The Effects of Factor Market Integration on the Macroeconomic Development in Unified Germany

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Abstract

German reunification provides a unique quasi-natural experiment to study the integration of two economies with perfect factor mobility. The reallocation of capital and labor is the most striking characteristic of German unification and may affect the macroeconomic development in East and West Germany in the long run. Therefore, a baseline two-region open economy model is set up to study the integration of two economies with perfect factor mobility. Adjustment costs in moving production factors determine the speed of convergence and affect the long run endowment of factor inputs. An extension of the baseline model includes wage-setting behavior as well as tax and transfer policies to consider the specific German case in the early 1990s. The numerical results reveal that the extended model is capable to reproduce observed income convergence and migration pattern in unified Germany. Moreover, wage-setting behavior and transfer policies decreased the speed of income convergence in East Germany during the early years.

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1 Introduction

The advent of German reunification marked a scarce quasi-natural experiment of economic integration with perfect factor mobility. At the beginning of the 1990s, the economic and political challenges for unified Germany were tremendous. During this time nearly one fifth of Germany's population were allocated in a geographical area which was artificially hold back over 40 years from an economic system they were suddenly confronted with. The fall of the Berlin Wall led to massive capital and labor movements between East and West Germany and the world markets over the last two decades. The movement of production factors did not only affect the economic development in the new states, it rather caused a structural change in East and West Germany as indicated by Burda (2006). In this paper several important indicators of macroeconomic development in unified Germany are highlighted to motivate a simple theoretical framework which accounts for factor movements in a decentralized two-region open economy. The model allows then to analyze the consequences of factor market integration on the macroeconomic development in two integrated economies. Adjustment costs in moving production factors determine the speed of convergence and affect the long run endowment of factor inputs. The key research question then concerns about the ability of the simple growth model to reproduce important empirical pattern of East-West convergence. As the numerical results show, the baseline model is able to reproduce regional income convergence and migration pattern in unified Germany on average over the first 10-15 years. In two extensions of the baseline model, wage-setting behavior and social transfer payments are introduced to account for the specific German case. In the early 1990s, when labor union agreements increased East German wage rates to prevent mass migration and to protect the western labor force from low-wage competition (cf. Sinn (2000, p. 310)), wage convergence predominated productivity convergence and led to massive lay-offs and high unemployment rates. Consequently, migration incentives increased further and put pressure on the German government to rise social transfer payments as unemployment and retirement benefits to prevent the eastern economy from a tremendous loss of population. As the numerical results reveal, the extended model is able to reproduce income convergence and migration pattern in unified Germany on average over nearly the observable time period. The sharp decline in income and wage convergence from 2004 onwards leads to an increasing deviation between empirical and theoretical results at the very end of the time period since the model exhibits full convergence on a stable equilibrium growth path. Moreover, the analysis shows that wage-setting behavior accompanied with social transfer payments decreased the speed of income convergence in East Germany during the early years.

The paper is structured as follows: Section 2 highlights important empirical pattern on East-West convergence between 1991 and 2010 to motivate the drivers of economic growth in the theoretical model. Section 3 sets up the baseline model and analysis the

stability of the dynamic system, the transitional dynamics, and the models ability to reproduce important empirical facts. Section 4 introduces wage-setting behavior into the baseline model and analysis the transitional dynamics and implications. Section 5 introduces additionally social transfer payments into the model and shows its ability to reproduce important empirical stylized facts of East-West convergence. Section 6 summarizes and concludes.

2   Empirical Stylized Facts on East-West Convergence and Related Literature

The macroeconomic development in unified Germany is summarized in Table 1 by the time series of real and nominal GDP per capita, labor productivity, and wage rates in East Germany (without Berlin) in ratios of West German levels (without Berlin).[1] From Table 1 follows that the income per capita gap has been closed on average by 3.6% and 4.1% for real and nominal values respectively. However, income per capita convergence was fast during the first and remarkably slow within the last ten years. Between 1991 and 2000 real GDP per capita growth rates were on average roughly 5 percentage points higher in East Germany than in West Germany but decreased down to 1.8 percentage points between 2001 and 2010.[2] Nevertheless, the process is remarkable as it nearly doubles the prediction by Robert Barro (1991), where he stated that "An extrapolation of the U.S. experience to the eastern regions of unified Germany implies that per-capita growth in the East would be initially 1 1/2 to 2 percentage points per year higher than in the West." Barro (1991, p. 1). However, this treatment should not hide the fact that eastern real GDP per capita convergence slowed down significantly since 2004, which leaves the question unanswered yet, if East Germany will experience full or even just limited income convergence in the long run.

Focusing on labor productivity convergence, again, catching-up was surprisingly fast during the first decade and slowed down in the second. Between 1991 and 2000 the initial real labor productivity gap has been closed by 4.4% on average each year, while the rate fell down to 1.4% between 2001 and 2010. Moreover, real and nominal productivity convergence stagnates on a 77% level since 2004, except for a small increase in 2009. In contrast, real and nominal wage convergence is described by a fast increase of eastern wages during the early 1990s, which was followed by decreasing convergence rates already from 1995 onwards. Over the last decade from 2000 to 2010, eastern wage convergence stagnates on a $77\% - 80\%$ level of its western equivalent.[3] However, note

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[1]   Note that the Statistische Bundesamt uses a different price index for East and West Germany respectively, as the deviation between real and nominal values indicate during the first six years.

[2]   The same picture holds true for nominal values. In the first decade the nominal GDP per capita gap was closed on average by 6% and in the second decade by 1.8%.

[3]   Note that the private consumption deflator is used as a price index for real wage rates. A different measure would be the GDP deflator since the consumer price index is only available for the whole of Germany.

Table 1: East-West Convergence from 1991 to 2010

|	Real GDP per Capita	Nominal GDP per Capita	Real Labor Productivity	Nominal Labor Productivity	Real Wages	Nominal Wages
	in ratios of West German levels					
1991	0.39	0.33	0.41	0.35	0.59	0.51
1992	0.44	0.41	0.52	0.48	0.67	0.62
1993	0.52	0.50	0.61	0.59	0.70	0.69
1994	0.58	0.57	0.65	0.65	0.73	0.72
1995	0.61	0.60	0.67	0.66	0.75	0.74
1996	0.63	0.62	0.69	0.68	0.75	0.75
1997	0.63	0.63	0.70	0.69	0.75	0.75
1998	0.62	0.62	0.69	0.69	0.76	0.76
1999	0.63	0.63	0.71	0.71	0.76	0.76
2000	0.63	0.63	0.72	0.72	0.77	0.77
2001	0.63	0.63	0.73	0.74	0.77	0.77
2002	0.65	0.65	0.75	0.76	0.77	0.77
2003	0.66	0.66	0.76	0.76	0.78	0.77
2004	0.67	0.67	0.77	0.77	0.78	0.77
2005	0.67	0.67	0.77	0.77	0.78	0.78
2006	0.68	0.68	0.77	0.77	0.78	0.78
2007	0.68	0.68	0.77	0.77	0.78	0.78
2008	0.68	0.68	0.77	0.78	0.78	0.78
2009	0.70	0.70	0.79	0.79	0.80	0.80
2010	0.69	0.69	0.77	0.78	n. a.	0.79

Note: According to Brenke (2009) real wages are based on the private consumption deflator for East and West Germany respectively, since the consumer price index published by the Statistische Bundesamt is only available for the whole of Germany.

Source: Statistisches Bundesamt (2011a,b)

that Table 1 reveals the significant divergence between eastern labor productivity and wage convergence during the first decade. One explanation is that the 1:1 exchange rate of the East German Mark to the Deutsch Mark in July 1990 led to a fast wage jump right after unification, while only 20% of the East German industry was internationally competitive at this conversion rate (cf. Akerlof et al. (1991, p. 28)). In addition, eastern wage rates were increased above productivity by labor union agreements to avoid mass migration and to prevent low-wage competition in West Germany. The high-wage policy resulted into massive lay-offs and was accompanied by massive social transfer payments in the form of unemployment and retirement benefits to the eastern population. Even though that the average speed of real labor productivity convergence doubled the average speed of real wage convergence during the first decade, according to Table 1 it was not about 2004 that productivity was able to catch up with wage rates.

Figure 1 shows the effect of interregional migration on the stock of the East German labor force since 1990. In the first year, roughly 2.4% of the eastern labor force migrated to the western states, which was followed by some lower rates during the mid 1990s.

Figure 1: The East German Labor Force
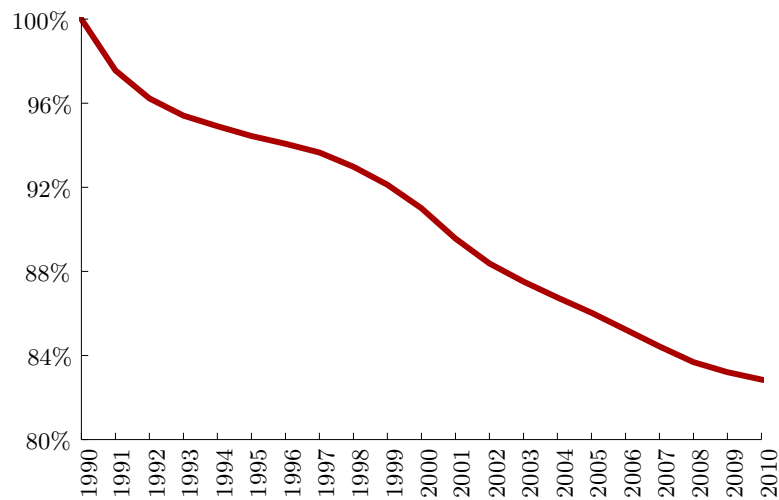
Note: The figure shows the consequences of interregional migration on the stock of the East German labor force (in % of 1990 levels) since unification. Source: Statistisches Bundesamt (2005, 2009).

This short-term decline of migration rates is a result of the high-wage policy and the social transfer payments as documented above but leaves the resulting high eastern unemployment rates undisclosed during this time. At the end of the 1990s, migration rates increased further at a time when wage convergence nearly stagnated and led to persistent migration rates in unified Germany. Between 1991 and 2010, interregional migration reduced the East German labor force by around 60,000 people on average each year. Until 2010, total net migration from East to West Germany amounted to nearly 1.2 million people, which is alarming because migration was especially concentrated among the higher educated youth (cf. Hunt (2006, p. 1027)). Nevertheless, even if the aggregate migration pattern displays a negative picture for the whole of East Germany, one should not overlook that single regions as Leipzig, Dresden, or Potsdam experienced positive net migration during the last years.

Between 1991 and 2004 private capital inflows amounted to 80-90 billion Euros on average each year (Burda 2006, p. 368). However, Sinn (2002, p. 119) shows that new capital investments have been mainly concentrated on buildings rather than on equipments in which only the latter is productive in itself and carries technological knowledge. Over the same time span, calculations by the Halle Institute for Economic Research discloses that on average around 40 billion Euros of private capital (without buildings) has been invested annually in the new German states without Berlin or equivalently 18% of regional GDP each year. Figure 2 displays the evolution of private gross capital investments (without buildings and 2000 prices) in East Germany in % of regional real GDP. Moreover, total gross investments without buildings sum up to around 535 billion Euro

Figure 2: Private Gross Capital Investments (without Buildings) in East Germany
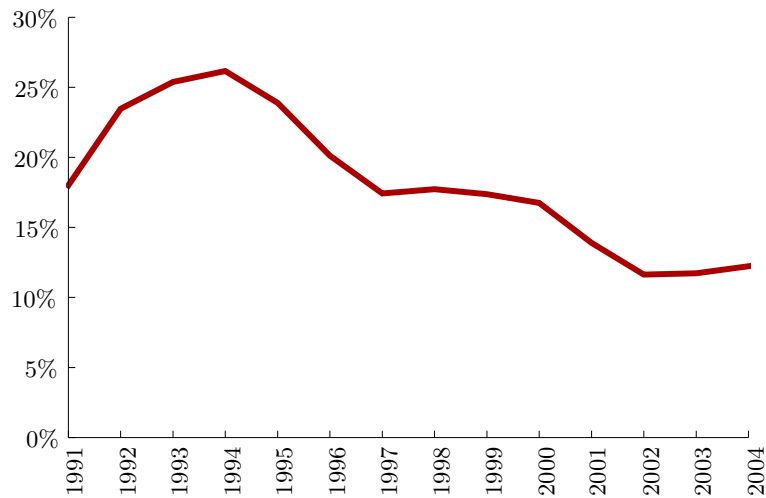
Note: The figure plots private gross capital investments without residential build-
ings in East Germany in percentage points of regional GDP. Source: Halle Institute
for Economic Research (IWH).

during the observable time period of 15 years. These massive investments are likely to
exert a first order impact on the evolution of the East German economy.

The German post-unification periods are marked by a massive reallocation of produc-
tion factors accompanied by fast income, productivity and wage convergence over the
first decade and a sharp decline of convergence rates over the second. Thereby, from to-
day's point of view it is not clear if (i) income per capita between both German regions
will fully convergence or if a persistent gap remains, as e. g. between different states
in West Germany. There are several contributions analyzing East Germany's macroe-
conomic development by presuming full convergence. For instance, Funke and Strulik
(2000) and Burda (2006) employ dynamic macroeconomic models and a selection of the
empirical stylized facts to investigate regional integration in unified Germany. How-
ever, there is only the contribution by Schäfer and Steger (2011) that offers a theoreti-
cal explanation for a possible persistent income gap in unified Germany. The authors
showed the dynamic consequences of comprehensive integration shocks when expecta-
tions matter for resulting equilibrium dynamics. (ii) High rates of East-West migration
will continue and therefore affect East Germany's future economic structure and de-
velopment. Sinn (2000) emphasizes the possibility of reversal migration when living
conditions in the source and host region are equalized. Moreover, he shows that migra-
tion pattern chosen by market forces are efficient and should not artificially reduced as
happened in unified Germany during the early 1990s. (iii) Government interventions
in form of tax and transfer policies can influence the adjustment path of the eastern
economy in the long run. In the analysis of Funke and Strulik (2000) the authors show

that actual transfers seem to be sufficient to equalize regional human wealth in East and West Germany. This paper contributes to the existing literature by introducing a basic theoretical framework of a two-region open economy model with capital adjustment and migration to analyze the general effects of factor market integration on the macroeconomic development in two integrated economies. Moreover, the baseline model is extended by wage-setting behavior and social transfer payments to evaluate the consequences of the high-wage policy on the macroeconomic development in unified Germany.

3   The Baseline Model

Consider a small open economy which comprises a rich region, west, and a poor region, east, in terms of initial capital intensity. Both regions are perfectly integrated into the world capital market and populated by a large number of identical households and firms. Capital supply and demand is perfectly elastic at a fixed interest rate $\bar{r}$. In each period $t \in [0,\infty)$ only interregional migration between east and west is permitted. Given a continuum of mass N_i, $i = E, W$ of households in both regions and denoting $M(t)$ as net migration from east to west, total population in region i at time t is given by initial population and migration until period t

$$N_W(t) \;=\; N_W(0) + \int\limits_0^t M(s)\,ds \qquad (1)$$

$$N_E(t) \;=\; N_E(0) - \int\limits_0^t M(s)\,ds \;. \qquad (2)$$

Differentiating equation (1) and (2) with respect to t yields

$$\dot{N}_W \;=\; -\dot{N}_E \;=\; M(t)\,, \qquad (3)$$

where $M(t) > 0$ implies net migration from east to west. Overall population P in the economy is assumed to be constant, such that $\bar{P} = N_W + N_E$.

Firms

There is mass one of identical firms in both regions operating under perfect competition. Each firm owns its private capital stock k_i and demands l_i units of labor to produce output y_i according to a neoclassical constant return production function. Since all firms are symmetric the aggregate output of region i reads

$$Y_i(t) \;=\; K_i(t)^\alpha \, L_i(t)^{1-\alpha}\,, \quad i = E, W\,, \quad 0 < \alpha < 1\,, \qquad (4)$$

where capital letters denote aggregate variables. The private capital stock in region i depreciates at a constant rate $\delta \geq 0$ and evolves according to

$$\dot{K}_i(t) \ = \ I_i(t) - \delta \, K_i(t) \, , \tag{5}$$

where $\dot{K}_i := dK_i/dt$. It is assumed that private capital investments induce external and convex adjustment costs according to Hayashi (1982). The representative firm in both regions maximizes the present value of its net cash flow subject to the law of motion for capital[4]

$$\max_{L_i, \, I_i} \quad \int\limits_0^\infty \left\{ K_i^\alpha \, L_i^{1-\alpha} - w_i \, L_i - I_i \left[1 + \frac{\gamma_K}{2} \left(\frac{I_i}{K_i} \right) \right] \right\} e^{-\bar{r} t} \, dt \tag{6}$$

$$s.t. \quad \dot{K}_i = I_i - \delta \, K_i \tag{7}$$

$$K_i(0) = L_{i0} \ \text{given}$$

$$L_i(0) = L_{i0} \ \text{given} \, , \qquad i = E, W \, ,$$

where $\gamma_K > 0$ measures the capital adjustment costs intensity. From the first-order necessary conditions together with (5) follows

$$w_i \ = \ (1 - \alpha) \left(\frac{K_i}{L_i} \right)^\alpha \tag{8}$$

$$\dot{K}_i \ = \ \left(\frac{q_i - 1}{\gamma_K} - \delta \right) K_i \tag{9}$$

$$\dot{q}_i \ = \ (\bar{r} + \delta) \, q_i - \alpha \left(\frac{K_i}{L_i} \right)^{\alpha - 1} - \frac{(q_i - 1)^2}{2 \gamma_K} \, , \tag{10}$$

where q_i denotes the shadow value of private capital in region i.[5] Equation (8) reveals that labor is paid its marginal product w_i and equation (9) shows the negative effect of capital adjustment costs on the evolution of the private capital stock. High values of γ_K will slow down capital accumulation and will affect the steady state capital level. The evolution of the shadow value is given by the arbitrage condition in (10), which equates the return of an additional unit of capital installed to its respective opportunity costs.

Households

The representative household in both regions maximizes utility from intertemporal consumption $c_i(t)$ subject to his budget constraint. Since all households of one region are symmetric in the sense that $c_i^m(t) = c_i^n(t)$ for $m \neq n$, aggregate consumption in

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4   The time index t is suppressed whenever no ambiguity arises.
5   The derivatives are shown in Appendix A.1.

region i is given by

$$C_i(t) \quad = \quad N_i(t)\, c_i(t)\,, \quad i = E, W\,. \tag{11}$$

Moreover, each household supplies labor inelastically to the labor market. Let $\ell_i(t)$ denote the employment rate of the representative household in region i at time t, such that aggregate labor supply is given by

$$L_i(t) \quad = \quad N_i(t)\, \ell_i(t)\,, \quad i = E, W\,. \tag{12}$$

Following Braun (1993), interregional migration is subject to migration costs mc_i, which accrue in terms of consumable goods by the migrant at the period of leaving his region. These costs are assumed to be an increasing function in the number of migrants $M(t)$ of the same period and therefore depend on the flow of migrants and not on the stock of formerly migrated individuals. This relationship reflects transaction costs in the process of moving, for instance, increasing transportation costs or agent's commission for housing in the host region. Another interpretation might be social costs in form of public opposition in the host region against massive immigration. The migration costs are explicitly given by

$$mc_i \quad = \quad \gamma_L M(t)\,, \quad i = E, W\,, \tag{13}$$

where $\gamma_L > 0$ denotes the migration cost intensity parameter. Thus, the household's budget constraint depends on the fixed interest rate $\bar{r}$, labor income, and consumption but differs in case of a resident and migrant with respect to migration costs

$$\dot{a}_i(t) \quad = \quad \bar{r}\, a_i(t) + w_i(t)\, \ell_i - c_i(t) \quad \text{(resident)} \tag{14}$$

$$\dot{a}_i(t) \quad = \quad \bar{r}\, a_i(t) + w_j(t)\, \ell_j - c_i(t) - mc_i \quad \text{(migrant)}\,, \tag{15}$$

where savings are devoted to the acquisition of new assets $\dot{a}_i := da_i/dt$ and $i, j = E, W, \quad i \neq j$.

The maximization problem of an household in region i is summarized by

$$\max_{c_i(s)} \quad U_i(t) = \int_t^\infty \frac{c_i(s)^{1-\sigma} - 1}{1 - \sigma} \, e^{-\rho \, [s-t]} \, ds \tag{16}$$

$$s.\,t. \quad \begin{cases} \displaystyle\int_t^\infty e^{-\bar{r}\,[s-t]} c_i(s) \, ds = a_i(t) + \int_t^\infty e^{-\bar{r}\,[s-t]} w_i(s) \, \ell_i(s) \, ds \quad \text{(resident)} \\[2em] \displaystyle\int_t^\infty e^{-\bar{r}\,[s-t]} c_i(s) \, ds = a_i(t) + \int_t^\infty e^{-\bar{r}\,[s-t]} w_j(s) \, \ell_j(s) \, ds - mc_i \quad \text{(migrant)} \end{cases} \tag{17}$$

$$a_i(t) = a_{it} \ \text{given}\,, \qquad\qquad i,j = E,W, \quad i \neq j \,,$$

where $\rho > 0$ denotes the time-preference rate, and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution.[6] From the maximization problem it becomes clear that the difference between a resident and migrant is given by the present value labor income differential and the migration costs. The first-oder necessary conditions deliver the well-known Keynes-Ramsey rule

$$\frac{\dot{c}_i}{c_i} \;=\; \frac{1}{\sigma} \left(\bar{r} - \rho \right) \,, \quad i = E,W \,, \tag{18}$$

which is independent of the migration decision. Equation (18) reveals that given a fixed interest rate, individual's consumption growth rate is constant in all periods t. Moreover, the knife-edge condition of the model requires equality of the interest rate $\bar{r}$ and the time-preference rate ρ, such that consumption growth rate is zero and implies a constant individual consumption level $\bar{c}_i$.[7] The level of consumption is obtained by using the budget constraints in (14) and (15) and reads

$$\bar{c}_i \;=\; \rho \left(a_i(t) + \int_t^\infty e^{-\bar{r}\,[s-t]} w_i(s) \, \ell_i(s) \, ds \right) \tag{19}$$

for residents as well as

$$\bar{c}_i \;=\; \rho \left(a_i(t) + \int_t^\infty e^{-\bar{r}\,[s-t]} w_j(s) \, \ell_j(s) \, ds - mc_i \right) \tag{20}$$

for migrants. Thus, consumption is proportional to initial and future wealth minus migration cost if the household decides to migrate.

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[6] The budget constraints are written in intertemporal form to emphasize the difference between migrants and residents. The intertemporal form is derived from equation (14) and (15) using the no-Ponzi game condition $\lim_{T \to \infty} a(T) e^{-\bar{r}\,[T-t]} = 0$. The derivation is shown in Appendix A.2.

[7] The knife-edge condition of the Open-Economy Ramsey model are extensively discussed e.g. in Barro and Sala-i Martin (2004, pp. 161).

Migration

As shown above, the only difference between a migrant and a resident regardless of region is given by the difference in the present value of labor income as well as migration costs. Thus, let $\lambda_j(t)$ denote the migration benefit of a migrant from the source region i at time $t \in [0,\infty)$, i.e.

$$\lambda_j(t) \;=\; \int_t^\infty \left[w_j(s)\,\ell_j(s) - w_i(s)\,\ell_i(s) \right] e^{-\bar{r}\,[s-t]} ds\,, \quad i,j = E, W, \quad i \neq j\,. \tag{21}$$

Then, households of region i are willing to migrate in period t if and only if their migration benefit is positive and outweighs the migration costs. In a competitive equilibrium costs must equal benefits such that

$$\lambda_j(t) \;=\; mc_i\,, \quad \forall\, t,\quad i,j = E, W,\ i \neq j\,. \tag{22}$$

Regarding equation (21) there are two points that are noteworthy: First, the equation implies perfect foresight since the labor income differential depends on the future amount of capital and labor employed in both regions. Second, a positive migration benefit for households of region i, implies the inverse for households of the other region j, i.e.

$$\lambda_j(t) \;=\; -\lambda_i(t)\,, \quad \forall\, t,\quad i,j = E, W,\ i \neq j\,. \tag{23}$$

An initial income differential between both regions implies a positive migration benefit for households from the region with less labor income. Throughout the transition process the positive benefit declines due to migration and capital investments until full convergence of labor income is achieved. In equilibrium the migration benefit is zero. From equation (15) in intertemporal form together with (21) and (22) it follows directly that a migrant faces the identical budget constraint as a resident and therefore chooses exactly the same optimal consumption level given in equation (19). Therefore, individual wealth is unaffected by migration and we can not distinguish between households of the same origin.[8]

At the aggregate level, migration decreases total consumption in the source region and leaves per capita consumption constant and equal to the optimal consumption level given in (19). However, total consumption in the host region increases due to migration, while per capita consumption is affected by the difference in the optimal consumption level between both regions. To see the latter, let $C_h(t)$ denote aggregate consumption in

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[8] However, the optimal consumption level might be different between eastern and western households, since initial wealth $a_i(t)$ can deviate between both regions. Moreover, note that migration lowers labor productivity in the host region and increases labor productivity in the source region, i.e. there is an externality on non-movers. This externality is identical for all remaining households such that symmetry is still guaranteed.

the host region at time t which is composed by total consumption of native households plus total consumption of migrants

$$C_h(t) = \bar{c}_n N_h(0) + \bar{c}_s \int_0^t M(s)\,ds\,, \tag{24}$$

where $\bar{c}_n$ and $\bar{c}_s$ denote the constant individual consumption levels of natives and migrants, respectively. Dividing both sides of equation (24) by the population level in the host region at time t and differentiating with respect to time yields the evolution of per capita consumption in the host region[9]

$$\dot{c}_h = -[c_h - c_s]\frac{M}{N_h(t)}\,. \tag{25}$$

As indicated above, the east is considered as the poor region in terms of initial capital intensity. Consequently, the initial benefit of migration for an eastern resident is positive and given by

$$\lambda_W(t) = \int_t^\infty [w_W(s)\,\ell_W(s) - w_E(s)\,\ell_E(s)]\,e^{-\bar{r}\,[s-t]}ds\,>\,0\,. \tag{26}$$

Consider the equilibrium condition in (22) and substitute equation (13) and (26) to get the evolution of the eastern population size

$$\dot{N}_E = -\gamma_L^{-1}\lambda_W\,, \tag{27}$$

where equation (3) is used. Positive values of λ_W imply east-west net migration. The lower the parameter γ_L, the lower population mobility and vice versa.

3.1 Dynamic System and Stability Analysis

The dynamics of the small open economy are described by a six-dimensional differential equation system

$$\dot{K}_E = \left(\frac{q_E - 1}{\gamma_K} - \delta\right)K_E \tag{28}$$

$$\dot{K}_W = \left(\frac{q_W - 1}{\gamma_K} - \delta\right)K_W \tag{29}$$

$$\dot{q}_E = (\bar{r} + \delta)\,q_E - \alpha\left(\frac{K_E}{N_E\ell_E}\right)^{\alpha-1} - \frac{(q_E - 1)^2}{2\gamma_K} \tag{30}$$

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[9] The derivation is shown in Appendix A.3.

$$\dot{q}_W = (\bar{r} + \delta)\, q_W - \alpha \left(\frac{K_W}{(P - N_E)\ell_W} \right)^{\alpha - 1} - \frac{(q_W - 1)^2}{2\gamma_K} \tag{31}$$

$$\dot{N}_E = -\gamma_L^{-1} \lambda_W \tag{32}$$

$$\dot{\lambda}_W = \bar{r}\lambda_W - (w_W \ell_W - w_E \ell_E) \ , \tag{33}$$

where $N_W = P - N_E$. The evolution of the eastern and western capital stock are characterized by the equations (28)-(31). Interregional migration is described by equation (32) and (33), where the latter is derived by differentiating (26) with respect to time t. The wage rates are given by equation (8).

The long run equilibrium of the dynamic system is defined by the constancy of the three state variables, namely $\bar{K}_E$, $\bar{K}_W$, and $\bar{N}_E$, and the three costate variables, namely $\bar{q}_E$, $\bar{q}_W$, and $\bar{\lambda}_W$. According to equation (32), constant population shares in both regions are only given if $\lambda_W = 0$. Thus, labor income in equation (33) must be equalized in equilibrium and therewith capital intensity.[10] From (28) and (29) follows the steady state value for the shadow prices of capital

$$\bar{q}_E = \bar{q}_W = 1 + \delta\gamma_K \ . \tag{34}$$

Substituting (34) into (30) and (31) yields the steady state capital stock in east and west

$$\bar{K}_i = \left(\frac{\alpha}{(\bar{r} + \delta)(1 + \gamma_K \delta) - \frac{\gamma_K \delta^2}{2}} \right)^{\frac{1}{1-\alpha}} \bar{N}_i \ell_i \ , \quad i = E, W \ . \tag{35}$$

For a stability analysis of the dynamic system the six equations (28)-(33) are linearized by means of a first-order Taylor approximation and written in matrix form to get[11]

$$
\begin{pmatrix} \dot{q}_E \\ \dot{q}_W \\ \dot{\lambda}_W \\ \dot{K}_E \\ \dot{K}_W \\ \dot{N}_E \end{pmatrix}
=
\underbrace{
\begin{pmatrix}
\bar{r} & 0 & 0 & -F_{KK}^E & 0 & -F_{KL}^E \\
0 & \bar{r} & 0 & 0 & -F_{KK}^W & -F_{KL}^W \\
0 & 0 & \bar{r} & F_{LK}^E & -F_{LK}^W & F_{LL}^E - F_{LL}^W \\
\bar{K}_E \gamma_K^{-1} & 0 & 0 & 0 & 0 & 0 \\
0 & \bar{K}_W \gamma_K^{-1} & 0 & 0 & 0 & 0 \\
0 & 0 & -\gamma_L^{-1} & 0 & 0 & 0
\end{pmatrix}
}_{M}
\begin{pmatrix} q_E - \bar{q}_E \\ q_W - \bar{q}_W \\ \lambda_E - \bar{\lambda}_W \\ K_E - \bar{K}_E \\ K_W - \bar{K}_W \\ N_E - \bar{N}_E \end{pmatrix}
\tag{36}
$$

Without loss of generality the employment rates ℓ_i, $i = E, W$ are set to unity to simplify expressions in this section, i. e. $L_i = N_i$, $i = E, W$. The analytic expression for the six eigenvalues of matrix M are given by[12]

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[10] Moreover, employment rates are equalized in steady state, i. e. $\ell_W = \ell_E$.

[11] To simplify notations, let F_{jk}^i denote the second derivative of the production function of region $i = E, W$ with respect to $j, k = K_i, L_i$. The derivations are given in Appendix A.4.

[12] The derivation of the eigenvalues is shown in Appendix A.5

$$\{\lambda_1,\ldots,\lambda_6\} = \begin{Bmatrix} 0, & \bar{r}, \\ \frac{1}{2}\left(\bar{r}+\sqrt{\bar{r}^2+2(A-\sqrt{A^2-4B})}\right), & \frac{1}{2}\left(\bar{r}+\sqrt{\bar{r}^2+2(A+\sqrt{A^2-4B})}\right) \\ \frac{1}{2}\left(\bar{r}-\sqrt{\bar{r}^2+2(A+\sqrt{A^2-4B})}\right), & \frac{1}{2}\left(\bar{r}-\sqrt{\bar{r}^2+2(A-\sqrt{A^2-4B})}\right) \end{Bmatrix} . \quad (37)$$

For non-negative values of the inner discriminant, A^2-4B, of the eigenvalues $\lambda_3,\ldots,\lambda_6$ the model implies meaningful solutions. Analog to Burda (2006), the model exhibits three strictly positive ($\lambda_2,\lambda_3,\lambda_4$) and two strictly negative eigenvalues (λ_5,λ_6). The constellation corresponds to a perfect foresight model with saddle-path stability as well as path dependency or hysteresis since one eigenvalue is zero. Moreover, in the long run the model's speed of convergence is determined by the smallest absolute negative eigenvalue λ_6.

3.2 Calibration Strategy and Solution Technique

The model parameters are specified to meet the empirical regularities of Germany's post-unification periods. The capital and labor elasticities of production are approximated by the average labor income share in unified Germany. Over the period 1991 - 2009 the average empirical value ranges between 0.7 and 0.78 according to the degree of considering self-employed income. The value of 0.75 is chosen, which is in line with the empirical literature on German data.[13] Thus, from the constant returns to scale assumption follows the capital elasticity of production $\alpha = 0.25$. The interest rate is set to $\bar{r} = 0.03$ and the time-preference rate correspondingly.[14]

For the migration cost intensity, γ_L, equation (32) is used together with the empirical observations in section 2.[15] In a first step, the migration benefit $\lambda_W(t)$ is calculated for an East German resident in 1991. Therefore, consider equation (26) and assume that the western wage rate is constant in each period, such that $w_W(s) = \bar{w}_W \forall s$, to get

$$\lambda_E(0) = \int_0^\infty [w_E(s) - \bar{w}_W]\, e^{-\bar{r}s} ds \,, \quad (38)$$

where $t = 0$ corresponds to 1991 in real-time and employment rates are set to unity.[16]

---

[13] Scheufele (2008) found the same value for the structural macroeconometric German Economy model by the Halle Institute for Economic Research. Other findings as the survey by the Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung (2007) estimated a value of 0.78 for the production elasticity of labor over the period 1970 - 2007, which includes observations of German dispartment. Willman (2002) estimated a value of around 0.7 over the period 1971 - 1997.

[14] The interest rate coincides with the real long-run risk free rate on treasury bills estimated by Mehra and Prescott (1985).

[15] The procedure basically follows Burda (2006).

[16] Remember that $\lambda_W(t) = -\lambda_E(t)$.

The eastern wage rate w_E equates labor productivity and evolves according to equation (8). Assume that productivity convergence between East and West Germany takes place at a constant rate κ.[17] Then, for a given initial and steady state value the following approximation holds

$$w_E(s) - \bar{w}_W \;\cong\; [w_E(0) - \bar{w}_W]\, e^{-\kappa s}, \; s \geq 0 \, . \tag{39}$$

By normalizing the western wage rate to unity, the time series of real labor productivity convergence in Table 1 is used to back out a value for κ

$$\kappa \;=\; \frac{\ln\left(\frac{w_E(s) - \bar{w}_W}{w_E(0) - \bar{w}_W}\right)}{s} \;=\; \frac{-\ln\left(\frac{-0.23}{-0.59}\right)}{19} \;\cong\; 0.05 \, . \tag{40}$$

Finally, substitute equation (39) into (38) and use the initial productivity gap of roughly 59% to get a value for the migration benefit in 1991[18]

$$\lambda_E(0) \;=\; \int_0^\infty [w_E(0) - \bar{w}_W]\, e^{-0.03\,s} e^{-0.05\,s} ds \;=\; \frac{-0.59}{0.08} \;\cong\; -7.375 \, . \tag{41}$$

For the last step, note that East Germany's population size accounted for roughly 20% of overall Germany in 1991. Therefore, the eastern population share N_E is normalized to unity, whereas overall population is set to $P = 5$. In the same year about 2.4% of the East German labor force migrated to West Germany. Given equation (32) the value for γ_L is given by

$$\gamma_L \;=\; \frac{\lambda_W}{-\dot{N}_E} \;=\; \frac{7.375}{-(-0.024)} \;=\; 307.29 \, . \tag{42}$$

For γ_K, the capital adjustment costs intensity, the steady state condition for the shadow price of capital given in (34) is used. Over the period 1994 - 2005, Dittmann et al. (2010) estimated an average shadow value of capital for German firms of $q = 1.54$ by using Tobin's Q approach. Moreover, the average private capital depreciation rate in Germany was roughly 5% over the period 1991 - 2008, such that γ_K is approximated by

$$\gamma_K \;=\; \frac{1 - \bar{q}}{\delta_K} \;=\; \frac{0.54}{0.05} \;=\; 10.8 \, . \tag{43}$$

The baseline set of parameters are listed in Table 2. The frictions γ_L and γ_K are the key determinants of slowing migration and capital adjustments in both regions, such that capital intensity do not instantaneously converge. For further analysis of the transitional dynamics in the described economy, the non-linear differential equation system

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[17] Note that the model's real convergence rate is not constant since the western wage rate is endogenously determined in each period and the dynamic system is of non-linear form.

[18] Note that our initial benefit for an East German resident is higher compared to the results in Burda (2006). His corresponding value is $\lambda_E(0) = -5$ compared to $\lambda_E(0) = -7.375$.

Table 2: Baseline Set of Parameter Values

Parameter	Value	Comment
α	0.25	Capital Elasticity of Production
$\bar{r}$	0.03	World Interest Rate
ρ	0.03	Time-preference Rate
δ	0.05	Private Capital Depreciation Rate
s	19	Periods of Observation
κ	0.05	Wage Convergence Rate
γ_L	307.29	Migration Cost Intensity
γ_K	10.8	Capital Adjustment Cost Intensity
P	5	Total Population

is solved by applying the relaxation algorithm introduced by Trimborn et al. (2008).[19]

3.3   Transitional Dynamics of the Baseline Model

Notice that the baseline model converges on a three-dimensional stable manifold. Moreover, given the set of parameters in Table 2 the eigenvalue constellation corresponds to a perfect foresight model with path dependency. For a simulation of the model's transition, the initial conditions of the three state variables K_E, K_W, and N_E need to be specified. Assume that West Germany developed on his balanced growth path before unification, then the initial value for K_W is given by equation (35) and the baseline set of parameters. Since the east's population size is already normalized to unity, the initial eastern capital stock K_E is left to be determined. Note that East Germany's real labor productivity was roughly 41% of West German levels in 1991. Given equation (8) and the initial value of the western capital stock, the initial value for K_E is given by[20]

$$K_E(1991) \;\; = \;\; \left[\frac{0.41\, w_W(1991)}{1-\alpha} \right]^{\frac{1}{\alpha}} \;\; \leq \;\; K_W(1991)\,, \qquad (44)$$

Table 3 lists the initial and steady state values of all variables. Given the calculated initial values of the state variables, the associated initial values of the three co-state variables are identified by the relaxation algorithm. The equilibrium endowments of capital and labor in east and west are shown in the third column of Table 3. Since both regions converge to the same capital intensity, but do not take on exactly the same value, the steady state of the economy is defined as an income per capita gap between

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[19]   Details regarding the code are available upon request.
[20]   See Akerlof et al. (1991) for an inspiring analysis of microeconomic data on East German firms viability after reunification.

Table 3: Initial & Steady State Values of the Baseline Model

Variables	Initial Value	Steady State Value	Comment
K_W	12	12.47	Capital Stock (West)
K_E	0.08	2.51	Capital Stock (East)
N_E	1	0.84	Eastern Population Size
q_W	1.55	1.54	Capital Shadow Value (West)
q_E	10.32	1.54	Capital Shadow Value (East)
λ_W	4.31	0.01	Migration Benefit

regions by less than 0.1%.[21]

The time path of the endogenous variables as well as the evolution of wages and income per capita are displayed in Figure 3. The blue solid lines show the equilibrium growth path of the eastern and the purple dashed lines of the western variables. Starting from the initial values at time zero the dynamic transition sets in at period one.[22] The panels in the first two rows display the evolution of three state and co-state variables. Panel (a) and (d) show the eastern capital stock and its associated shadow value. A relatively low initial capital endowment implies capital investment incentives, since the marginal product and therewith the shadow value are initially above their steady state levels. With each unit of capital installed the marginal product declines, such that the eastern capital stock increases over time with a decreasing rate until the steady state is achieved. Moreover, panel (g) reveals that the initial eastern wage rate is lower compared to its western equivalent, which induces east-west migration. Panel (f) shows the positive but decreasing migration benefit, while panel (c) displays the decreasing eastern population size until capital intensities are equalized and with it wage rates. In the steady state the migration benefit is zero. Since east-west migration increases the western population size, the marginal product of the western capital stock increases with each migrant. Thus, capital investment incentives are expressed by the positive deviation of the associated shadow price in panel (e). As panel (b) points out, the western capital stock increases over time until the steady state is achieved. Repercussion effects of regional integration are illustrated in panel (g) and (h). Wages rates and income per capita in the western region slightly decrease during the first periods since the negative effect of migration on capital intensity can not be fully compensated by capital investments right from the start.

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[21] Note that the initial migration benefit λ_W identified by the algorithm deviates significantly from the calibrated value. The deviation stems from the constant rate of wage convergence assumption in equation (39) to simplify calibration. However, the model presented here is of non-linear form, i. e. the convergence rate is not constant over all periods rather the rate is high at the beginning and slows down as the model converges to its steady state.

[22] Previous periods on the negative x-axis are associated with the initial value of the respective variable to highlight the economy's take off.

Figure 3: Transitional Dynamics of the Baseline Model
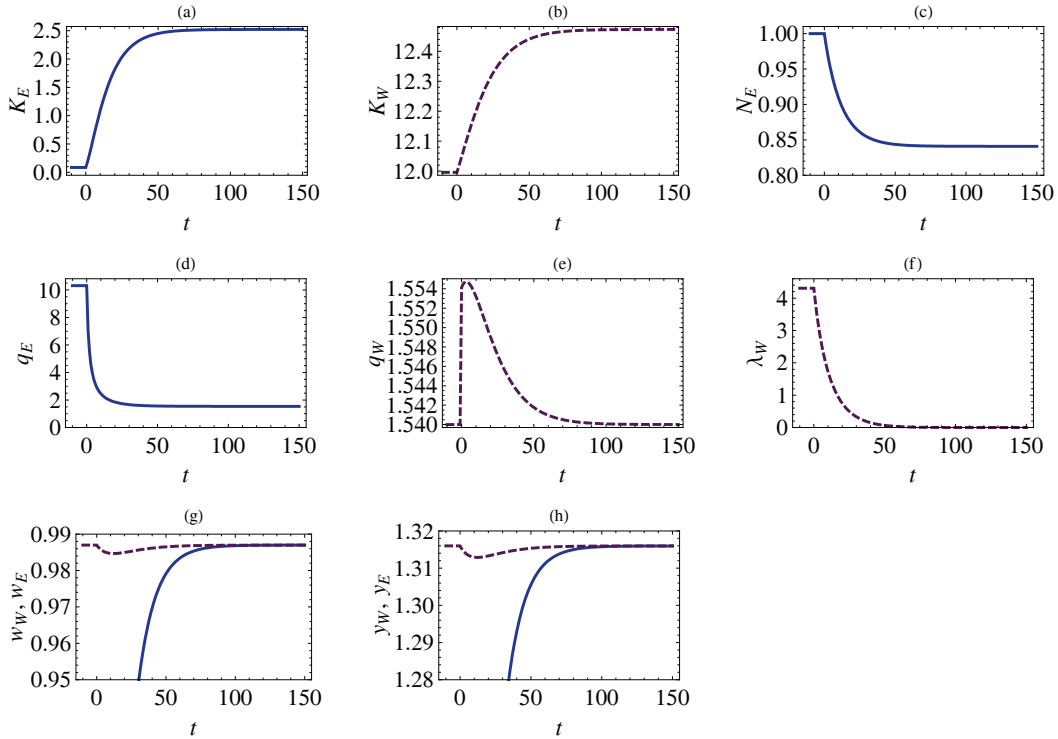
Note: The blue solid lines plot the trajectories of the eastern and the purple dashed lines of the western variables respectively.

3.4 Results

In a final step of this section the model is taken to the data to evaluate its ability to reproduce some important empirical pattern on East-West convergence since 1991. The red solid lines in Figure 4 represent the empirical time series, while the dashed lines show the theoretical results. As panel (a) reveals, the baseline model can very good reproduce real labor productivity convergence on average over the first 10 years after unification. However, from 2004 onwards the deviation between empirical and theoretical results increases over time due to the indicated stagnation of productivity convergence and the model's underlying assumption of full income convergence. Regarding the observed migration pattern in unified Germany, panel (b) indicates that the baseline model offers a good approximation roughly for the first 10 years after reunification. From 2000 onwards the empirical migration rates are significantly higher compared to the model's prediction. However, the widening gap is consistent with the results above. As capital intensity fully converges to western levels over time and therewith labor productivity and wage rates, the theoretical migration benefit $\lambda_W(t)$ decreases and slows down east-west migration.

As Table 1 already indicated, eastern real wage convergence predominated productiv-

Figure 4: Evidence and Theoretical Results for the Baseline Model
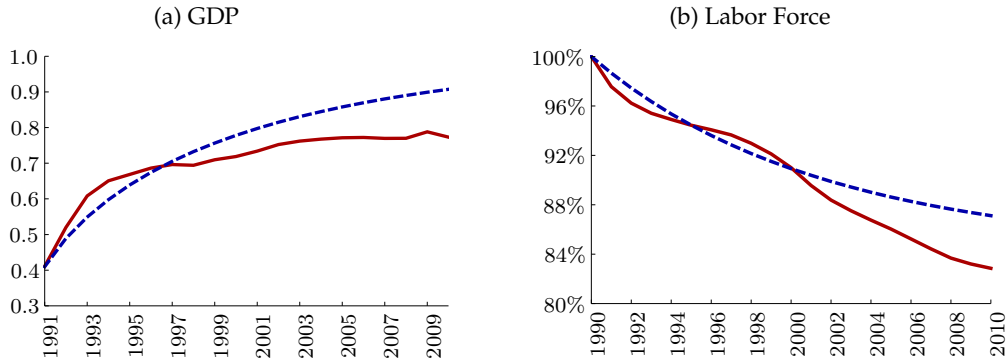
(a) GDP

(b) Labor Force

Note: The blue dashed line represents theoretical results and the red solid line empirical evidence. Panel (a) compares the models prediction of real GDP per person employed convergence in East Germany since unification in comparison to the empirical evidence given in Table 1 (in ratios of West German levels). Panel (b) shows the observed and predicted evolution of the East German labor force with respect to interregional migration.

ity convergence particularly in the early 1990s. According to (Sinn 2000, p. 310) the high-wage policy was motivated to prevent mass migration and to protect the western labor force from possible low-wage competition. However, the policy resulted in massive lay-offs and forced the German government to increase social transfer payments as unemployment and retirement benefits to prevent further migration. This specific aspect of German unification is not covered by the baseline model. Extending the model by wage-setting behavior and social transfers in the following sections makes it possible to study the consequences of the high-wage and transfer policy in unified Germany.

4   Introducing Wage-Setting Behavior

When introducing wage-setting behavior into the neoclassical framework it is obvious that labor markets are not necessarily cleared. In the following extension of the baseline model labor is still paid its marginal product according to equation (8), but the average employment rate in the eastern region ℓ_E varies according to the wage-setting behavior. This assumption seems reasonable in a representative agent framework since the considered agent represents the average household of the respective region. Let $\tilde{w}_E$ denote the eastern wage rate imposed by a behavioral function, where $\tilde{w}_E > w_E$. Moreover, assume that there is full employment normalized to unity in the western region for native households and immigrants at any point in time, such that $\ell_W(t) = 1 \, \forall \, t$, and therefore $L_W = N_W$ and $L_E = N_E \ell_E$. Thus, during transition it holds that $\ell_E < 1$, which therefore reflects underemployment in the eastern region relative to western lev-

els. The migration benefit for an eastern household is then given by

$$\lambda_W(t) = \int_t^\infty [w_W(s) - \tilde{w}_E(s)\ell_E(s)] e^{-\bar{r}[s-t]} ds.$$ (45)

According to Funke and Strulik (2000) the evolution of the eastern wage rate $\tilde{w}_E$ is assumed to be a function of its western equivalent and takes the form of

$$\tilde{w}_E(t) = \theta^\beta w_W(t), \qquad 0 < \beta < 1,$$ (46)

where $\theta = y_E/y_W$ denotes the east's relative GDP per person employed with $y_E = (K_E/N_E)^\alpha \ell_E^{1-\alpha}$ and $y_W = (K_W/(P-N_E))^\alpha$. For $\theta < 1$ the wage-setting behavior in (46) implies that $\tilde{w}_E$ outweighs market wages w_E until capital intensities are equalized. Dividing both sides of (46) by the western wage rate at time t and normalizing $w_W(t)$ and $y_W(t)$ to unity, the empirical results in Table 1 are used to back out $\beta \cong 0.8$.[23] Finally, the evolution of the eastern employment rate ℓ_E over time is specified. According to the assumption that the aggregate employment rate adjusts itself in a way such that wages still equal marginal product it must hold that

$$\tilde{w}_E = (1-\alpha) \left(\frac{K_E}{N_E \ell_E} \right)^\alpha.$$ (47)

Substituting equation (46) into (47) and solving for ℓ_E yields the equilibrium eastern employment rate at each point in time

$$\ell_E^*(t) = \left(\frac{\frac{K_E(t)}{N_E(t)}}{\frac{K_W(t)}{P-N_E(t)}} \right)^{\frac{\alpha-\alpha\beta}{(1-\alpha)\beta+\alpha}},$$ (48)

where the time path only depends on the evolution of the three state variables. Plugging $\ell_E^*(t)$ back into (47) yields the eastern wage rate depending on capital and labor employed in both regions

$$\tilde{w}_E(\ell_E^*) = (1-\alpha) \left(\frac{K_E}{N_E} \right)^{\frac{\alpha\beta}{(1-\alpha)\beta+\alpha}} \left(\frac{K_W}{P-N_E} \right)^{-\frac{\alpha^2(\beta-1)}{(1-\alpha)\beta+\alpha}}.$$ (49)

From equation (48) follows that $\bar{\ell}_E = 1$ in steady state when capital intensities are equalized and therefore $\bar{\theta} = 1$ and $\tilde{w}_E = \bar{w}_W$. The dynamic system of equation is then

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[23] The approximation implies $\beta = \ln\left(\frac{\tilde{w}_E}{w_W}\right)/\ln\left(\frac{y_E}{y_W}\right)$. The average values are chosen such that $\tilde{w}_E$ can trace on average the convergence path of the relative East German real wage rate between 1991 and 2009.

given by

$$\dot{K}_E = \left(\frac{q_E - 1}{\gamma_K} - \delta \right) K_E \tag{50}$$

$$\dot{K}_W = \left(\frac{q_W - 1}{\gamma_K} - \delta \right) K_W \tag{51}$$

$$\dot{q}_E = (\bar{r} + \delta) q_E - \alpha \left(\frac{K_E}{N_E \ell_E^*} \right)^{\alpha - 1} - \frac{(q_E - 1)^2}{2\gamma_K} \tag{52}$$

$$\dot{q}_W = (\bar{r} + \delta) q_W - \alpha \left(\frac{K_W}{P - N_E} \right)^{\alpha - 1} - \frac{(q_W - 1)^2}{2\gamma_K} \tag{53}$$

$$\dot{N}_E = -\gamma_L^{-1} \lambda_W \tag{54}$$

$$\dot{\lambda}_W = \bar{r}\lambda_W - (w_W - \tilde{w}_E(\ell_E^*)\ell_E^*) \ , \tag{55}$$

where the western wage rate evolves according to (8). For the computation of the transitional dynamics the baseline set of parameters given in Table 2 are taken except for the values specifying γ_L. Now, the eastern wage rate predominates GDP per person employed during transition, such that labor income $\tilde{w}_E\ell_E$ is approximated by the time series of real wage convergence in Table 1. The corresponding values are now given by $\kappa = 0.04$, $\lambda_E(0) = -5.86$ and $\gamma_L = 244.05$. Note that the constant rate of wage convergence and the initial migration benefit are lower compared to the baseline scenario which yields a lower migration cost intensity. The initial eastern capital endowment $K_E(0)$ is chosen such that the initial real GDP per person employed gap is equal to the observed disparity of 41% between East and West Germany in 1991.

Figure 5 plots the transitional dynamics of the model with wage-setting behavior. In comparison to the baseline scenario the eastern region experiences higher migration rates, panel (c), and lower capital investment incentives, panel (d). The former is due to the higher migration benefit $\lambda_W(t)$ along the transition, as shown in panel (f). The blue solid line in panel (g) plots the trajectory of the eastern wage rate in the absence of wage-setting behavior to highlight the difference to the high-wage policy represented by the the red solid line. Even that wage-setting behavior increased eastern wage rates, labor income $\tilde{w}_E\ell_E$ is lower compared to the market wage scenario. This is because the elasticity of labor demand is greater than unity for a Cobb-Douglas production function, which yields that $\tilde{w}_E\ell_E < w_E$ and results into higher rates of east-west migration. With each migrant the marginal product of eastern capital increases and leads to a lower shadow value for capital and within lower capital investment rates in the eastern region. The initial employment rate is roughly 83% and converges to full employment as capital intensity equalizes between both regions.

Again, the model is taken to the data to evaluate its ability to reproduce empirical ev-

Figure 5: Transitional Dynamics of the Model with Wage-Setting Behavior
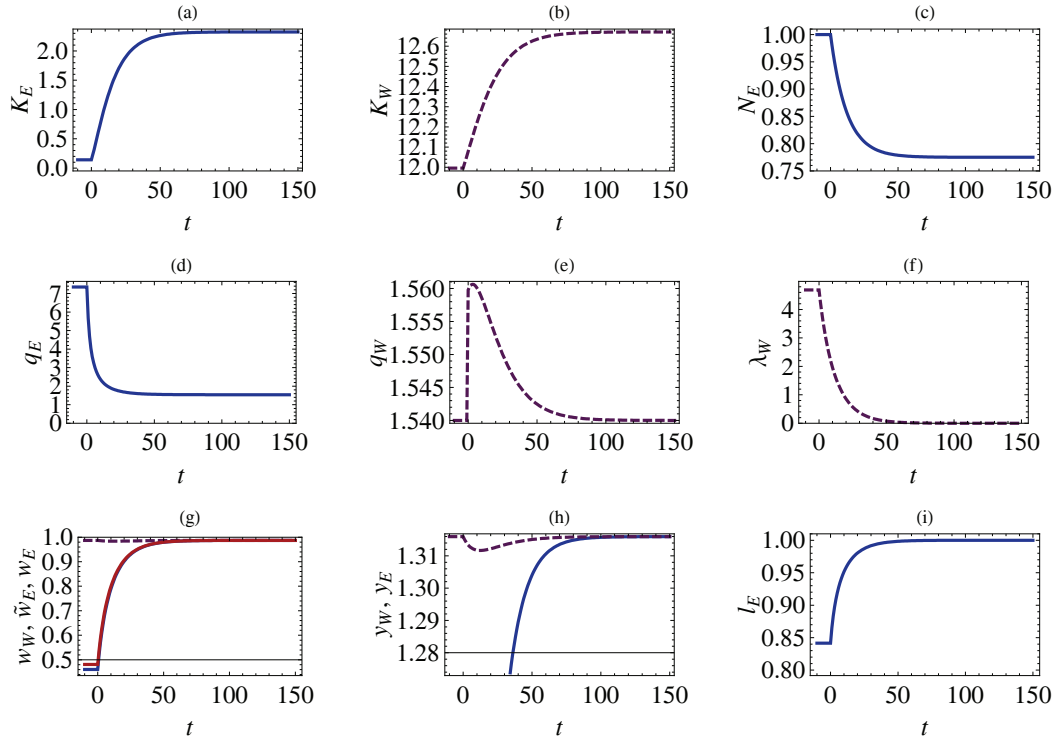
Note: The blue solid lines plot the trajectories of the eastern and the purple dashed lines of the western variables respectively. The red solid line represents the trajectory of $\tilde{w}_E$.

idence. Panel (a) of Figure 6 reveals that the model can roughly reproduce income convergence on average over nearly the observable time periods. The slowdown of income convergence during the last years leads to an increasing deviation between evidence and theoretical results as analog to the baseline model. However, panel (b) discloses an interesting picture. In case of wage-setting behavior the model can only reproduce migration rates for the first couple of years after unification. The following years are marked by decreasing migration rates in unified Germany in reply to massive social transfer payments to East Germany due to high unemployment rates. Thus, the results of the theoretical model seems to indicate a possible migration pattern in case of a high-wage policy without social transfer payments. At the end of the observable time period the theoretical model nearly predicts the same migration rates as evidence reveals.

Figure 6: Evidence and Theoretical Results for the Model with Wage-Setting Behavior
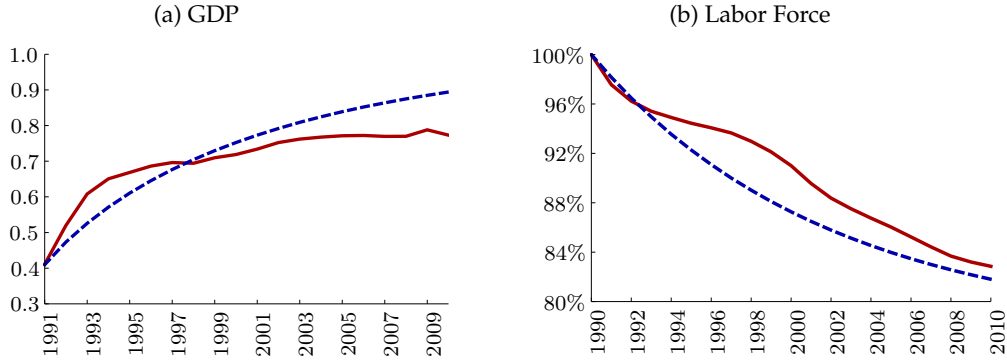
(a) GDP

(b) Labor Force

Note: The blue dashed line represents theoretical results and the red solid line empirical evidence. Panel (a) plots the prediction for real GDP per person employed convergence of the extended model with wage-setting behavior. Panel (b) shows the observed and predicted evolution of the East German labor force with respect to interregional migration.

5 The Baseline Model with Wage-Setting Behavior and Social Transfer Payments

The following extension introduces social transfer payments in addition to wage-setting behavior to the baseline model. Assume that a government taxes firms capital income in both regions with an identical but time-varying tax rate $\tau(t)$.[24] The government runs a balanced budget which is given by

$$\tau(t)\left[\alpha Y_W(t) + \alpha Y_E(t)\right] = Z_E(t) , \tag{56}$$

where $Z_E(t) = z_E(t)N_E(t)$ denotes aggregate government spendings. Furthermore, assume that tax earnings are spent on regional and interregional transfer payments exclusively to the eastern households to compensate for their loss of labor income due to regional unemployment $\ell_E < 1$, i.e.

$$z_E = (1 - \ell_E^*) \, \tilde{w}_E , \tag{57}$$

---

[24] In addition, taxation may be extended to labor income, which however would not change the results. A capital income tax is sufficient for analytically tractable solutions.

where $\tilde{w}_E$ is given by equation (49). Then, the new maximization problem of the representative firm in both regions is given by[25]

$$\max_{L_i, I_i} \quad \int_0^\infty \left\{ (1-\tau) \left[K_i^\alpha L_i^{1-\alpha} - w_i\,L_i \right] - I_i \left[1 + \frac{\gamma_K}{2} \left(\frac{I_i}{K_i} \right) \right] \right\} e^{-\bar{r}t}\,dt \qquad (58)$$

$$s.t. \qquad (7)\,,$$

where only the resulting evolution of the shadow price of capital is affected by the tax rate and changes to

$$\dot{q}_i \;=\; (\bar{r}+\delta)\,q_i - (1-\tau)\,\alpha \left(\frac{K_i}{L_i} \right)^{\alpha-1} - \frac{(q_i-1)^2}{2\gamma_K}\,, \qquad i = E, W\,. \qquad (59)$$

On the households side, the budget constraint for an eastern resident and migrant are now given by

$$\int_t^\infty e^{-\bar{r}[s-t]} c_E(s)\,ds \;=\; a_E(t) + \int_t^\infty e^{-\bar{r}[s-t]} \left[\tilde{w}_E(s)\ell_E^*(s) + z_E(s) \right]\,ds \quad \text{(resident)} \quad (60)$$

$$\int_t^\infty e^{-\bar{r}[s-t]} c_E(s)\,ds \;=\; a_E(t) + \int_t^\infty e^{-\bar{r}[s-t]} w_W(s)\,ds - mc_E \quad \text{(migrant)}\,, \qquad (61)$$

while the western budget constraints remain unchanged. Substituting the expression for z_E from (57) into the migrants' budget constraint simplifies the eastern labor income to $\int_t^\infty e^{-\bar{r}[s-t]} \tilde{w}_E(s)\,ds$ and changes the migration benefit to

$$\lambda_W(t) \;=\; \int_t^\infty \left[w_W(s) - \tilde{w}_E(s) \right] e^{-\bar{r}[s-t]}\,ds\,. \qquad (62)$$

Finally, for the tax rate $\tau(t)$ multiply both sides of equation (57) by the eastern population size at time t and substitute the equation into (56) to get

$$\tau(t) \;=\; \frac{\left[\tilde{w}_E(t) - \ell_E^*(t)\tilde{w}_E(t) \right] N_E(t)}{\alpha Y_W(t) + \alpha Y_E(t)}\,, \qquad (63)$$

where ℓ_E^* and $\tilde{w}_E$ evolve according to equation (48) and (49) respectively. The dynamic system of the model is then given by

$$\dot{K}_E \;=\; \left(\frac{q_E - 1}{\gamma_K} - \delta \right) K_E \qquad (64)$$

---

[25] Note that a capital income tax is equivalent to a tax on net cash flow when capital adjustment costs are not deductible.

$$\dot{K}_W = \left(\frac{q_W - 1}{\gamma_K} - \delta \right) K_W \tag{65}$$

$$\dot{q}_E = (\bar{r} + \delta) q_E - (1 - \tau)\alpha \left(\frac{K_E}{N_E \ell_E^*} \right)^{\alpha - 1} - \frac{(q_E - 1)^2}{2\gamma_K} \tag{66}$$

$$\dot{q}_W = (\bar{r} + \delta) q_W - (1 - \tau)\alpha \left(\frac{K_W}{P - N_E} \right)^{\alpha - 1} - \frac{(q_W - 1)^2}{2\gamma_K} \tag{67}$$

$$\dot{N}_E = -\gamma_L^{-1} \lambda_W \tag{68}$$

$$\dot{\lambda}_W = \bar{r}\lambda_W - [w_W - \tilde{w}_E(\ell_E^*)] \ . \tag{69}$$

For the computation of the transitional dynamics the same parameter values and initial conditions are used as in the former section. Figure 7 shows the equilibrium growth path of the dynamic variables. Note that the migration benefit in panel (g) is lower compared to the former model without transfers since total income of an eastern household is composed of labor income and social transfers. This leads to lower east-west migration rates on the equilibrium growth path, panel (c), and induces higher capital investments in the east. Again, the initial eastern employment rate is roughly 83% and converges to unity in equilibrium. Panel (d) and (h) display the evolution of the economy's tax rate and the individual transfer payments, respectively. An initial tax of roughly 5% is necessary to compensate eastern households for their loss of labor income with respect to wage-setting behavior. Moreover, unification induces negative repercussion effects on the western economy. During the early years after unification, western capital investment incentives sharply decline, panel (f), due to capital income taxation to finance transfer payments. Moreover, western wage rates, panel (i), and income per person employed, panel (j), slightly deviate from their steady state value over several years which indicates the negative affect of unification on the rich region.

Finally, Figure 8 plots the model's prediction of GDP per person employed and migration in contrast to the observed empirical pattern. As both panels reveal, the model is capable to reproduce two important empirical stylized facts of East-West convergence over the first 15-20 years after unification. The observed income and wage stagnation as described in section 2 leads to an increasing deviation between evidence and prediction. Moreover, Figure 9 compares the baseline model with the extended model. As panel (a) reveals, wage-setting behavior accompanied with social transfer payments decreases the speed of income convergence in the poor region during the early years. The lower employment rate $\ell_E < 1$ reduces the marginal product of capital in the east and within capital investment incentives which can be seen by comparing the shadow value of the eastern capital stock in Figure 3 and 7. Additionally, the capital income tax τ decreases investment incentives and therefore adds to a lower speed of income convergence.

Figure 7: Transitional Dynamics of the Model with Wage-Setting Behavior and Social Transfer Payments
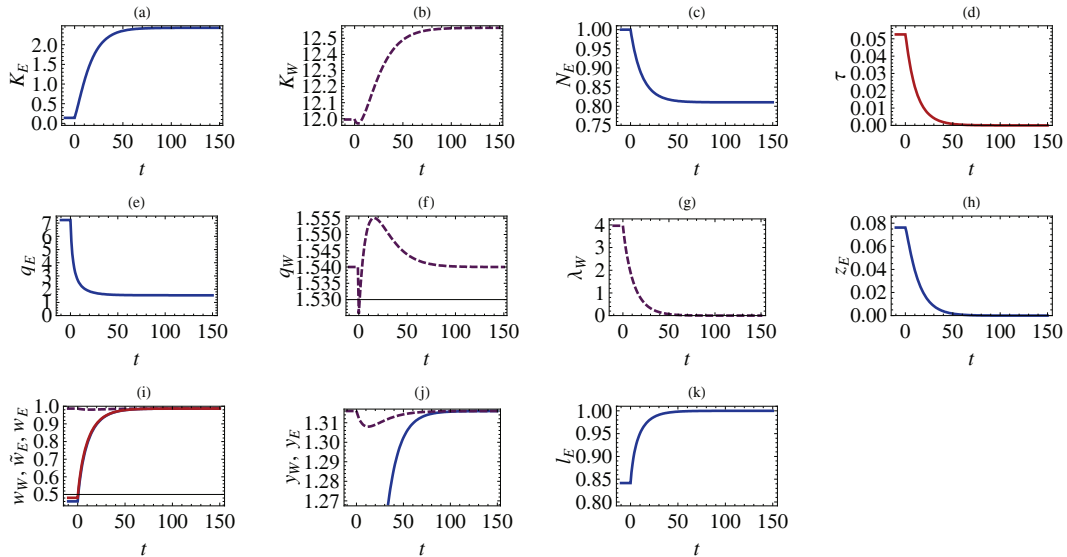

Note: The blue solid lines plot the trajectories of the eastern and the purple dashed lines of the western variables respectively. The red solid line represents the trajectory of $\tilde{w}_E$.

Figure 8: Evidence and Theoretical Results for the Model with Wage-Setting Behavior and Social Transfer Payments
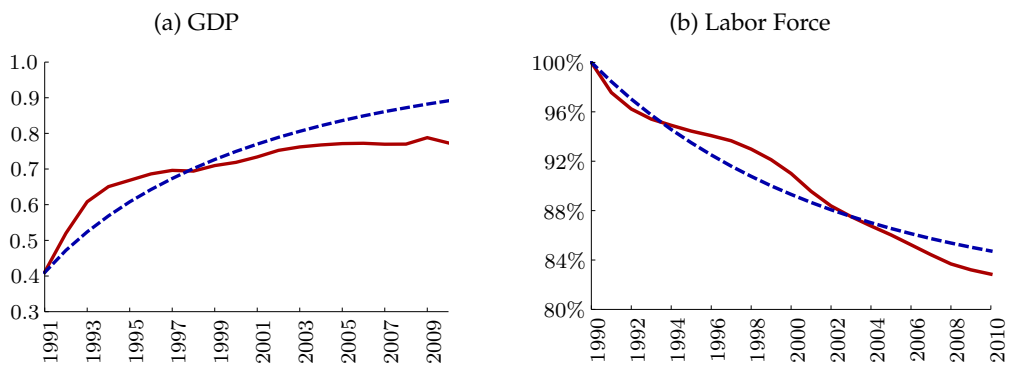
Note: The blue dashed line represents theoretical results and the red solid line empirical evidence. Panel (a) plots the prediction for real GDP per person employed convergence of the extended model with wage-setting behavior and social transfer payments. Panel (b) shows the observed and predicted evolution of the East German labor force with respect to interregional migration.

Figure 9: Comparison of the Baseline Model with the Extended Model

(a) GDP

(b) Labor Force
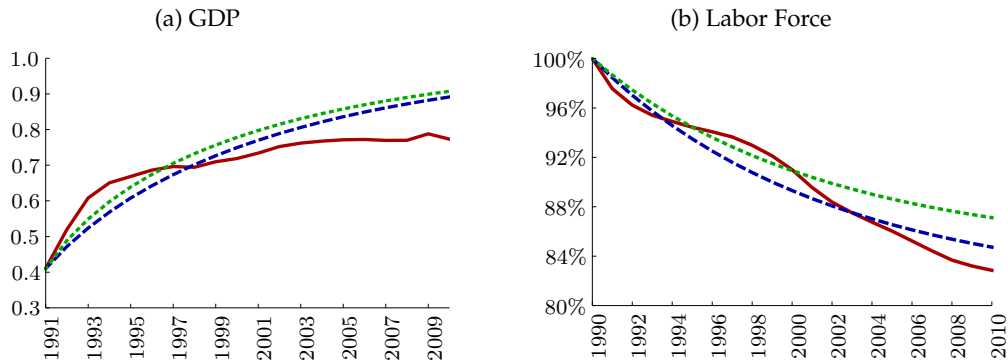
Note: The blue dashed line represents theoretical results of the extended model with wage-setting behavior and social transfer payments analog to Figure 8. The green dotted line shows the results of the baseline model analog to Figure 4 and the red solid line empirical evidence.

6   Summary and Conclusion

This paper focuses on 21 years of factor market integration in unified Germany. It shows that convergence in income per capita, labor productivity, and wages between the two formerly separated regions was fast during the first decade and remarkably slow during the second. The catching-up of the eastern economy was accompanied amongst others by high rates of private capital investments, negative net migration from east to west and drastic wage and transfer policies. Regarding the latter, labor union agreements in the early 1990s kept East German wage rates above productivity to oppose high rates of early migration. Nevertheless, in the last decade, between 2000 and 2009, net migration from East to West Germany was about 60.000 people on average each year, or 0.4% annually. The reallocation of capital and labor is the most striking characteristic of German unification and may affect the macroeconomic development in East and West Germany in the long run. Therefore, a baseline two-region open economy model with private capital accumulation and migration is set up. Both regions differ in their initial capital endowment, such that the poor region exhibits lower wage rates and higher marginal returns on capital investments on the equilibrium growth path. In the long run, migration between regions and capital investments lead to an equalization of factor rewards. The model is calibrated with German data after 1990 and a numerical solution technique developed by Trimborn et al. (2008) is used to simulate the dynamic transition of the East German economy after unification. In an extension of the baseline model, wage-setting behavior and social transfer payments are introduced to consider the specific German case in the early 1990s. Then, the extended model exhibits several important implications: (i) the model can reproduce observed income convergence and migration pattern in unified Germany over the first 15-20 years. (ii) A high-wage policy accompanied with social transfer payments de-

creases the speed of income convergence in the poor region during the early years. (iii) Capital accumulation, migration, as well as wage and transfer policies alone are not capable to explain observed stagnation of income convergence together with persistent high rates of East-West migration. The latter result indicates the need to further extend the theoretical framework, for instance, by human capital endowment and government spendings to investigate the real long term perspectives of the macroeconomic development in unified Germany.

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A Technical Appendix

A.1 The Firm's Maximization Problem

From the maximization problem in (6) follows the current-value Hamiltonian function for region $i \in [E, W]$

$$H_i = K_i^\alpha L_i^{1-\alpha} - w_i L_i - I_i \left[1 + \frac{\gamma_K}{2} \left(\frac{I_i}{K_i} \right) \right] + q_i (I_i - \delta K_i) \ .$$

The first-order necessary conditions describing the optimum are given by

$$\frac{\partial H_i}{\partial L_i} = (1-\alpha) K_i^\alpha L_i^{-\alpha} - w_i \overset{!}{=} 0$$

$$\Rightarrow \ w_i = (1-\alpha) \left(\frac{K_i}{L_i} \right)^\alpha \tag{70}$$

$$\frac{\partial H_i}{\partial I_i} = -1 - \gamma_K \left(\frac{I_i}{K_i} \right) + q_i \overset{!}{=} 0$$

$$\Rightarrow \ \frac{I_i}{K_i} = \frac{q_i - 1}{\gamma_K} \tag{71}$$

$$\dot{q}_i = \bar{r} q_i - \frac{\partial H_i}{\partial K_i} = \bar{r} q_i - \alpha K_i^{\alpha-1} L_i^{1-\alpha} - \frac{\gamma_K}{2} \left(\frac{I_i}{K_i} \right)^2 + \delta q_i \overset{!}{=} 0$$

$$\Rightarrow \ \dot{q}_i = (\bar{r} + \delta) q_i - \alpha \left(\frac{K_i}{L_i} \right)^{\alpha-1} - \frac{\gamma_K}{2} \left(\frac{I_i}{K_i} \right)^2 \tag{72}$$

Finally, substitute equation (71) into the law of motion for capital given in (5) as well as into (72).

A.2 The Households Budget Constraint in Intertemporal Form

Consider equation (14), where we suppressed the regional index i to simplify expressions[26]

$$\dot{a}(t) = \bar{r} a(t) + w(t) \ell(t) - c(t) . \tag{73}$$

---

[26] The intertemporal form of equation (15) can be easily derived in the same way.

The forward solution of the budget constraint reads[27]

$$a(t) = a(T)e^{-\bar{r}\,[T-t]} - \int_t^T e^{-\bar{r}\,[s-t]} \left[w(s)\,\ell(s) - c(s) \right] ds \tag{74}$$

$$a(t) = a(T)e^{-\bar{r}\,[T-t]} - \int_t^T e^{-\bar{r}\,[s-t]} \left[w(s)\,\ell(s) \right] ds + \int_t^T e^{-\bar{r}\,[s-t]} c(s) ds \,, \tag{75}$$

where $a(T)$ is given. Let the future point in time T go to infinity and take the limit of the above equation to get

$$a(t) = \lim_{T \to \infty} a(T)e^{-\bar{r}\,[T-t]} - \int_t^\infty e^{-\bar{r}\,[s-t]} \left[w(s)\,\ell(s) \right] ds + \int_t^\infty e^{-\bar{r}\,[s-t]} c(s) ds \,. \tag{76}$$

The no-Ponzi game or solvency condition requires $\lim_{T \to \infty} a(T)e^{-\bar{r}\,[T-t]} = 0$ and we end up with the households budget constraint in intertemporal form

$$\int_t^\infty e^{-\bar{r}\,[s-t]} c_i(s) ds = a_i(t) + \int_t^\infty e^{-\bar{r}\,[s-t]} \left[w_i(s)\,\ell_i(s) \right] ds \,. \tag{77}$$

In the same way we can rewrite the budget constraint of a migrant given in (15)

$$\int_t^\infty e^{-\bar{r}\,[s-t]} c_i(s) ds = a_i(t) + \int_t^\infty e^{-\bar{r}\,[s-t]} \left[w_j(s)\,\ell_j(s) \right] ds - mc_i(t) \,. \tag{78}$$

A.3 Evolution of Consumption in the Host Region

Consider equation (24) and divide both sides by the population level in the host region at time t to get

$$\frac{C_h(t)}{N_h(t)} = \bar{c}_n \frac{N_h(0)}{N_h(t)} + \bar{c}_s \frac{\int_0^t M(s)\, ds}{N_h(t)} \,. \tag{79}$$

---

[27] Compare e. g. Wälde (2010), Chapter 4.3.

Differentiating with respect to time and reorganizing the equation yields the evolution of total consumption in the host region

$$\frac{\dot{C}_h N_h(t) - C_h(t)\dot{N}_h}{N_W(t)^2} = -\bar{c}_n \frac{N_h(0)\dot{N}_h}{N_h(t)^2} + \frac{\bar{c}_s M N_h(t) - \bar{c}_s \int_0^t M(s)\,ds\,\dot{N}_h}{N_h(t)^2} \quad (80)$$

$$\dot{C}_h N_h(t) - C_h(t)\dot{N}_h = -\underbrace{\left[\bar{c}_n N_h(0) + \bar{c}_s \int_0^t M(s)\,ds\right]}_{C_h(t)} \dot{N}_h + \bar{c}_s M N_h(t) \quad (81)$$

$$\dot{C}_h = \bar{c}_s M \quad (82)$$

For per capita consumption divide both side by $N_h(t)$ to get

$$\dot{c}_h = -[c_h - c_s]\frac{M}{N_h(t)} . \quad (83)$$

where we used equation (3) and (11) as well as

$$\frac{\dot{C}_h}{N_h(t)} = \dot{c}_h + \frac{C_h(t)\dot{N}_h}{N_h(t)^2} \quad (84)$$

from $c_h(t) = C_h(t)/N_h(t)$.

A.4   First-order Taylor Approximation

The linear approximation of equation (28) and (29) is given by

$$\dot{K}_i(q_i, K_i) = \left.\frac{\partial \dot{K}_i}{\partial q_i}\right|_{q_i=\bar{q}_i}(q_i - \bar{q}_i) + \left.\frac{\partial \dot{K}_i}{\partial K_i}\right|_{K_i=\bar{K}_i}\left(K_i - \bar{K}_i\right)$$

$$\dot{K}_i = \bar{K}_i\gamma_K^{-1}(q_i - \bar{q}_i) + \left[\frac{\bar{q}_i - 1}{\gamma_K} - \delta\right]\left(K_i - \bar{K}_i\right), \qquad i = E, W .$$

Substituting equation (34) yields

$$\dot{K}_E = \bar{K}_E\gamma_K^{-1}(q_E - \bar{q}_E) \quad (85)$$

$$\dot{K}_W = \bar{K}_W\gamma_K^{-1}(q_W - \bar{q}_W) . \quad (86)$$

Equation (32), the evolution of the eastern population size, is linearized by

$$\dot{N}_E(\lambda_W) = \left.\frac{\partial \dot{N}_E}{\partial \lambda_W}\right|_{\lambda_W=\bar{\lambda}_W}\left(\lambda_W - \bar{\lambda}_W\right)$$

$$\dot{N}_E = -\gamma_L^{-1}\left(\lambda_W - \bar{\lambda}_W\right) \quad (87)$$

The evolution of the shadow value of the eastern capital stock is given by equation (30) and linearized by

$$
\dot{q}_E(q_E, K_E, K_W, L_E) = \left. \frac{\partial \dot{q}_E}{\partial q_E} \right|_{q_E = \bar{q}_E} (q_E - \bar{q}_E) + \left. \frac{\partial \dot{q}_E}{\partial K_E} \right|_{K_E = \bar{K}_E} (K_E - \bar{K}_E)
$$

$$
+ \left. \frac{\partial \dot{q}_E}{\partial K_W} \right|_{K_W = \bar{K}_W} (K_W - \bar{K}_W) + \left. \frac{\partial \dot{q}_E}{\partial L_E} \right|_{L_E = \bar{L}_E} (L_E - \bar{L}_E) \ .
$$

To simplify notation let F^i_{jk} denote the second derivative of the production function of region $i = E, W$ with respect to $j, k = K_i, L_i$. Thus, linearizing equation (30) yields

$$
\dot{q}_E = \bar{r}(q_E - \bar{q}_E) - F^E_{KK}(K_E - \bar{K}_E) - F^E_{KL}(L_E - \bar{L}_E) \ , \tag{88}
$$

where we used equation (34). The second derivatives are explicitly given by

$$
F^E_{KK} = \alpha(\alpha - 1)K_E^{\alpha-2}L_E^{1-\alpha} < 0
$$

$$
F^E_{KL} = \alpha(1 - \alpha)K_E^{\alpha-1}L_E^{-\alpha} > 0 \ .
$$

The law of motion for the shadow value of the western capital stock is linearized by

$$
\dot{q}_W(q_W, K_E, K_W, L_E) = \left. \frac{\partial \dot{q}_W}{\partial q_W} \right|_{q_W = \bar{q}_W} (q_W - \bar{q}_W) + \left. \frac{\partial \dot{q}_W}{\partial K_E} \right|_{K_E = \bar{K}_E} (K_E - \bar{K}_E)
$$

$$
+ \left. \frac{\partial \dot{q}_W}{\partial K_W} \right|_{K_W = \bar{K}_W} (K_W - \bar{K}_W) + \left. \frac{\partial \dot{q}_W}{\partial L_E} \right|_{L_E = \bar{L}_E} (L_E - \bar{L}_E)
$$

and yields

$$
\dot{q}_W = \bar{r}(q_W - \bar{q}_W) - F^W_{KK}(K_W - \bar{K}_W) - F^W_{KL}(L_E - \bar{L}_E) \ . \tag{89}
$$

The second derivatives are explicitly given by

$$
F^W_{KK} = \alpha(\alpha - 1)K_W^{\alpha-2}(P - L_E)^{1-\alpha} < 0
$$

$$
F^W_{KL} = -\alpha(1 - \alpha)K_W^{\alpha-1}(P - L_E)^{-\alpha} < 0 \ .
$$

The evolution of the migration benefit in (33) is linearized by

$$
\dot{\lambda}_W(\lambda_W, K_E, K_W, L_E) = \left. \frac{\partial \dot{\lambda}_W}{\partial \lambda_W} \right|_{\lambda_W = \bar{\lambda}_W} (\lambda_W - \bar{\lambda}_W) + \left. \frac{\partial \dot{\lambda}_W}{\partial K_E} \right|_{K_E = \bar{K}_E} (K_E - \bar{K}_E)
$$

$$
+ \left. \frac{\partial \dot{\lambda}_W}{\partial K_W} \right|_{K_W = \bar{K}_W} (K_W - \bar{K}_W) + \left. \frac{\partial \dot{\lambda}_W}{\partial L_E} \right|_{L_E = \bar{L}_E} (L_E - \bar{L}_E)
$$

and yields

$$\dot{\lambda}_W = \bar{r}\left(\lambda_W - \bar{\lambda}_W\right) + F_{LK}^E\left(K_E - \bar{K}_E\right) - F_{LK}^W(K_W - \bar{K}_W)$$

$$-(F_{LL}^W - F_{LL}^E)\left(L_E - \bar{L}_E\right) . \tag{90}$$

The second derivatives are explicitly given by

$$F_{LK}^E = \alpha(1-\alpha)K_E^{\alpha-1}L_E^{-\alpha} > 0$$

$$F_{LL}^E = -\alpha(1-\alpha)K_E^{\alpha}L_E^{-\alpha-1} < 0$$

$$F_{LK}^W = \alpha(1-\alpha)K_W^{\alpha-1}(P-L_E)^{-\alpha} > 0$$

$$F_{LL}^W = \alpha(1-\alpha)K_W^{\alpha}(P-L_E)^{-\alpha-1} > 0 .$$

Note that $F_{KL}^E = F_{LK}^E$, while $F_{KL}^W = -F_{KL}^W$. The linearized system is described by equation (85), (86), (87), (88), (89), and (90).

A.5 Analytic Derivation of the Models Eigenvalues

Note that M is a 2×2 block matrix with four 3×3 matrices. Since both matrices on the diagonal of M are diagonal in itself the eigenvalues of M can be calculated analytically in a simple straight forward way. Let F denote the quadratic upper right matrix of M and C the quadratic lower left matrix so we can rewrite M to

$$M = \begin{pmatrix} \bar{r}\,Id & F \\ C & 0\,Id \end{pmatrix}, \tag{91}$$

where Id denotes the identity matrix. The matrix eigenvalues are given by the solution of the characteristic polynomial $\rho_M(\lambda) = \det(M - \lambda\,Id) \overset{!}{=} 0$ with

$$M - \lambda\,Id = \begin{pmatrix} (\bar{r}-\lambda)\,Id & F \\ C & -\lambda\,Id \end{pmatrix}. \tag{92}$$

Since the diagonal of M is given by two diagonal matrices the characteristic polynomial $\rho_M(\lambda)$ simplifies to

$$\rho_M(\lambda) = -\det\left(FC + (\bar{r}-\lambda)\lambda\,Id\right) \overset{!}{=} 0 , \tag{93}$$

and implies the two roots λ_1 and λ_2. Let $-\varphi$ be the solution to $(\bar{r}-\lambda)\lambda$ such that

$$\lambda_{1,2} = \frac{1}{2}\left(\bar{r} \pm \sqrt{\bar{r}^2 + 4\varphi}\right) . \tag{94}$$

Then, computation further simplifies to the solution of the characteristic polynomial of the 3×3 matrix FC

$$\rho_{FC}(\varphi) \quad = \quad -\det\left(FC - \varphi\, Id\right) \overset{!}{=} 0 \,, \tag{95}$$

where

$$FC \quad = \quad \begin{pmatrix} -F^E_{KK}\bar{K}_E\gamma_K^{-1} & 0 & F^E_{KL}\gamma_L^{-1} \\ 0 & -F^W_{KK}\bar{K}_W\gamma_K^{-1} & F^W_{KL}\gamma_L^{-1} \\ F^E_{LK}\bar{K}_E\gamma_K^{-1} & -F^W_{LK}\bar{K}_W\gamma_K^{-1} & -(F^E_{LL}-F^W_{LL})\gamma_L^{-1} \end{pmatrix} \,.$$

The solution to (95) yields the three roots φ_1, φ_2, and φ_3 and is given by

$$\rho_{FC}(\varphi) \quad = \quad \det(F)\det(C) - B\varphi + A\varphi^2 - \varphi^3 \overset{!}{=} 0 \,, \tag{96}$$

where

$$A \quad := \quad \mathrm{tr}(FC) > 0 \tag{97}$$

$$B \quad := \quad [\det(FC_{11}) + \det(FC_{22}) + \det(FC_{33})] > 0 \,, \tag{98}$$

and $\det(FC_{ii})$ is the minor of matrix FC formed by eliminating row i and column i from FC.[28] Note that the determinant of matrix F is zero for a Cobb-Douglas production function with constant returns to scale such that $\det(F)\det(C) = 0$. Then, the solution to (95) reduces to

$$\rho_{FC}(\varphi) \quad = \quad \varphi\left(\varphi^2 - A\varphi + B\right) \overset{!}{=} 0 \,, \tag{99}$$

and is solved by

$$\varphi_1 \quad = \quad 0 \tag{100}$$

$$\varphi_{2,3} \quad = \quad \frac{1}{2}\left(A \pm \sqrt{A^2 - 4B}\right) \,. \tag{101}$$

Finally, plugging the results from (100) and (101) into equation (94) yields the six eigenvalues of matrix M.

---

[28] The elements of matrix F exhibit the following properties $F^i_{KL} = F^i_{LK} > 0$ and $F^i_{KK} < 0$, $i = E, W$ as well as $F^W_{LL} > 0$ and $F^E_{LL} < 0$.