde{m}_1 B$: **Welfare loss** (ie the difference between the total surpluses) is the shaded area under the inverted money demand curve (in terms of $\widetilde{m}$), ie the combined net loss of surpluses extracted by the gov't and consumers when moving from i_0 up to i_1

III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

Finding 4 *(see Figures 5 and 6 from Lucas, 2000):*

Lucas (2000) quantifies the partial equilibrium welfare gains from reducing the nominal interest rate from $i_1 > 0$ to $i_0 = 0$ by evaluating the consumer surplus expression

$$\int_0^{i_1} \widetilde{m}(x)dx - i_1 \widetilde{m}(i_1)$$

for his money demand estimates

$\rightarrow$ For the benchmark log-log specification with $\eta = 0.5$ a **permanent reduction of short-term nominal interest rates from 10% to 0%** leads to a welfare gain of $\approx 1.6\%$ of annual US real GDP

$\rightarrow$ Assuming that for the US over the 95 year horizon a nominal interest rate of 3% on average leads approximately to price stability, a **permanent reduction of short-term nominal interest rates from** 3% **to** 0% still leads to a sizeable welfare gain of $\approx 0.9\%$ of annual US real GDP

III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

Comments:

- For the **log-log specification** $\lim_{i \to 0} \widetilde{m}(i) \to \infty$
- This makes welfare gains at very low levels of i relatively **'large'**

→ Gains like 1.6% or 0.9% of US real GDP may seem small, but they would be available every year, ie the discounted **present value** of them is **substantial**
→ However, these gains need to be compared with **costs of disinflation** (see Ball, 1993). Such costs typically arise because of (short-run) nominal rigidities and unexpected disinflation surprises
→ Such features are not addressed in the flex-price rational expectations approach considered so far

III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

Comments:

- For the **semi-log specification**, $\widetilde{m}(i=0)$ is a finite number (ie $\widetilde{A}$)
- This makes welfare gains at very low levels of i relatively **'small'**

$\rightarrow$ What is meant by **'large' vs. 'small'?** The numbers can be computed from the expressions (using $A = 0.05$, $\widetilde{A} = 0.35$):

$$\textbf{Log-log specification} \quad : \quad \int_0^{i_1} \widetilde{m}(x)dx - i_1 \widetilde{m}(i_1) = A \cdot \frac{\eta}{1-\eta} i_1^{1-\eta}$$

$$\textbf{Semi-log specification} \quad : \quad \int_0^{i_1} \widetilde{m}(x)dx - i_1 \widetilde{m}(i_1) = \frac{\widetilde{A}}{\xi} \cdot [1 - e^{-\xi \cdot i_1}(1 + \xi \cdot i_1)]$$

$\rightarrow$ **Upshot:** Under the semi–log specification virtually all gains are realised if one stops at $i = 3\%$, while under the log-log specification there remain sizeable gains if one goes all the way to $i = 0\%$

III Welfare cost of inflation

Partial equilibrium estimates of welfare costs of inflation:

Lucas (2000): Criticism

→ Estimated money demand specifications of type (4) or (6) can well be satisfactory in isolation, but they lack a general equilibrium perspective

→ To use such estimates for welfare comparisons, in general, can be misleading since policy changes affect all equilibrium relationships (while the partial equilibrium approach going back to Bailey relies on the ceteris paribus assumption)

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

Lucas (2000): To address this conceptual challenge, consider as an alternative the following general equilibrium strategy:

$\longrightarrow$ Start out from the estimated money demand relationship

$$\widetilde{m} = \frac{m}{y} = A \cdot i^{-0.5} \quad \text{with: } A = 0.05 \tag{7}$$

$\longrightarrow$ Find an appropriately adjusted version of the basic MIU-model such that eqn (7) can be recovered from a first-order optimality condition
$\longrightarrow$ Solve for the steady state of this model
$\longrightarrow$ **Identify the (permanent) welfare cost associated with $i > 0$ as the percentage increase in annual steady-state consumption that would be needed to make the representative household indifferent between a steady state with $i = 0$ and $i > 0$**

[$\longrightarrow$ For **consistency** *we use from now on the* exposition in **Walsh**, *Section 2.3]*

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

Simplifications and adjustments:

1) Since the basic MIU model displays steady-state superneutrality, c and y are independent of i, π
$\rightarrow$ Simplify the set-up and consider an economy with an exogenous and constant endowment (period by period) y, yielding the resource constraint

$$y = c$$

2) For consistency with the general first-order condition of the MIU-model

$$\frac{u_m}{u_c} = \frac{i}{1+i} \tag{8}$$

replace i in eqn (7) by the alternative opportunity cost measure $\frac{i}{1+i}$

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

- As a point of departure for finding a utility function $u(c, m)$ which displays a first-order condition consistent with eqn (7) consider the general specification

$$u(c, m) = \frac{1}{1-\sigma}\{[c \cdot \varphi(\frac{m}{c})]^{1-\sigma} - 1\}, \tag{9}$$

  with $\varphi(\frac{m}{c})$ to be determined below
- Define $x \equiv \frac{m}{c}$
- For eqn (9) the general first-order condition of the MIU-model (8) becomes

$$\frac{u_m}{u_c} = \frac{\varphi\prime(x)}{\varphi(x) - x\varphi\prime(x)} = \frac{i}{1+i} \tag{10}$$

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

- Choose the function $\varphi(x)$ such that eqn (10) will be consistent with the estimated money-demand function (7)
- Accordingly, let

$$\varphi(x) = \frac{1}{1 + A^2 \cdot x^{-1}} = \frac{1}{1 + A^2 \cdot \frac{c}{m}}, \tag{11}$$

implying

$$\frac{\varphi\prime(x)}{\varphi(x) - x\varphi\prime(x)} = \frac{A^2}{(\frac{m}{c})^2} \tag{12}$$

- Combining eqns (8) and (12) yields

$$m = c \cdot A \cdot (\frac{i}{1+i})^{-0.5}, \tag{13}$$

which confirms eqn (7), since we assumed $y = c$ and replaced i by $\frac{i}{1+i}$

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

Comment:

- Using (9) and (11), the analysis will be based on

$$u(c,m) = \frac{1}{1-\sigma}\{[c \cdot \varphi(\frac{m}{c})]^{1-\sigma} - 1\}, \quad \text{with: } \varphi(\frac{m}{c}) = \frac{1}{1 + A^2 \cdot \frac{c}{m}}$$

  and not on the utility function (2), ie

$$u(c,m) = [ac^{1-b} + (1-a)m^{1-b}]^{\frac{1}{1-b}}$$

- Does this matter? **no**

  (since the utility functions are monotonic transformations of each other, leaving all welfare results established below unaffected)

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

Initial steady state (with $i \longrightarrow 0$):

- Normalize the initial steady-state consumption s.t. $c^* = 1$
- Let m^* denote the level of m yielding the highest possible level of utility if $i \longrightarrow 0$
- From eqn (13), ie $m = c \cdot A \cdot (\frac{i}{1+i})^{-0.5}$, it is clear that

$$m^* \longrightarrow \infty$$

- This implies

$$\varphi(\frac{m^*}{1}) = 1 \quad \text{and} \quad u(1, m^*) = 0$$

from the definition of φ in eqn (11) and the general utility specification (9), respectively

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

New steady state (with $i > 0$):

- Define the **welfare cost** $w(\frac{i}{1+i})$ **associated with** $i > 0$ **implicitly** via

$$u\left(1 + w(\frac{i}{1+i}),\ m(\frac{i}{1+i})\right) = u(1, m^*) = 0 \tag{14}$$

  which, using the definition of m via (13), is equivalent to

$$u\left(1 + w,\ (1+w) \cdot A \cdot (\frac{i}{1+i})^{-0.5}\right) = u(1, m^*) = 0$$

- Use eqn (9), ie $u(c, m) = \frac{1}{1-\sigma}\{[c \cdot \varphi(\frac{m}{c})]^{1-\sigma} - 1\}$, to see that this implies

$$(1+w) \cdot \varphi(\frac{m}{1+w}) = 1$$

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

New steady state (with $i > 0$):

- From the definition of φ the last step, ie $(1+w)\cdot\varphi(\frac{m}{1+w}) = 1$, is equivalent to

$$(1+w)\cdot\frac{1}{1+A^2\cdot\frac{1+w}{(1+w)\cdot A\cdot(\frac{i}{1+i})^{-0.5}}} = 1$$

which can be rearranged to obtain the desired welfare measure $w(\frac{i}{1+i})$ of the cost of inflation

$$w(\frac{i}{1+i}) = A\cdot\sqrt{\frac{i}{1+i}} \tag{15}$$

III Welfare cost of inflation

General equilibrium estimates of welfare costs of inflation:

Finding 5 *(Lucas, 2000):*

- Use the estimated value of $A = 0.05$ to calculate the welfare gains from a **permanent reduction of short-term nominal interest rates from 10% to 0%** from the expression

$$w\left(\frac{i}{1+i}\right) = A \cdot \sqrt{\frac{i}{1+i}}$$

- The **welfare gain** is $0.05 \cdot \sqrt{\frac{0.1}{1.1}} \approx 1.5\%$ of **annual steady-state consumption** (which is identical to output because of the assumption $c = y$)

IV Comments

→ **Quantitatively**, the derived **welfare costs** of inflation from a **partial** and **general equilibrium perspective** are very **similar**. This reflects the steady-state superneutrality of money in the basic MIU model.

→ **Conceptually**, Lucas' point that one needs to distinguish between partial and general equilibrium welfare measures, of course, is not affected by this finding
→ To the contrary, the simple MIU model had been chosen to show that under special and well understood assumptions the general and the partial equilibrium welfare measures can approximately coincide
→ In larger models with richer interactions such coincidence is likely to disappear (but researchers are invited to motivate their findings from transparent benchmark models, to make sure that we understand the driving forces behind their findings)

IV Comments

→ Recall from above that **Ireland (2009)** finds for **post-1980 US-data** a much improved fit of the **semi-log specification**, calling for some caution when using welfare costs from a log-log specification

→ See **Figures 1** and **2** from Ireland (2009) which extend the Figures from Lucas until 2006

→ Moreover, for about the same period FED policy was arguably implemented via an **interest rate rule** (and not, for example, a monetary aggregates policy). This improves the fit of estimated money demand equations like (4) or (6) which use the interest rate as an instrument. This reinforces the caution expressed by Ireland

Appendix: Figures

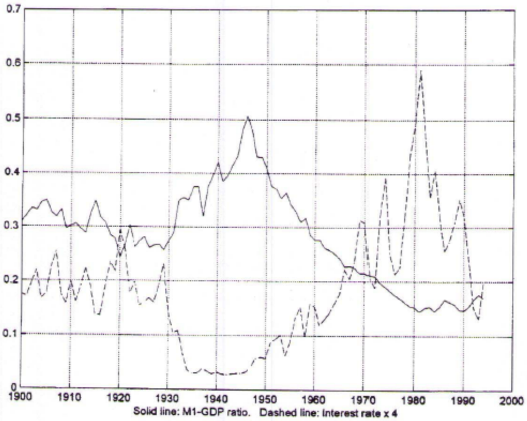

FIGURE 1.—U.S. money, income, and interest rates.

Source: Lucas, R., Inflation and welfare, *Econometrica*, 68, 2, 247-274, 2000.

Appendix: Figures

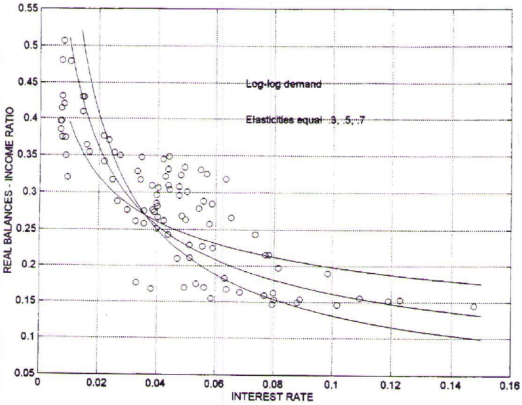

FIGURE 2.—U.S. money demand, 1900–1994.

Source: Lucas, R., Inflation and welfare, *Econometrica*, 68, 2, 247-274, 2000.

Appendix: Figures

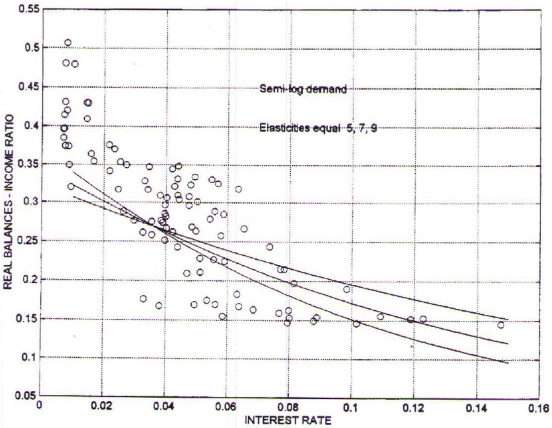

FIGURE 3.—U.S. money demand, 1900–1994.

Source: Lucas, R., Inflation and welfare, *Econometrica*, 68, 2, 247-274, 2000.

Appendix: Figures

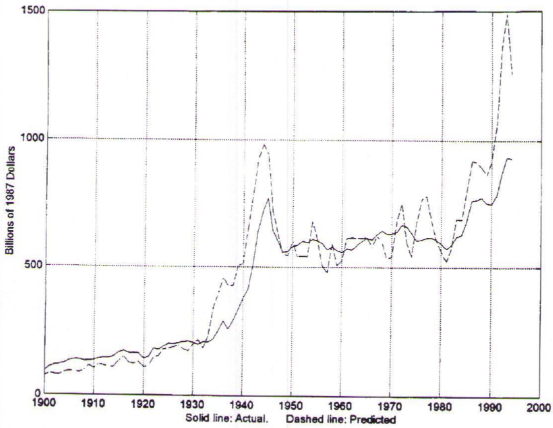

FIGURE 4.—Actual and predicted real balances, 1900–1994.

Source: Lucas, R., Inflation and welfare, *Econometrica*, 68, 2, 247-274, 2000.

Appendix: Figures

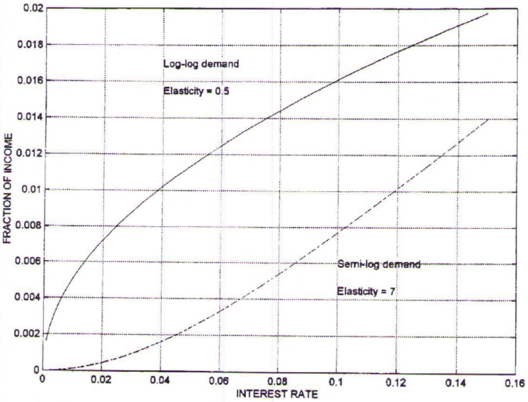

FIGURE 5.—Welfare cost functions.

Source: Lucas, R., Inflation and welfare, *Econometrica*, 68, 2, 247-274, 2000.

Appendix: Figures

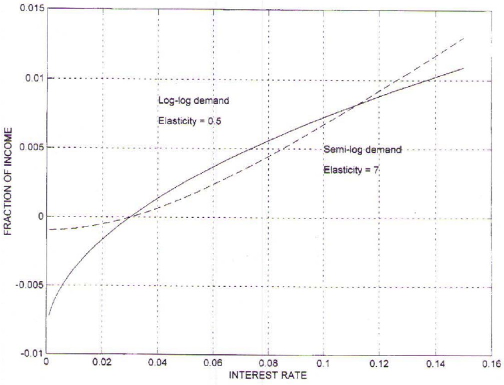

FIGURE 6.—Welfare cost relative to 3% interest.

Source: Lucas, R., Inflation and welfare, *Econometrica*, 68, 2, 247-274, 2000.

Appendix: Figures

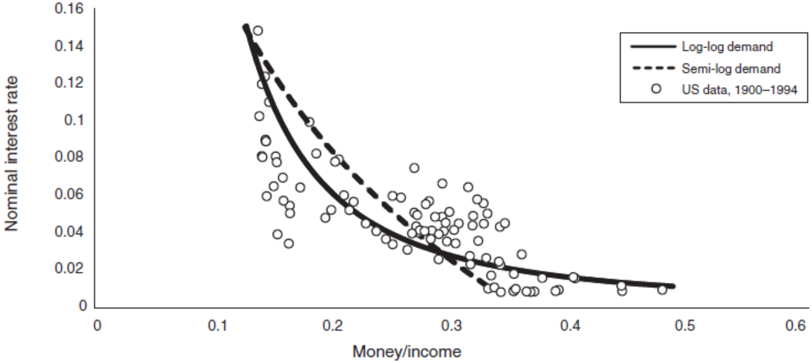

FIGURE 1. US MONEY DEMAND, 1900–1994

Source: Ireland, P. On the welfare cost of inflation and the recent behavior of money, *American Economic Review*, 99, 3, 1040-1052, 2009.

Appendix: Figures

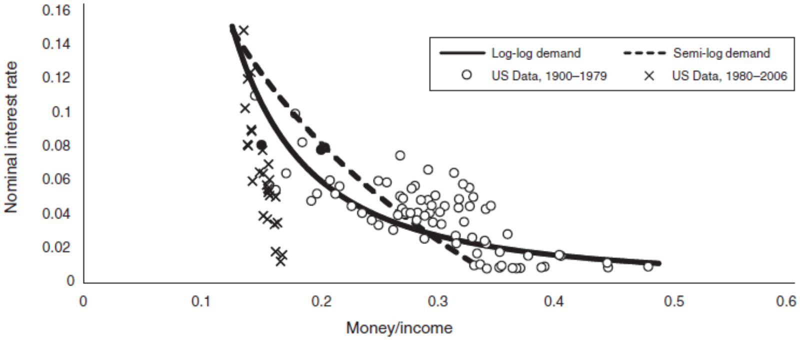

FIGURE 2. US MONEY DEMAND, 1900–2006

Source: Ireland, P. On the welfare cost of inflation and the recent behavior of money, *American Economic Review*, 99, 3, 1040-1052, 2009.