Technology in the Global Economy:

A Framework for Quantitative Analysis

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Introduction

Technology is central to international economics. Nearly two centuries ago, Ricardo posited that differences in technologies across countries generate comparative advantage, and the basis for mutually gainful trade. More recently, economists have attributed differences in technologies across individual producers for the observed heterogeneity in their productivity and size, and the correlation between productivity, size, and export participation. Improvements in technologies over time are the main explanation for the growth of nations. Hence barriers to the international diffusion of technology are responsible for persistent income differences across countries.

Our goal is a unified theory of technology in the world economy that speaks to these issues. Aside from drawing the theoretical connections between technology's role in the these various phenomena, we seek a tighter link between theory and measurement.

CHAPTER 1 — MANUSCRIPT

In international economics, progress in theory and in empirics has tended to proceed down separate paths, with bends in one occasionally reflecting those in the other. New theories emerge after too many facts contradict an old one (as the "new trade theory" arose to explain the prevalence of intraindustry trade and trade between like countries). Econometricians test a theory to see if they can find a correlation in the data that the theory predicts (as with the various tests of factor endowments theory). But, for the most part, researchers have shied away from building models that can both replicate qualitative features of the data and also capture basic quantitative features. We will not be shy in our attempt to do just that.

Over the past decade we have developed approaches for addressing a number of questions about trade and innovation in a multicountry world. We began by developing a multicountry growth model that we could combine with research indicators to quantify the extent of international technology diffusion. This work appeared as Eaton and Kortum (henceforth EK), 1999. We then realized that our framework delivered a static model that was readily amenable to the quantitative analysis of bilateral trade flows (EK 2002). Having heard Andrew Bernard and J. Bradford Jensen's new findings on the export behavior of U.S. plants, we jointly saw a connection between our trade model and their facts, which we exploited in Bernard, Eaton, Jensen, and Kortum (henceforth, BEJK, 2003). Having looked at data on innovation around the world, in turning to trade data we saw the intimate connection between a country's R and D intensity and its specialization in the production of equipment. We used a variant of our framework to quantify the role of trade in capital goods in generating income differences (EK 2001).

Our work proceeded piecemeal: We didn't always see the connections among the pieces and, in retrospect, didn't necessarily take the shortest route from here to there. We didn't fully see the big picture as we worked on various pieces of it. We've taken the opportunity in this book to restructure our approach in a more unified, simplified way. At the same time, a vast number of issues that our approach might shed light on remain unexplored. Our hope is to make this framework accessible to a wider audience, and to lower the barrier to entry for future research.

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1.1 Modeling the International Economy

National borders create barriers to the flow of technology, either because they impede the movement of ideas themselves or because they impede the movement of goods produced using those ideas. We seek to measure these barriers by exploiting various types of aggregate data, such as production, bilateral trade, and patent statistics, but also data on individual producers.

In pursuing this task, we need to be aware that the division of the global economy into individual countries colors our understanding of how the world works. We necessarily rely heavily on statistics that national governments provide about what goes on within their jurisdictions. A challenge for academic researchers is to draw the correct connections between theoretical concepts and what the data actually measure. In the particular case of international data, what are we to make of the nation as the unit of observation?

In the first formal model of international trade, Ricardo provided a stark answer which became the basis for nearly all subsequent trade theory: Factor markets are national while commodity markets are potentially international. Workers don't cross borders, but goods can if governments let them. Ricardo treated technologies themselves as national. All workers have access to the domestic technology for producing a good, but not a foreign one. Differences in national technologies determine comparative advantage, the basis of the gains from trade.

Factor endowments theory stuck with Ricardo's assumption that national frontiers segment factor markets, but not commodity markets. But, it switched to the opposite assumption about technologies, treating them as commonly available to all countries of the world. Differences in factor endowments across countries and in factor intensities across goods then provide the incentives to trade.

The much more recent literature on growth has faced the same quandary: Should we think of the forces driving growth as national or as international in scope? Should we think of all countries sharing the world's best technologies? Again, different models make opposite assumptions.

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A component of our research measures the speed with which ideas are adopted at home and abroad. We thus encompass the case of no cross-border diffusion and equal rates of diffusion everywhere. Calibrating our model to measures of productivity, research intensity, and international patent applications, we find ideas to be about two-thirds as potent abroad as at home.

Another component of our research assesses the degree to which borders segment markets for commodities, both in the aggregate and for individual producers. Using data on international prices, production, bilateral trade, and features of geography, we find significant geographic barriers to the flow of goods between countries. Using data on U.S. manufacturing plants, we infer the extent to which overcoming geographic barriers requires an efficiency advantage that leads to greater size and higher observed productivity at home.

A third component of our research combines the two questions. A country can benefit from a foreign idea without actually knowing it by importing goods that embody the idea. The major research economies are also major exporters of equipment. Using data on bilateral trade in equipment we trace the flow of knowledge from these economies to the rest of the world as technology is embodied in their exports of capital goods. We find that developing countries' inability to access the best equipment explains about a quarter of the difference between the incomes of the richest and poorest countries.

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1.2 A Brief Outline

We divide our book into four parts.

Part I, Foundations, with 2 chapters, sets the stage for our analysis. Chapter 2 provides an overview of some basic features of the data on trade, research, and productivity that we seek to capture. Chapter 3 then reviews several previous models of international trade and economic growth on which we build.

The four chapters of Part II, Framework, develop the analytic structure underlying our various applications to data. We present our core assumption about the distribution of ideas in Chapter 4, and then derive the properties of this distribution that we use throughout the rest of the analysis. In Chapter 5 we complete the specification of the closed economy by making specific assumptions about how goods are aggregated in preferences, and consider the determination of price indices, income distribution, and the real wage under a variety of market structures. How the framework can readily accommodate trade among an arbitrary number of countries separated by geographic barriers is the topic of Chapter 6. Here we show how we can use the framework to make connections between data on prices and on trade shares, and use them to infer parameter values and such magnitudes as the gains from trade. In Chapter 7 we introduce dynamics. We calculate the value of an idea under alternative market structures, and gauge the incentive to innovate in each. We do so first for a closed economy, and then for a world in which ideas diffuse across borders with arbitrary lags.

Part III, Applications, returns to the four measurement issues that motivated our original work, not in the order in which we pursued them. Chapter 8 connects the framework to data on prices and bilateral trade in manufactures among members of the Organization of Economic Cooperation and Development (OECD). The calibrated model is then used to assess the gains from trade, from reductions in regional trade barriers, and from technological improvements across countries. Chapter 9, written jointly with Andrew Bernard and J. Bradford Jensen, connects aggregate data on bilateral trade flows in manufacturing among the United States and its 47 major trade partners with observations on the export participation of U.S. manufacturing plants. The calibrated model is then used to assess the effect of lower trade barriers and a dollar appreciation on plant entry and exit and on manufacturing productivity. In Chapter 10 we turn to data on trade in equipment among both developed and selected developing countries to assess the role of trade in capital goods as a conduit of international technology diffusion. Finally, in Chapter 11 we return to the issue that originally motivated us, the calibration of a multicountry model of growth and technology diffusion to data from the five major research economies.

In Part IV, Extensions, we pursue two theoretical issues suggested by our framework. One is the connection between diffusion and trade. The other the optimal degree of intellectual property protection.

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Part I

Foundations

Our work builds both on observations about the world from a number of different perspectives, and on a wide body of knowledge in international economics. The next chapter surveys the key features of the data that our framework is trying to come to terms with. Chapter 3 lays out the theoretical approaches that have contributed most directly to our own work.

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Chapter 2

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Empirical Foundations

While much of this book is theoretical, the structure of the theory is guided by a set of observations. This chapter presents the key ones motivating our analysis. We classify them into those pertaining to (i) international trade, (ii) research and development, and (iii) aggregate output and growth.

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2.1 International Trade

During the past decade, detailed data on international trade have become much more widely available. This development has had a dramatic effect on the research agenda in international trade and on how economic models of international trade are evaluated. In this sense, international trade is becoming more like the fields of economic growth and industrial organization in which there is a constant interplay between theory and data. Here we lay out facts about trade, both at the level of individual countries and at the level of individual producers, that we think any model of trade must contend with. But before looking at the numbers themselves, we need to discuss some issues in measurement.

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2.1.1 Data Construction

Looking across the major sectors of the economy, differences in technology over time and distance appear to be most pronounced in manufactures. Technology obviously plays an important role in agriculture and minerals as well, but differences in climate, soil and natural resources also matter fundamentally. While trade in services has been growing, with technology playing an important role, changes and differences in technology do not appear to be as dramatic.[1] Moreover, data on service trade is much more limited. Hence our focus is on trade in manufactures.

Feenstra, Lipsey, and Bowen (1997) and Feenstra (2000) have assembled data

[1] Calculations from the World Bank Development Indicators (2005) reveal that, for the World as a whole, value added in agriculture contributed 4 percent of GDP, with imports of agricultural raw materials equaling about 12 percent of value added. Value added in services constituted 67 percent of GDP, with service imports equaling 8 percent of service value added. While manufacturing value added contributed only 19 percent to GDP, manufacturing imports equalled 87 percent of value added. In absolute terms, services are more than three times bigger than manufactures in value added, but manufactures are more than three times bigger in trade.

on bilateral international shipments of merchandise, giving us, in particular, a measure of total imports of manufactures for each year from 1970 to 1997 for most countries of the world. We denote country n's imports of manufactures by I_n. We denote country i's total exports as E_i (our convention is to denote the exporting country by i and the importer by n). Each of these measures is translated from foreign currencies to current U.S. dollars at the prevailing exchange rates.

To obtain a complete picture of the production and shipment of goods around the world we augment imports and exports with a measure of what countries produce and retain for their own use. Here we are forced to make some difficult choices. Since trade data necessarily include intermediates, our measure of production should includes intermediates as well. The United Nations Industrial Development Organization (UNIDO, 2001) reports a measure of gross production of the manufacturing sector for many years and countries (although the data are less complete and probably less precise than the international trade data). We denote country i's gross production of manufactures, in U.S. dollars, by Y_i. Unlike a measure of value added, gross production does not net out the production of intermediates that are sold to other producers. Unlike value added measures, gross production rises with the extent of vertical fragmentation in production across manufacturing establishments located in the same country, just as the amount of trade increases with the extent of vertical fragmentation in production across establishments in different countries.[2]

With these measures in hand, we can calculate what country n purchases from its own producers, $X_{nn} = Y_n - E_n$. Country n's total purchases of manufactures, sometimes referred to as its absorption or market size, is simply $X_n = X_{nn} + I_n$. Standard measures of country n's participation in international trade are its imports as a fraction of its absorption, I_n/X_n or its exports as a fraction of production E_n/Y_n.

Feenstra et al. (1997) and Feenstra (2000) also report bilateral trade, what each country n purchases from each other country i, which we denote X_{ni}, for $n \neq i$ (again, measured in current U.S. dollars). When merged with what each country buys from itself X_{nn}, these data can be summed up into what each country n purchases, $X_n = \sum_{i=1}^{N} X_{ni}$ and what each country i produces, $Y_i = \sum_{n=1}^{N} X_{ni}$, where N is the total number of countries.

We begin our analysis using data on a set of about 100 of the world's largest countries. We present the data in the form of two cross sections, one an average over the years 1970-1972 and the other an average over the years 1995-1997, referring to them, respectively, as the "early" and "late" periods. Table 1 supplies details about the sample of countries.

[2] Yi (2003) analyzes the implications of vertical fragmentation for trade volumes.

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2.1.2 The International Trade of Countries

The "gravity equation" of international trade provides a useful means of organizing the facts. It relates bilateral trade volumes (here measured as country n's imports from country i, X_{ni}) to a measure of importer size, exporter size, and some index of the distance between exporter and importer, τ_{ni}:

$$X_{ni} = \kappa \frac{X_n Y_i}{\tau_{ni}}, \tag{2.1}$$

where the constant κ sucks up units of measurement.[3] In applying the gravity equation to observation $i = n$, we will set $\tau_{nn} = 1$. We explore the role of importer size, exporter size, and distance, in turn.

To examine the role of importer market size, Figures 1 and 2 (for the early and late periods) plot total imports I_n against absorption X_n (a measure of economic size) across countries. There is a strong tendency for bigger countries to import more. Yet imports clearly increase less than proportionally with market size. Larger countries are more likely to buy from their own domestic suppliers. The largest countries only import about 10 percent of their purchases. A simple cross-country regression shows that the elasticity of imports with respect to market size was 0.74 in the early 1970's. This elasticity rose to 0.90 in the mid to late 1990's. Thus, trade grew faster in big

[3] Early applications of the gravity equation were by Tinbergen (1962) and Pöyhönen (1963). There have been countless subsequent implementations. In the following chapter we provide several theoretical derivations of the equation.

markets than in small markets.

To examine the role of exporter size we look at the extent to which countries that produce more also have greater penetration of their domestic and export markets. In particular, Figure 3 plots country i's market share in each destination n, X_{ni}/X_n, against country i's total production, Y_i. The figure also includes observations on country i's penetration of its own domestic market, X_{ii}/X_i (appearing with a + rather than a circle). Note how the observations of domestic market penetration all appear along the top of the Figure. In a benchmark world with no geographic barriers and complete specialization, all observations would lie along a 45 degree line (on a log scale): A country's market share in each destination would correspond to its share in world production, so that in each destination n country i would have a share Y_i/Y_W, where Y_W is world production, independent of n. A regression through the scatter of Figure 3 does in fact deliver a coefficient of one, but note the huge variation across destinations.[4]

[4] Adding up constraints imply a slope of one unless there is some covariance between size as captured by $\ln Y_i$, and higher moments of export patterns, as reflected by differences across i in $\sum_{n=1}^{N} \ln e_{ni}$ where e_{ni} is the fraction of output that i ships to n. To see this result note that we can decompose the variable on the y-axis as:

$$\ln(X_{ni}/X_n) = \ln e_{ni} + \ln Y_i - \ln X_n.$$

For each observation $\ln Y_i$ on the x-axis there are N observations of $\ln(X_{ni}/X_n)$ with $\ln Y_i$ common to each. Since $\ln X_n$ is the same for each $\ln Y_i$, any deviation from a slope of 1 can only be due to covariance between $\ln Y_i$ and $\sum_{n=1}^{N} \ln e_{ni}$. Our finding of a slope of one thus indicates that there is no

An obvious explanation is that trade costs depend on geography.

To isolate the role of geography we construct an index of bilateral trade defined as $B_{ni} = \sqrt{(X_{ni}X_{in})/(X_{ii}X_{nn})}$. This index appropriately adjusts for the effect of size by normalizing with the home sales of each country in the pair, and treats the countries in the pair symmetrically. According to the gravity equation (2.1), the bilateral trade index is tightly linked to the geographic barriers between two countries, $B_{ni} = 1/\sqrt{\tau_{ni}\tau_{in}}$, or simply $1/\tau_{in}$ if these barriers are symmetric.

We plot this index against the distance between countries i and n in Figures 4 and 5. (The home-country observations are dropped since this index equals 1 for each of them.) Distances, from Haveman (2005), are between capital cities. Although the relationship is quite noisy, bilateral trade patterns are quite strongly associated with the simple distance between countries. Countries far apart trade much less with each other. Comparing Figures 4 and 5, between the earl 1970's and the mid to late 1990's, we see that the index of bilateral trade increased by nearly an order of magnitude at all distances. While the scatter shifts up over time, the slope of this relationship remains fairly stable (the slopes are -1.03 and -1.17, respectively).[5] International trade has

systematic covariance.

[5] The relationship is even tighter among counties in the OECD, but the elasticity with respect to distance remains very close to -1. Xavier Gabaix has drawn our attention to this coefficient of -1 as a fundamental fact in search of a theory. While the theory we put forward in this book will allow for that coefficient, it does not predict it.

risen substantially over the past 25 years in relation to purchases from domestic producers, but in terms of where countries import from, the curse of distance remains as strong as ever.

Taken together these figures point to how far the world is from theoretical models in which all countries buy the same basket of goods. Rather, geography plays a crucial role in trade patterns. While we cannot learn about the magnitude of impediments to trade from these pictures, we do learn that reasonable predictions about bilateral trade must take geography into account.

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2.1.3 The International Trade of Firms

How are these patterns of trade between countries reflected in the export behavior of individual producers? Bernard and Jensen (1995) began to document facts about producer-level exporting based on total foreign shipments of individual U.S. manufacturing plants. Chapter 9 contains our joint work with them exploring the implications of these producer-level facts. In Eaton, Kortum, and Kramarz (2004) we exploit unique data on the export sales of individual French firms to each of over 100 countries. Our current analysis of the French data is limited to a single cross-section of manufacturing firms for the year 1986 (details of the data construction are described in Eaton, Kortum, and Kramarz, 2005).

The most striking fact emerging from these data is how little most producers participate in export markets. Less than 15 percent of U.S. manufacturing plants reported exporting in 1987 (this percentage rose to 21 by 1992, see Table 1 of Chapter 9), and of those that did export, most reported sending less than 10 percent of their shipments abroad. In these respects, French firms appear very similar to U.S. plants. Only 17% of these French manufacturing firms report exporting in 1986.

The French data, however, also provide evidence on the geographic scope of exports. Figure 6 plots the number of French firms according to how many national markets they penetrate. The left most observation is simply all French firms (since, to exist as a firm it must be selling in at least one market) while the second from the left is the number of exporters selling to at least two markets, etc. We see not only that a small minority export, but among those that do, most sell in only a few export destinations. Only about 1000 firms sell in more than 30 export markets. In terms of popularity, France itself is almost always a destination while Belgium is the most popular foreign market, although around half of exporters don't sell in Belgium.

How does the limited participation of firms in export markets align with the data on bilateral trade? The gravity equation implies that France will sell more in countries with large markets, but its share of a market will vary according to geographical factors. We want to see how the participation of individual French firms relates to these two factors. We thus regress the number of French firms selling to a country on the market size of the country and on France's market share in the country. Based on a logarithmic specification, the estimated elasticity of exporters with respect to market size is 0.62 and with respect to market share is 0.88. We can illustrate the relationship by plotting (on a log scale) the number of French firms selling to a market (relative to France's share of it, implicitly imposing an elasticity of 1 with respect to market share) against market size. Under a wide range of assumptions, the ratio plotted on the vertical axis, the number of French exporters to a market relative to the French share of the market, is an estimate of the total number of firms selling to the market. Figure 7 shows that this estimate of the number of firms selling to a market lines up very neatly with market size, increasing with an elasticity of about two-thirds.

In summary, looking either across destinations or across producers, the microlevel data reinforce the conclusion that national markets are highly fragmented. The producer level data show that this fragmentation reflects primarily the limited entry of exporters into foreign markets.

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2.2 Research and Invention

Measuring the creation of technology and its diffusion around the world raises more serious conceptual challenges than measuring the production of manufactures and their movement across borders. Nevertheless, a number of indirect indicators portray some striking regularities.

The Organization for Economic Cooperation and Development (OECD) assembles data on employment of research scientists and engineers, as well as spending on research and development (R&D), for over 30 countries (essentially the world's richest for reasons other than natural resources). Since our concern is with market-oriented technology, we focus on research activity in the business enterprise sector rather than in government or in universities.[6]

To what extent do countries specialize in innovative activity? Table 2, taken from Eaton and Kortum (2004), reports the number of research scientists and engineers employed in business per 1000 workers for the year 2000 (or nearest available earlier year), ranked from most to least research intensive. As the first two entries (Finland and the United States) suggest, there is no particular tendency for large countries to be more specialized in research than small ones. Research-intensive countries tend to be rich, but many rich countries, such as Denmark, do little research. (Note that very few workers anywhere are designated research scientists and engineers. Only two countries report having more than one percent of their industrial workers engaged in R&D.)

The sheer scale of a nation's research enterprise is more relevant than its specialization in gauging its contribution to the world's inventive activity. Our analysis turns to data on R&D expenditures (although data on research employment tell the

[6]The data are based on periodic surveys of enterprises. OECD (2004) details discrepancies in how the data are reported across countries. For instance, Japan counts all researchers, not just the full-time equivalent. Hence we look at data on research scientists and engineers, research expenditure, and patenting, as a package, to get a sense of who is doing research.

same story). An advantage of expenditure is that we can measure just that part of business enterprise research activity that is privately financed, thus cutting out research activity for military purposes, for example. Figure 8 plots the R&D numbers, which we have averaged over the years 1997-1999 and have presented as each country's share of the OECD total. The overriding feature of these data is the concentration of R&D spending in just three countries, the United States, Japan, and Germany, which together account for more than three-fourths of the OECD total.

To what extent does this feature of concentration hold when we look beyond the OECD? For this task, we use patents granted by the U.S. Patent and Trademark Office (USPTO) in the year 2000 to residents of foreign countries. This patent measure counts the inventions that foreign researchers deemed worth protecting in the U.S. market and that USPTO examiners deemed to have made an inventive step. Figure 9 shows that this patent measure lines up very closely with R&D expenditures across OECD countries, which we take as evidence that foreign patenting in the United States is likely to be a good proxy for R&D for countries outside the OECD.

Figure 10 indicates patents in the United States from all foreign countries whose residents were granted at least 200 US patents. These data also convey a concentration of inventive activity in Japan and Germany, which, together, account for nearly 60 percent of foreign patenting in the United States.[7] Taiwan is the one big

[7] Since there is a strong tendency to patent at home, this patent measure would have an upward

contributor to the world's inventive activity outside the OECD. Our focus on countries granted over 200 patents by the USPTO is not very restrictive; all other countries in the world, taken together, account for less than US patents granted to Swedish applicants.

How do ideas and technologies flow from innovating countries to the rest of the world? Measuring the flows of ideas is much harder than measuring the flow of goods. Patent data provide one possible indicator since a patent granted in one country contains information about the country of residence of either the inventor or (more commonly) the applicant. The fact that an inventor seeks patent protection in a destination suggests that he anticipates his invention abroad could be useful there. Since the early 1980's, investigators such as Bosworth (1984) and Evenson (1984) noted the potential for international patent statistics to trace flows of knowledge. Slama (1981) exploited the analogy to the gravity equation of international trade, an idea that we pursue here.

We look at the frequency with which inventors from country i obtain patents on their inventions in country n. Here we use data on patents issued (granted) in 2000, as reported by World Intellectual Property Organization (WIPO).[8] Let G_{ni} indicate the number of patents granted by country n to inventions from country i. The diagonal

bias if we included US patents to US residents. Below we will compare the United States with other countries in terms of their patenting in Germany.

[8]We use patents granted rather than patent applications due to recent problems with WIPO's patent application data raised by the European patent. See Eaton, Kortum, and Lerner (2004).

elements G_{nn} represent the patents granted by country n to local inventors.

Because facts about the bilateral patent data are less well known than facts about bilateral trade, we begin with two very simple plots. Figure 11 shows which countries account for most of the world's patents, as measured by patents obtained in either the United States or in Germany. Thus, for destinations g = Germany and u = United States we plot G_{gi} against G_{ui}. We can see that, as measured by patents granted either in Germany or in the United States, the three countries leading the world as sources of patentable inventions are Germany, Japan, and the United States. Other countries are far behind these three leaders in patenting (WIPO does not have data for Taiwan). Note the bias toward patenting in the domestic market. US inventors obtain many more patents at home than in Germany while German inventors obtain somewhat more in Germany than in the United States.

Figure 12 looks at which countries are the most popular for obtaining patent protection. Thus for sources g = Germany and u = United States we plot G_{ng} against G_{nu}. As a destination for patents the United States stands out, while Japan is less popular (for inventors from the United States and Germany) than Germany, France, and Great Britain.

Figure 13 is the analog for bilateral patent data to Figures 4 and 5 for bilateral trade data. It plots the bilateral patenting statistic $\sqrt{(G_{ni}G_{in})/(G_{ii}G_{nn})}$ against the distance between i and n. While bilateral trade is very much influenced by distance, bilateral patenting is much less so. Gravity has a much more modest effect on ideas than on goods. Nevertheless, one does observe a distinct drop in patenting between countries farther apart from each other.

Note that all home country observations (i.e., for $i = n$) appear as one point on Figure 13: their bilateral patenting index is 1 (and we have arbitrarily set within-country distance to 100). Yet, for nearly all foreign country pairs, the bilateral patenting index is far below 1. Thus, the patent data suggests a world in which, while technology does spread between countries, diffusion is far from perfect. To pursue this interpretation, of course, one needs a model of the patenting decision. That is the topic of Chapter 11.

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2.3 Productivity

How does research activity and the flow of ideas around the world feed into countries' aggregate productivity? While we would like to answer that question, it is beyond the scope of this data summary to contend with the issues of growth accounting and causality it entails. Instead we simply present an impressionistic account of the evolution of the most basic measure of aggregate productivity, GDP per capita, across a wide swath of countries over a wide swath of time.

The data on GDP per capita (in 1990 dollars) at international prices are from Maddison (2003). We start by choosing the 49 countries for which there is some data on GDP per capita prior to World War II. We then break this sample into the 24 most productive and the 25 least productive countries as of 2001. We include data back to 1870 when available.

Figure 14 plots the data for the countries that are currently the most productive. These countries display a clear upward trend in productivity throughout the period, but with substantial heterogeneity in productivity levels across countries. Countries sometimes move up or down in the pack, but overall there is substantial persistence in rankings. Over the post World War II period these countries were clearly converging to much more similar levels of productivity, a tendency that could largely be an artifact of our having selected them for their current high levels of productivity (see Baumol, 1986, and DeLong, 1988). But, even for this sample, we see roughly parallel growth in the pre-World War II period (see Bernard and Durlauf, 1995).

Figure 15 plots the data for the 25 least productive, with the United States included for perspective. In the post-WW II period these countries also appear to be converging, but to a level of productivity far below that of the United States. Yet, the bottom of the pack is growing roughly in parallel with the United States. When combined with Figure 14 the whole set of countries display remarkably parallel growth over the entire 130 year span.

In summary, these data show remarkable stability in the growth rate, both over time and across countries. Certainly, no country is leaving all the others behind.

The Figure suggests a world in which a single process is driving world growth, although countries have a clear pecking order in terms of their relative positions.[9]

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2.4 Conclusion

In summary, data on trade, patenting, innovation, and growth demonstrate some remarkable regularities. Our goal is a framework that can weave them together into a coherent whole.

[9]Parente and Prescott (1993) elaborate on this view, painting a very clear picture of the central facts about the behavior of per capita GDP across countries over time.

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Chapter 3

CT

Analytic Foundations

Our analysis in the next chapters draws on several literatures. Our quantitative analysis builds on the "gravity" approach to modeling bilateral trade flows. Our theoretical analysis builds on the theory of trade with monopolistic competition, the Ricardian model of trade with a continuum of goods, and the literature on growth in the global economy. We don't attempt to cover each of these areas in depth, but rather refer the reader to some recent, very thorough, surveys. Instead we present some basic results, first from international trade and then from economic growth, that our work builds upon.

A

3.1 International Trade

We consider, in turn, the Armington model of trade, the monopolistically competitive model of trade, and the Ricardian model with a continuum of goods. In our analysis of each, we emphasize the role of trade costs for general equilibrium outcomes.

B

3.1.1 Armington and the Gravity Equation

The Armington model is built on the idea that international trade reflects consumers' desire for goods from different countries.[1] Because the force for trade comes from consumer preferences, we can simplify our analysis by ignoring the production of goods altogether. Instead, we can use the Armington model to focus on the role of trade costs in a general equilibrium analysis of international trade. Many of the relationships that arise in this simple model will appear again, in some guise, when we turn to more realistic models.

Following Anderson (1979), we can also use the Armington model to derive the gravity equation (2.1). As shown in the previous chapter, the gravity equation is a good statistical representation of bilateral trade flows. There is thus something to be said for a theory that is consistent with it.[2] Anderson and van Wincoop (2003) show

[1] This assumption, named for Armington (1969), has been a workhorse in the quantitative analysis of international trade.

[2] Deardorff (1998) provides a nice explanation of how the gravity equation relates to other theories of international trade.

that a theoretical derivation of the gravity equation can resolve some puzzles that have arisen in interpreting estimates of it. We summarize their arguments below.

Consider N countries. Each country i has a quantity y_i of a good unique to it. We can name this good after the country it comes from, "good i." We can think of y_i simply as an endowment. Alternatively, treating input supplies and technology as exogenous, we can think of y_i as the output of a good (or composite of goods) which the country completely specializes in producing. Specialization itself is not modeled, as it is in the monopolistically competitive and Ricardian cases taken up below.

Consumers everywhere have identical constant elasticity of substitution (CES) preferences, with a preference weight $\alpha_i > 0$ on good i. The elasticity of substitution between goods from different countries is σ. Welfare in country n is thus:

$$U_n = \left[\sum_{i=1}^{N} \alpha_i^{1/\sigma} y_{ni}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

where y_{ni} is country n's consumption of the good i. For most of our discussion of the Armington model we need only restrict $\sigma \geq 0$, although, as we see below, for the model to match basic features of the data described in the previous chapter, requires $\sigma > 1$.

In place of a transport sector, we adopt Samuelson's (1952) "iceberg" assumption. Delivering a unit of i's good to n requires shipping $d_{ni} \geq 1$ (with $d_{ii} = 1$) units from i. Anderson and van Wincoop refer to d_{ni} as the "bilateral resistance" to trade between n and i.

The assumption that trade costs augment production costs multiplicatively is very common in the general equilibrium modeling of international trade. A natural alternative is to treat the transport cost as additive, but a reformulation of the analysis in this chapter and those that follow under the additive alternative would be vastly more complicated, as the reader embarking on such a task can quickly verify. Evidence on the form of trade costs is mixed.[3]

Consumption around the world of good i is constrained by the world's endowment of it. Taking account of the iceberg transport technology and summing across

[3]Hummels and Skiba (2004) shed some light on how trade costs vary with production cost by regressing freight costs on f.o.b. prices in a set of destinations within a wide range of narrowly defined product categories. They find that freight costs increase with f.o.b. price with an elasticity strictly below one (the elasticity implied by the multiplicative assumption) but well above zero (the elasticity implied by a purely additive specification). Their results indicate the need for both more theory and measurement of trade barriers. For one thing, they do not provide evidence on how freight costs vary with f.o.b. prices across product categories. For another, freight costs constitute only one component of the geographic barriers to trade, which also include the cost of searching for a supplier, negotiating a purchase, and servicing the product subsequently. Rauch (1999) provides important indirect evidence on the role of trade barriers that arise for reasons other than shipping. He divides internationally traded goods into three categories: (1) goods for which there are organized exchanges, (2) goods offered for sale at a posted reference price by the supplier, and (3) differentiated products. Estimating gravity equations for goods in different categories, he finds that distance and differences in language are most inhibiting for trade among goods in the third category. His interpretation is that trade in differentiated products requires search and negotiation, which are facilitated by proximity and common language.

destinations, the resource constraint for each good i is

$$y_i = \sum_{n=1}^{N} d_{ni} y_{ni}.$$

Due to bilateral resistance, the law of one price will not hold, i.e. the price of good i will differ across markets n. We denote the price of good i in country n by p_{ni}. Taking account of these price differences, country i's total income is

$$Y_i = \sum_{n=1}^{N} p_{ni} y_{ni},$$

of which $Y_i - p_{ii} y_{ii}$ is income from exports. The budget constraint for country n spending X_n is:

$$X_n = \sum_{i=1}^{N} p_{ni} y_{ni},$$

of which $X_n - p_{nn} y_{nn}$ is spending on imports.

We consider a competitive equilibrium. In particular, we look for a set of prices p_{ni} and consumption amounts y_{ni} such that: (i) given prices, each country i sells its endowment so as to maximize its income Y_i subject to the resource constraint and (ii) given income Y_n and prices, the representative consumer in each country n allocates spending across goods i so as to maximize utility U_n subject to its budget constraint. Much of what we say holds whatever the trade deficit $D_n = X_n - Y_n$, although to solve for the general equilibrium below we assume balanced trade $X_n = Y_n$.[4]

[4] In a static model, D_n can be thought of as a transfer to n from the rest of the world. The budget

The solution for a consumer with total spending X_n is to spend:

$$X_{ni} = \alpha_i \left(\frac{p_{ni}}{P_n}\right)^{-(\sigma-1)} X_n, \tag{3.1}$$

on good i, where P_n is the CES price index in country n:

$$P_n = \left[\sum_{k=1}^{N} \alpha_k (p_{nk})^{-(\sigma-1)}\right]^{-1/(\sigma-1)}. \tag{3.2}$$

This solution is standard except that we express it in terms of expenditures (rather than quantities) to make a more explicit link to the data.[5]

For any finite prices, each country n demands some of good i. Country i will be willing to sell positive amounts to each country n only if p_{ni}/d_{ni} is the same

constraints imply that transfers must net out to zero around the world:

$$\sum_{n=1}^{N} D_n = 0.$$

Dekle, Eaton, and Kortum (2007, 2008) show how to incorporate deficits into this sort of model and also show how changes in deficits impinge on the trade equilibrium.

[5] The representative consumer in n chooses y_{ni} ($i = 1, \ldots, N$) to maximize U_n given prices p_{ni} ($i = 1, \ldots, N$) and spending X_n, subject to the budget constraint $X_n = \sum_{i=1}^{N} p_{ni} y_{ni}$. The first-order conditions for y_{ni} can be written:

$$\alpha_i^{1/\sigma} y_{ni}^{-1/\sigma} U_n^{1/\sigma} = \lambda_n p_{ni}$$

or:

$$y_{ni} = \alpha_i U_n \lambda_n^{-\sigma} p_{ni}^{-\sigma},$$

where λ_n is the Lagrange multiplier on the budget constraint. Multiplying each side of the second version of the first-order condition by p_{ni} and taking the ratio for purchases from countries k and i

across markets. Thus, the competitive equilibrium implies $p_{ni}/d_{ni} = p_{ii}$ or, equivalently, $p_{ni} = d_{ni}p_{ii}$ for all n.

Since a similar expression keeps coming up in different models, we assemble what we have so far into an expression for the fraction of spending from n devoted to goods from i:

$$\frac{X_{ni}}{X_n} = \frac{\alpha_i \left(p_{ii}d_{ni}\right)^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}}. \tag{3.3}$$

Country i's trade share in n is its contribution to the sum $\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}$. Its contribution reflects (i) its importance in preferences α_i, (ii) the local price of its goods

yields:

$$\frac{X_{nk}}{X_{ni}} = \frac{\alpha_k}{\alpha_i}\left(\frac{p_{nk}}{p_{ni}}\right)^{-(\sigma-1)}.$$

Summing both sides of this expression over $k = 1, \ldots, N$:

$$\frac{X_n}{X_{ni}} = \frac{\sum_{k=1}^{N} \alpha_k \left(p_{nk}\right)^{-(\sigma-1)}}{\alpha_i \left(p_{ni}\right)^{-(\sigma-1)}} = \frac{P_n^{-(\sigma-1)}}{\alpha_i \left(p_{ni}\right)^{-(\sigma-1)}},$$

which, inverted, delivers (3.1) with the price index (3.2). The price index P_n is the deflator that converts expenditures X_n into utility. To see this result, start with the first version of the first-order condition above and multiply each side by y_{ni} to get:

$$\alpha_i^{1/\sigma} y_{ni}^{(\sigma-1)/\sigma} = \lambda_n X_{ni} U_n^{-1/\sigma}.$$

Plugging these conditions for each i into the utility function yields $U_n = \lambda_n X_n$. Substituting this result into the second version of the first-order condition above and multiplying each side by p_{ni} gives:

$$X_{ni} = \alpha_i X_n \left(\lambda_n p_{ni}\right)^{-(\sigma-1)}.$$

Comparing this expression with (3.1) shows that $\lambda_n = 1/P_n$. Hence $U_n = X_n/P_n$.

p_{ii}, and (iii) the cost of getting the goods from i to n, as determined by d_{ni}. Note how the elasticity of substitution σ governs the sensitivity of trade shares to trade costs. If $\sigma < 1$ then the value of trade between countries rises with trade costs. In Figures 4, and 5 of the previous chapter we saw that trade falls systematically with distance. Since it's natural to think that trade costs rise with distance, for the Armington model to capture the relationship in these figures requires elastic demand. Thus, to avoid a taxonomy of cases, we impose $\sigma > 1$ in the remainder of our analysis of the Armington model.

To get each country's income Y_i, multiply both sides of the resource constraint by p_{ii} and apply the result about international price differences to obtain:

$$p_{ii}y_i = \sum_{n=1}^{N} p_{ni}y_{ni} = Y_i. \tag{3.4}$$

This equation states that country i's income is simply the value of its endowment at local prices or, equivalently, its sales around the world. Noting that $X_{ni} = p_{ni}y_{ni}$ we substitute (3.3) into (3.4) to obtain country $i's$ income as:

$$Y_i = \sum_{n=1}^{N} \frac{\alpha_i \left(p_{ii}d_{ni}\right)^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}} X_n. \tag{3.5}$$

Since $Y_i = p_{ii}y_i$, this expression allows us to solve for prices as a function of each country's spending levels X_n as well as α_n, y_n, and d_{ni} (whether or not trade is balanced).

C Gravity Results

Even without solving for local prices p_{ii} we can use (3.5) to derive the gravity equation, rewriting it as:

$$\begin{aligned} Y_i &= \alpha_i p_{ii}^{-(\sigma-1)} \sum_{n=1}^{N} \left(\frac{d_{ni}}{P_n}\right)^{-(\sigma-1)} X_n \\ &= \alpha_i p_{ii}^{-(\sigma-1)} \Xi_i, \end{aligned} \tag{3.6}$$

where:

$$\Xi_i = \sum_{m=1}^{N} \left(\frac{d_{mi}}{P_m}\right)^{-(\sigma-1)} X_m.$$

Substituting into (3.1), with $p_{ni} = d_{ni} p_{ii}$, gives:

$$X_{ni} = \frac{Y_i X_n}{\Xi_i} \left(\frac{d_{ni}}{P_n}\right)^{-(\sigma-1)}. \tag{3.7}$$

Setting $\tau_{ni} = d_{ni}^{\sigma-1}$ in (3.7) yields an expression with all the ingredients of the simple gravity equation (2.1).

But, as Anderson and van Wincoop point out, equation (3.7) has two additional terms that reflect the proximity of third countries. One is the price index P_n in the destination. Given its own cost of shipping to a destination, country i will fare better in countries that are more remote from other suppliers, since i faces less competition there. A destination's remoteness is reflected in a high price index there. The other term is Ξ_i, often called the source's "market potential." If source i is itself more remote from other countries, as implied by a smaller value of Ξ_i, it will sell more in market n given its cost of shipping there.[6]

[6] Anderson and van Wincoop go on to show that if trade is balanced, $X_i = Y_i$, and if trade costs are symmetric, meaning that $d_{ni} = d_{in}$, then $\Xi_i = cP_i^{-(\sigma-1)}$, where c, determined below, is a constant which does not vary with i. Countries that have lower prices also have greater market potential (by virtue of their proximity to other countries both as suppliers and as consumers). In this case the gravity equation simplifies to:

$$X_{ni} = \frac{1}{c} Y_i Y_n \left(\frac{d_{ni}}{P_i P_n}\right)^{-(\sigma-1)}.$$

Given the numeraire $p_{NN} = 1$, the constant c can be computed as:

$$c = \frac{y_N}{\alpha_N} P_N^{\sigma-1}.$$

Since the derivation is not trivial we include it here. Using the definition of Ξ_i, imposing trade balance, substituting in (3.6), and employing symmetric trade costs:

$$\begin{aligned}
\Xi_i &= \sum_{m=1}^{N} \left(\frac{d_{mi}}{P_m}\right)^{-(\sigma-1)} Y_m \\
&= \sum_{m=1}^{N} \left(\frac{d_{mi}}{P_m}\right)^{-(\sigma-1)} \alpha_m p_{mm}^{1-\sigma} \Xi_m \\
&= \sum_{m=1}^{N} \alpha_m \left(\frac{p_{mi}}{P_m}\right)^{-(\sigma-1)} \Xi_m
\end{aligned}$$

If we conjecture that $\Xi_m = cP_m^{-(\sigma-1)}$ then

$$\begin{aligned}
\Xi_i &= \sum_{m=1}^{N} c\alpha_m p_{mi}^{-(\sigma-1)} \\
&= cP_i^{-(\sigma-1)},
\end{aligned}$$

thus confirming the conjecture. From (3.6) applied to country N:

$$Y_N = y_N = \alpha_N c P_N^{-(\sigma-1)},$$

which can be used to solve for c.

Anderson and van Wincoop refer to the effects of P_n and Ξ_i as reflecting country i's and n's "multilateral resistance" to trade. A high Ξ_i means that i has good selling opportunities outside market n, so will, other things equal, sell less to n. A lower P_n means that country n has good buying opportunities elsewhere than from i, so will, other things equal, buy less from i. Since a larger country has a big home market its Ξ_i is likely higher and its P_i lower, reducing its bilateral trade. As we saw in Figures 1 and 2 of the previous chapter, the raw elasticity of imports with respect to absorption is less than one. Moreover, since larger countries also tend to be farther from their neighbors, the standard formulation of the gravity equation with these multilateral resistance terms omitted yields estimates that overstate the negative effect of distance on bilateral trade.

Even correcting for multilateral resistance, however, distance has a dampening effect on trade. Figures 4 and 5 of the previous chapter relate (the square root of) the statistic $X_{ni}X_{in}/X_{nn}X_{ii}$ to distance. From Equation (3.1):

$$\frac{X_{ni}X_{in}}{X_{nn}X_{ii}} = (d_{ni}d_{in})^{-(\sigma-1)},$$

purging the theoretical gravity relationship of anything but its strictly bilateral component, including multilateral resistance, thus addressing the Anderson-van Wincoop critique.[7]

[7] Suppose we posit that trade costs are related to the distance k_{ni} between n and i according to $d_{ni} = d_{in} = \beta_0(k_{ni})^{\beta_1}$ for $n \neq i$. Our findings in Figures 4 and 5 suggest that $(\sigma - 1)\beta_1 \approx 1$ has

C General Equilibrium Results

While we can derive the gravity equation without solving for prices p_{nn}, we need them to address questions about the gains from trade, to which we now turn. Imposing trade balance ($X_n = Y_n$) in (3.5) we obtain a set of equations determining prices p_{ii} in terms of the primitives α_i, y_i, and d_{ni}:

$$p_{ii}y_i = \sum_{n=1}^{N} \frac{\alpha_i \left(p_{ii}d_{ni}\right)^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}} p_{nn}y_n. \tag{3.8}$$

It is illuminating to rewrite these conditions for equilibrium prices into ones for equilibrium incomes $Y_i = p_{ii}y_i$. To do so we define

$$s_n = \alpha_n^{1/(\sigma-1)} y_n$$

which summarizes how a country's endowment and the preference for it combine to determine its economic size. We can then turn expression (3.8) into:

$$Y_i = \sum_{n=1}^{N} \frac{s_i^{\sigma-1}(Y_i d_{ni})^{-(\sigma-1)}}{\sum_{k=1}^{N} s_k^{\sigma-1}(Y_k d_{nk})^{-(\sigma-1)}} Y_n. \tag{3.9}$$

These equations, one for each country i (with, by Walras Law, one redundant), can be solved for the $N-1$ incomes Y_i. If we make $p_{NN} = 1$ the numeraire, $Y_N = y_N$. Having solved for the Y_i we can recover the prices from $p_{ii} = Y_i/y_i$. Except for a few special cases, discussed below, there are not analytical solutions to (3.8) or (3.9). For most

remained constant while β_0 has fallen over time, leading to the rise in international trade.

cases they must be solved numerically.[8]

For calculating welfare note that:

$$U_n = \frac{p_{nn} y_n}{P_n};$$

welfare of country n is increasing in the quantity of its physical endowment y_n and its price p_{nn} relative to the overall price level P_n.

For a back-of-the-envelope calculation of the gains from trade, we can simply evaluate (3.1) for $i = n$. Solving the result for p_{nn}/P_n and multiplying by the endowment yields:

$$U_n = s_n \left(\frac{X_{nn}}{X_n} \right)^{-1/(\sigma-1)}. \tag{3.10}$$

Given s_n, country n is better off if, in equilibrium, a smaller share of its expenditure is devoted to its own good. As trade barriers rise, this share rises to one and welfare approaches s_n. The welfare gain from trade, relative to autarky, is $(X_{nn}/X_n)^{-1/(\sigma-1)}$. However, this measure does not relate the gains from trade to underlying parameters since X_{nn}/X_n is endogenous.[9]

In a couple of special cases we can solve equations (3.8) explicitly. The first case, frictionless trade, is when $d_{ni} = 1$ for all n and i. We then obtain $p_{nn} =$

[8] As discussed above, we can reinterpret this model as one in which each country i has an endowment L_i of labor specialized in the production of the country's distinct good. With output per worker a_i we replace y_i with $a_i L_i$ and p_{ii} with w_i/a_i, where w_i is the wage.

[9] For $\sigma < 1$, spending on imports actuall rises with trade barriers, but since the exponent switches signs, (3.10) still captures the gains from trade.

$[(\alpha_n/y_n)/(\alpha_N/y_N)]^{1/\sigma}$. Notice that the terms of trade turn against a country with a larger endowment. Plugging this result into the equation for the price index and rearranging delivers:

$$U_n = s_n \left[\sum_{i=1}^{N} \left(\frac{s_i}{s_n} \right)^{(\sigma-1)/\sigma} \right]^{1/(\sigma-1)}.$$

Country n's welfare is increasing in its own size and in the size of its trading partners.

The other special case is when trade costs are common, $d_{ni} = d$, for all $n \neq i$, and countries are the same size s. In this case $p_{ii} y_i = Y_i = Y$ is common across countries, as is the price level $P_n = P$.[10] Solving for welfare gives:

$$U = \frac{Y}{P} = s \left[1 + (N-1) d^{-(\sigma-1)} \right]^{1/(\sigma-1)}.$$

Note that welfare is decreasing in the trade cost.[11]

For either special case we make no statement about the effect of the number

[10] The price index is

$$\begin{aligned} P_n^{-(\sigma-1)} &= \sum_{k \neq n}^{N} \alpha_k \left(p_{kk} d \right)^{-(\sigma-1)} + \alpha_n p_{nn}^{-(\sigma-1)} \\ &= s^{(\sigma-1)} \left[\sum_{k \neq n}^{N} \left(Y_k d \right)^{-(\sigma-1)} + Y_n^{-(\sigma-1)} \right] \\ &= \left(\frac{s}{Y} \right)^{(\sigma-1)} \left[(N-1) d^{-(\sigma-1)} + 1 \right]. \end{aligned}$$

[11] With both symmetry *and* frictionless trade home share in purchases would be just $1/N$. Using (3.10) a measure of the benefit of moving from the status quo to frictionless trade would be: $(N X_{nn}/X_n)^{1/(\sigma-1)}$.

of trading partners on welfare, since the rules of Armington don't allow us to change this number without changing preferences.

The Armington framework provides an excellent tool to focus purely on the role of trade costs without having to model the forces that shape specialization. Given that much of the policy interest in trade concerns exactly issues of industrial structure, we now turn to theoretical frameworks in which trade has nontrivial implications for who makes what.

B

3.1.2 Monopolistic Competition

Dixit and Stiglitz (1977) revitalized the theory of monopolistic competition, providing a framework for incorporating it into general equilibrium analysis. A series of papers by Krugman (1979a, 1980) and, most thoroughly, in a book by Helpman and Krugman (1985) explored the theoretical implications of the framework for international trade. As we see below, monopolistic competition delivers a formulation for bilateral trade flows that mirrors that implied by the simpler Armington assumption.

In its simplest version the model does not focus on differences in factor intensity, so we can posit a single factor of production, which we call labor. Each country i has a given endowment L_i.[12] Workers are free to engage in different activities at home,

[12] More generally, we could posit a composite input bundle, but differences in intensities across individual inputs would not play a role in trade and specialization.

but don't move between countries. The wage w_i is thus the same across all activities in i, but can differ between countries.[13]

In its original formulation, differences in efficiency across goods and countries were not a focus, so one can posit a common output per worker z. Setting up the production of a good requires an additional F workers. The range of goods produced and consumed in a country arises endogenously through entry into production. Each producer makes a different good. The space of goods is most easily modelled as a continuum. We index goods by j.

A representative consumer in country n has preferences of the form:

$$U_n = \left[\int y_n(j)^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)}$$

where $y_n(j)$ is consumption of good j and σ is the elasticity of substitution. Since some goods are not available, so that $y_n(j) = 0$, for consumers to get any utility from goods that are available requires $\sigma > 1$. We impose this restriction.

If total spending in country n is X_n, spending on commodity j in country n

[13] Throughout this chapter we treat products as final goods. Essentially equivalent results emerge if products are instead intermediates used to produce a nontraded final good according to a CES production function, as demonstrated by Ethier (1979) for monopolistic competition. We will not continue to point out this alternative interpretation, but ask the reader to remain aware that with appropriate redefinitions the goods in question could be final, intermediate, or both. Intermediate goods will emerge in subsequent chapters.

is:

$$x_n(j) = X_n \left(\frac{p_n(j)}{P_n} \right)^{-(\sigma-1)}$$

where $p_n(j)$ is the price of good j in country n, and P_n is the price index:

$$P_n = \left[\int p_n(k)^{-(\sigma-1)} dk \right]^{-1/(\sigma-1)}.$$

We can think of a good that isn't available in country n as having an infinite price.[14]

The market structure is monopolistic competition. Each good is produced by a separate monopolist who takes total spending X_n and the price index P_n in each market as given. Markets are segmented so that producers can set a different price in each national market. Profit maximization results in a price markup over unit cost,

[14] To establish these relationships we proceed much as we did with Armington. The representative consumer in n chooses $y_n(j)$ to maximize U_n given prices $p_n(j)$ and subject to the budget constraint $X_n = \int p_n(j) y_n(j) dj$. The first-order condition for $y_n(j)$ gives:

$$y_n(j) = U_n \lambda_n^{-\sigma} p_n(j)^{-\sigma},$$

where λ_n is the Lagrange multiplier on the budget constraint. Multiplying each side by $p_n(j)$ gives:

$$x_n(j) = U_n \lambda_n^{-\sigma} p_n(j)^{-(\sigma-1)}.$$

Integrating across all goods j and rearranging yields:

$$U_n \lambda_n^{-\sigma} = X_n P_n^{(\sigma-1)}.$$

Substituting back into the previous expression yields the result. As in the Armington case $U_n = X_n / P_n$.

inclusive of transport, of $\overline{m} = \sigma/(\sigma-1)$.[15] Thus any firm in i will charge a price $p_{ni} = \overline{m}w_i d_{ni}/z$ when selling in n. Hence the revenue of a representative firm from i in n is:

$$x_{ni} = X_n \left(\frac{\overline{m}w_i d_{ni}}{zP_n}\right)^{-(\sigma-1)} \tag{3.11}$$

while its profit in market n is:

$$\Pi_{ni} = \frac{x_{ni}}{\sigma}. \tag{3.12}$$

Denoting the measure of goods produced in i as H_i, the price index in market n is:

$$P_n = \overline{m}\left[\sum_{i=1}^{N} H_i (w_i d_{ni}/z)^{-(\sigma-1)}\right]^{-1/(\sigma-1)}.$$

In the basic formulation the fixed cost $w_i F$ applies to a firm in i establishing a product, not to entering a market. Since free entry eliminates profit, all income goes to labor (either directly in production or for setting up firms) so that $X_n = w_n L_n$.

Two conditions determine the vector of wages and the measure of products produced in each country. One is the zero profit condition enforced by free entry, which

[15] A firm with unit cost c_n in market n charging a price p_n earns a profit (gross of fixed cost)

$$\Pi_n = (1 - c_n/p_n)\, X_n \left(\frac{p_n}{P_n}\right)^{-(\sigma-1)}$$

there. Maximizing with respect to p_n delivers:

$$p_n = \frac{\sigma}{\sigma - 1} c_n.$$

establishes that:

$$w_i^\sigma \sigma F = (\overline{m}/z)^{-(\sigma-1)} \Xi_i \quad i = 1, ..., N, \tag{3.13}$$

where:

$$\Xi_i = \sum_{n=1}^{N} \left(\frac{d_{ni}}{P_n} \right)^{-(\sigma-1)} X_n$$

is equivalent to the "market potential" term for the Armington case. The other is that total spending on country i's production equal its wage bill, which establishes that:

$$w_i L_i = H_i \sum_{n=1}^{N} x_{ni}$$

which, using (3.11), can be written:

$$w_i^\sigma L_i = (\overline{m}/z)^{-(\sigma-1)} H_i \Xi_i \quad i = 1, ..., N \tag{3.14}$$

Dividing (3.13) by (3.14) yields:

$$H_i = \frac{L_i}{\sigma F}, \tag{3.15}$$

implying that the measure of products a country produces is proportional to its labor force. Note that, even though H_i is endogenous, it does not depend on the extent of trade barriers. In particular, trade does not reduce the measure of goods that a country produces, as it typically does in the Ricardian model taken up next.

Relative wages are given by the solution to the system of equations:

$$w_i L_i = \sum_{n=1}^{N} \frac{L_i (w_i d_{ni})^{-(\sigma-1)}}{\sum_{k=1}^{N} L_k (w_k d_{nk})^{-(\sigma-1)}} w_n L_n \quad i = 1, ..., N. \tag{3.16}$$

Note the role for geography in determining wages. Countries with lower market potential, i.e., more distant from large markets, need to have lower relative wages in order to compete abroad.

Like the Armington model developed above, monopolistic competition readily yields an expression for bilateral trade. The value of exports from i to n is:

$$X_{ni} = H_i x_{ni} = \frac{H_i(\overline{m} w_i d_{ni}/z)^{-(\sigma-1)}}{P_n^{-(\sigma-1)}} X_n. \tag{3.17}$$

Analogous with equation (3.3) for Armington model, we have the following expression for the fraction of n's expenditure devoted to goods from i:

$$\frac{X_{ni}}{X_n} = \frac{L_i(w_i d_{ni})^{-(\sigma-1)}}{\sum_{k=1}^{N} L_k(w_k d_{nk})^{-(\sigma-1)}}. \tag{3.18}$$

The labor forces L_i replace the preference terms α_i in the Armington model, while wages replace the local prices p_{ii}. Otherwise the expression is the same. The major difference is that under the Armington assumption the share of a country's goods in preferences is exogenous while, under monopolistic competition, it rises with the labor force (since larger countries endogenously produce a greater variety of distinct goods).[16]

To get an expression more in line with the gravity equation, we use (3.14) to

[16] Interpreting the Armington model as one with specialized production, an important difference with monoplistic competition is the implication of having relatively more labor. Given the preference terms α_i in the Armington model, having more workers, by raising y_i, worsens the terms of trade. In monopolistic competition more workers at home is good, as it means that a greater variety of products can be purchased without having to incur trade costs. (Once trade costs disappear, relative size is

write:

$$Y_i = w_i L_i = (\overline{m} w_i / z)^{-(\sigma-1)} H_i \Xi_i.$$

Substituting this expression into (3.17) gives:

$$X_{ni} = \frac{Y_i X_n}{\Xi_i} \left(\frac{d_{ni}}{P_n} \right)^{-(\sigma-1)}, \tag{3.19}$$

exactly the same expression yielded by the Armington analysis, (3.7). Anderson and van Wincoop's message about the importance of including multilateral resistance stands.

But the monopolistic competition framework has quite different implications for how aggregate trade volumes vary at the extensive margin (number of goods shipped) and the intensive margin (amount of each good shipped). In Armington, all variation is at the intensive margin: A larger country exports more because it exports more of a given good, while it also imports more of each of a given set of goods. Under monopolistic competition, a larger country exports more because it exports a greater variety. Hence size-induced variation in export volumes across countries is purely at the extensive margin. But, as in Armington, size-induced variation in import volumes is purely at the intensive margin. Since every country imports every good, a larger country imports more purely because it is buying more of each one.

Monopolistic competition has provided the basis of a number of empirical studies of bilateral trade patterns, based on variants of equation (3.19). An early

a matter of indifference). Fixing relative labor supplies, a bigger world (holding fixed the number of countries) is a matter of indifference in Armington, but a good thing with monopolistic competition.

example is Helpman (1987), followed by many others.[17] Following most closely the analysis here is Redding and Venables (2004). They also includes a role for intermediate inputs, which also play an important part in our own work, as we discuss in Chapter 6.

An additional feature of monopolistic competition as a framework for analyzing trade flows is that it identifies a nontrivial role for individual producers. As data are becoming much more available on plant and firm participation in international trade, this feature is a major plus.[18] The basic framework makes stark predictions about how bilateral trade flows break down into the number of producers selling and how much each one sells. Looking across exporting countries, large countries export more because they have proportionately more firms, but an individual firm from a large country is not predicted to sell more abroad than one from a small country. Differences in export volumes per firm are dictated by geography rather than by country size. Looking across importing countries, large countries buy more because they purchase proportionately more from each foreign producer. All destinations purchase from the full range of individual producers.

Figure 7 of the previous chapter shows that in this last prediction the basic model falls flat on its face in two respects: Large markets attract systematically more

[17] Notable contributions include Hummels and Levinsohn (1995) and Debaere (2005).

[18] In contrast, trade theories based on perfect competition, such as the Ricardian one we turn to next, make no prediction about what to expect at the level of the individual producer to guide the analysis of the data.

firms, while differences in an export country's market share across destinations are almost all due to the number of its firms that sell there.[19]

Melitz (2003) provides an important extension to the monopolistically competitive model of trade which can potentially loosen its tight implications about the margin of entry. He introduces two innovations. First, he assumes that there is heterogeneity across potential producers in the unit cost of production. Second, as in Romer (1994), he posits a fixed cost of entering a foreign market. These assumptions have implications for variation in trade volumes at the extensive and intensive margins. More recently, Helpman, Melitz and Rubinstein (2008) have adapted his approach to the quantitative analysis of bilateral trade flows. In particular, unlike most theoretical formulations of the gravity model, their analysis allows for observations of zero trade. Since this analysis is intimately connected with our own in the following chapters, we postpone further discussion of these contributions for later chapters.

Unlike the Armington approach in which each country produces a different set of goods for exogenous reasons, under monopolistic competition producers in each country endogenously choose to produce a different set of goods. But the model does not deliver implications for how trade might shift specialization across industries.

[19] Hummels and Klenow (2005), using data on detailed product categories, which may proxy for the number of individual producers, look at the export breakdown. They find that the elasticity of the number of varities exported with respect to size is about .6; large, but less than the 1 predicted by the basic model.

B

3.1.3 Ricardo with a Continuum of Goods

Ricardo (1821) provided a model of the effects of trade on specialization in general equilibrium. To restate the canonical example, two countries (say H and F) have endowments of labor and constant-returns-to-scale technologies for producing each of two commodities (say C and W) using only labor. We can describe these technologies in terms of output per worker $z_i(j)$, $i = H, F$, and $j = C, W$. Workers are perfectly mobile between activities within a country, but not between countries. Goods are costlessly traded.

Ricardo showed that if country H has a comparative advantage in good C:

$$\frac{z_H(C)}{z_F(C)} > \frac{z_H(W)}{z_F(W)} \tag{3.20}$$

then H exports C and imports W, and at least one of the countries is better off (and neither worse off) due to this trade. Details to be worked out were if the equilibrium involved country H producing only C, country F producing only W, or both.[20]

For nearly two centuries Ricardo's formulation has served as an extremely useful vehicle for illustrating the gains from trade and specialization. Until very recently, however, it did not provide a basis for the quantitative analysis of bilateral trade flows.[21] The impediment is the vast array of possible types of equilibria it throws out (depending

[20] As Chipman (1965) documents, working out these details took almost a century.

[21] A literature initiated by MacDougall (1951,1952) looked at the relationship between measured productivity across industries and export specialization. This approach was limited to considering a pair of countries as exporters to the rest of the world, however, so could not deal with the simultaneous

on who is completely specialized, or not, in what) in a realistic multicountry, multigood setting.[22].

Something of a breakthrough occurred with Dornbusch, Fischer, and Samuelson (1977, henceforth DFS). By treating the space of goods as a continuum, it was no longer necessary to consider outcomes with complete and incomplete specialization separately. The pivotal good that might potentially be produced in both countries has zero measure, so can be ignored.

Since the DFS model is a special case of our own formulation of trade, we present a synopsis of their model in terms of elements of the approach that we develop in subsequent chapters.

Consider a unit continuum of goods indexed by $j \in [0, 1]$ which can be produced in the home country (H) or the foreign country (F). A worker in country $i = H, F$ can produce $z_i(j)$ units of good j. DFS perform their analysis using the ratio

determination of bilateral trade flows around the world.

[22]Relaxing twoness either in the number of countries or in the number of goods is relatively straighforward, as shown by Jones (1961). It's relaxing both together that causes trouble. Jones provides the criterion for the efficient assignment of goods to countries in higher dimensions, showing that the way to generalize Ricardo's criterion for the assignment of goods to countries is to reformulate inequality (3.20) as the assignment that maximizes the product of labor productivities. But with many countries and goods, even once this assignment is found there are many possible patterns of complete and incomplete specialization that have to be considered to solve for the equilibrium.

of efficiencies in H and F, defined as:

$$A(j) = \frac{z_H(j)}{z_F(j)}$$

where goods are ordered so that, for any j and j' such that $j' \geq j$, $A(j') \leq A(j)$. DFS impose the additional requirement that A is continuous and *strictly* decreasing in j.

Preferences are Cobb-Douglas and identical in each country, with each good j having equal share.[23] Each country i has an endowment of L_i workers. Perfect competition prevails.

As before, each country i has a wage w_i and iceberg costs are d_{ni}. The cost of good j in market n if purchased from country i is $w_i d_{ni}/z_i(j)$. We take F's labor as numeraire but leave w_F in the equations for ease of interpretation. Since goods are bought from the lowest cost source, the home country will produce for itself the range of goods $[0, \bar{j}]$, where $\bar{j}$ satisfies:

$$A(\bar{j}) = \frac{w_H}{d_{HF} w_F}. \tag{3.21}$$

Similarly, the foreign country will produce for itself the range of goods $[\underline{j}, 1]$ where $\underline{j}$ satisfies:

$$A(\underline{j}) = \frac{d_{FH} w_H}{w_F}. \tag{3.22}$$

[23] In contrast to monopolistic competition, the range of goods is given and every good is produced and consumed in equilibrium. Hence we don't need to worry about individual goods coming and going. Since the original model was formulated with Cobb-Douglas preferences, we stick with that here.

Since $A(j)$ is a strictly decreasing function, both $\underline{j}$ and $\overline{j}$ are decreasing in w_H, with $\underline{j} < \overline{j}$ if either trade cost is strictly greater than 1.

Income and expenditure in each country n is $w_n L_n$. Thus H's sales at home are just $\overline{j} w_H L_H$ while its export revenues are $\underline{j} w_F L_F$. Full employment in H thus requires that:

$$w_H L_H = \underline{j} w_F L_F + \overline{j} w_H L_H. \tag{3.23}$$

Together, equations (3.21), (3.22), and (3.23) determine $\underline{j}$, $\overline{j}$, and the relative wage $\omega = w_H/w_F$. The solution is unique since the right hand side of the expression:

$$L_H = A^{-1}\left(\omega d_{FH}\right) L_F/\omega + A^{-1}\left(\omega/d_{HF}\right) L_H \tag{3.24}$$

is strictly decreasing in ω while the left-hand side is independent of ω. With the relative wage $\omega = w_H/w_F$ determined by (3.24), $\overline{j}$ and $\underline{j}$ are nailed down by (3.21) and (3.22).

DFS's Ricardian analysis, unlike Armington or monopolistic competition, captures how trade can alter the set of goods a country produces. In the absence of trade ($d_{FH} = d_{HF} = \infty$) the solution is $\underline{j} = 0$, $\overline{j} = 1$, and we can normalize $w_H = w_F = 1$. Each country produces each good on the unit interval. Lowering the iceberg trade cost leads countries to: (i) cease production of the goods in which they have strongest comparative disadvantage, goods $j \in [\overline{j}, 1]$ for H and goods $j \in [0, \underline{j}]$ for F, (ii) specialize production in the goods in which they have a comparative advantage, and (iii) export those in which their comparative advantage is strongest. But, with positive trade costs (and given that $A(j)$ is a continuous function) the world will not be one of perfect specialization. A middle range of goods $j \in [\underline{j}, \overline{j}]$ countries produce for themselves and do not trade.[24]

A particular parameterization of relative productivity foreshadows the analysis in the following chapters. We posit:

$$A(j) = \frac{z_H(j)}{z_F(j)} = \left(\frac{T_H}{T_F}\right)^{1/\theta} \left(\frac{j}{1-j}\right)^{-1/\theta}. \tag{3.25}$$

The parameters T_H and T_F capture each country's absolute advantage while the parameter $\theta > 0$ governs the strength of comparative advantage. The elasticity of $A(j)$ with respect to j is proportional to $-1/\theta$. Hence the lower θ the larger a given increase in j reduces H 's comparative advantage.[25]

We can now proceed as in DFS, with $A(j)$ taking this particular form, to derive trade patterns and the relative wage. Country H's total expenditure is $X_H = w_H L_H$,

[24] A shortcoming of the DFS approach as a framework for quantitative analysis is its limitation to two countries. Wilson (1980) provides an important conceptual generalization of DFS to many countries. Like DFS, Wilson represents technologies in each country i as a function $z_i(j)$ defined over $j \in [0, 1]$. Rather than working with ratios of efficiencies, however, his analysis uses the $z_i(j)$ functions directly. Note that one is allowed an overall reordering of the goods to obtain well behaved $z_i(j)$ functions, but the ordering of goods must be common to all countries $i = 1, \ldots, N$. While Wilson's analysis provides a number of comparative static results, it remains to be shown whether there is a parameterization of the functions $z_i(j)$ that makes it amenable to the quantitative analysis of trade flows between many countries.

[25] The first derivative is

$$A'(j) = \frac{-1}{\theta} \frac{1}{j(1-j)} A(j)$$

of which a share $\bar{j}$ is spent on goods produced at home. Substituting (3.25) into (3.21) to solve for $\bar{j}$ yields:

$$\bar{j} = \frac{X_{HH}}{X_H} = \frac{T_H(w_H)^{-\theta}}{T_H(w_H)^{-\theta} + T_F(d_{HF}w_F)^{-\theta}} = \frac{T_H(w_H)^{-\theta}}{\Phi_H} \tag{3.26}$$

where:

$$\Phi_H = T_H(w_H)^{-\theta} + T_F(d_{HF}w_F)^{-\theta}. \tag{3.27}$$

This expression looks very similar to the trade share expressions from Armington (3.3) and monopolistic competition (3.18). The key difference is that the parameter θ, which governs comparative advantage, has replaced the parameter σ from the Dixit-Stiglitz preferences. The reason is that trade responds to wages at the extensive margin. With higher θ, relative productivities don't fall as much as j rises. Hence a given increase in w_F renders the foreign country uncompetitive in a wider range of goods.

For the special case in which $d_{HF} = d_{FH} = 1$, substituting this expression into (3.24) gives:

$$\omega = \frac{w_H}{w_F} = \left(\frac{T_H/L_H}{T_F/L_F}\right)^{1/(1+\theta)}. \tag{3.28}$$

Since prices are the same in both countries, ω measures welfare in the H relative to F. It is increasing in home's overall level of technology T_H relative to its labor force.

so that for any j, $-A'(j)j/A(j)$ is larger the lower θ. The second derivative is:

$$A''(j) = \frac{-1}{\theta}\frac{1}{j(1-j)}A'(j)[1 + \theta(1-2j)].$$

yielding an ogee shape: concave for small values of j, but turning convex for $j > (1+\theta)/(2\theta)$.

In order to calculate the gains from trade we need to consider the model's implications for prices.[26] Given Cobb-Douglas preferences, the price index in the home country is:$P_H = \exp\left\{\int_0^1 \ln p_H(j)dj\right\}$.With trade P_H becomes:

$$P_H = \exp\left\{\int_0^{\bar{j}} \left[\ln w_H - \ln z_H(j)\right] dj + \int_{\bar{j}}^1 \left[\ln\left(d_{HF} w_F\right) - \ln z_F(j)\right] dj\right\}$$

with $\bar{j}$ given by (3.26).

Under autarky, the price index is just:

$$P_H^A = \exp\left\{\int_0^1 \left[\ln w_H^A - \ln z_H(j)\right] dj\right\}$$

where w_H^A denotes H's autarky wage. We can calculate the gains from trade as:[27]

$$\begin{aligned}\frac{w_H/P_H}{w_H^A/P_H^A} &= \left(\frac{T_H w_H^{-\theta}}{\Phi_H}\right)^{-1/\theta} \\ &= \left[1 + \frac{T_F}{T_H}\left(\frac{d_{HF} w_F}{w_H}\right)^{-\theta}\right]^{1/\theta},\end{aligned}$$

which are greater the large is F's productivity advantage and the lower its wage. For the special case $d_{HF} = d_{FH} = 1$, we can exploit (3.28) to solve for relative wages:

$$\frac{w_H/P_H}{w_H^A/P_H^A} = \left[1 + \left(\frac{T_F}{T_H}\right)^{1/(1+\theta)}\left(\frac{L_F}{L_H}\right)^{\theta/(1+\theta)}\right]^{1/\theta}.$$

[26] Matsuyama (2008) sees how far the DFS model can be pushed in terms of its welfare implications without imposing a parametric form on $A(j)$.

[27] Getting here takes some work. Taking logs and differencing we calculate:

$$\ln(w_H/P_H) - \ln(w_H^A/P_H^A) = -\int_{\bar{j}}^1 \left[\ln\left(\frac{d_{HF} w_F}{w_H}\right) + \ln A(j)\right] dj + \ln T_H - \ln j dj + \int_{\bar{j}}^1 \left[-\theta \ln(d_{HF} w_F) + \ln T_F - \ln(1-j)\right] dj.$$

Solving the integral and substituting in the expression for $\bar{j}$gives the result.

Trade gains are greater the more productive is F and the larger its labor force. The special case of symmetry ($d_{HF} = d_{FH} = 1$, $T_H = T_F$, $L_H = L_F$, so that $w_H = w_F$) delivers simply:

$$\frac{w_H/P_H}{w_H^A/P_H^A} = \left[1 + d^{-\theta}\right]^{1/\theta}.$$

which is greater the closer is d to one. Note that the gains fall with θ. The less technological heterogeneity, the less benefit there is to replacing domestic technologies with foreign ones.

While DFS posit the $A(j)$ function as a primitive, a way of generating it is to think of each country i's efficiency at making any good j as the realization of a random variable Z_i drawn independently a probability distribution $F_i(z)$. For any j one can calculate the ratio of these realizations $z_H(j)/z_F(j)$. Sorting the j's in decreasing order of this ratio yields $A(j)$.[28]

[28] This probabilistic approach places no restrictions on the relative productivity curve $A(j)$ of DFS. To see why, equate good $j \in [0, 1]$ with the probability that the relative productivity of H to F exceeds $A(j)$:

$$j = \Pr[\frac{Z_H}{Z_F} \geq A(j)].$$

For simplicity, suppose that efficiency in country F is 1 for all j (meaning that $F_F(z)$ has a single step at $z_F = 1$). Then we have

$$j = \Pr[Z_H \geq A(j)] = 1 - F_H(A(j)).$$

Thus, for any relative productivity curve $A(j)$, there is a cumulative distribution function $F_H(z)$ that

Say that the Z_i are drawn from a particular family of distributions:[29]

$$F_i(z) = \Pr[Z_i \leq z] = \exp[-T_i z^{-\theta}] \qquad z \geq 0. \tag{3.29}$$

The parameter $T_i > 0$ governs country i's overall level of efficiency (absolute advantage) while $\theta > 0$ (common across countries) governs variation in productivity across different goods (comparative advantage). A higher value of T_i means that country i has, on average, higher efficiency draws, while a higher θ means draws are less dispersed.

---

delivers it, satisfying:

$$1 - j = F_H(A(j)).$$

[29] This distribution is called the Type II extreme value (or Fréchet) distribution. It is closely related the more familiar exponential distribution:

$$\Pr[X \leq x] = 1 - e^{-Tx}.$$

If $Z = X^{-1/\theta}$ then

$$\begin{aligned} \Pr[Z &\leq z] = \Pr[X^{-1/\theta} \leq z] \\ &= \Pr[X \geq z^{-\theta}] \\ &= e^{-Tz^{-\theta}} \end{aligned}$$

To derive the relative productivity function $A(j)$:

$$
\begin{aligned}
j &= \Pr\left[\frac{Z_H}{Z_F} \geq A(j)\right] = \Pr\left[Z_H \geq A(j)Z_F\right] \\
&= \int_0^\infty \left\{1 - \exp\left[-T_H(A(j)z)^{-\theta}\right]\right\} dF_H(z) \\
&= 1 - \int_0^\infty \exp\left[-T_H(A(j)z)^{-\theta}\right] \theta z^{-\theta-1} T_F \exp[-T_F z^{-\theta}] dz
\end{aligned}
$$

and thus

$$
\begin{aligned}
1 - j &= \int_0^\infty \exp\left[-(T_H A(j)^{-\theta} + T_F) z^{-\theta}\right] \theta z^{-\theta-1} T_F dz \\
&= \frac{T_F}{T_H A(j)^{-\theta} + T_F} \int_0^\infty \exp\left[-(T_H A(j)^{-\theta} + T_F) z^{-\theta}\right] \theta z^{-\theta-1} \left(T_H A(j)^{-\theta} + T_F\right) dz \\
&= \frac{T_F}{T_H A(j)^{-\theta} + T_F}.
\end{aligned}
$$

The last simplification follows from the fact that the integral in the second to the last expression is over the entire range of the density of the distribution given in (3.29), with $T_i = T_H A(j)^{-\theta} + T_F$, so has value 1. Solving for $A(j)$ we get

$$
A(j) = \left(\frac{T_H}{T_F}\right)^{1/\theta} \left(\frac{j}{1-j}\right)^{-1/\theta}
$$

as above.

What is the probability that country H finds the local producer cheapest? It is:

$$\begin{aligned}
\Pr\left[\frac{w_H}{Z_H} \leq \frac{w_F d_{HF}}{Z_F}\right] &= \Pr\left[Z_F \leq \frac{w_F d_{HF} Z_H}{w_H}\right] \\
&= \int_0^\infty F_F\left(\frac{w_F d_{HF} z_H}{w_H}\right) dF_H(z_H) \\
&= \int_0^\infty \exp\left\{-\left[T_F\left(\frac{w_F d_{HF}}{w_H}\right)^{-\theta} + T_H\right] z_H^{-\theta}\right\} \theta T_H z_H^{-\theta-1} dz_H \\
&= \frac{T_H (w_H)^{-\theta}}{\Phi_H}.
\end{aligned}$$

where the last step comes from turning the last integral into one over a Fréchet distribution with parameter $T_F\left(\frac{w_F d_{HF}}{w_H}\right)^{-\theta} + T_H$. Note that this last expression is the same as the expression for country H's home share $\bar{j}$ given in (3.26).

Note that our probabilistic derivation of the probability that H was the low cost supplier (and equivalently home share in expenditure) did not require that we order goods according to $z_H(j)/z_F(j)$. The benefit is that we can generalize the analysis to an arbitrary integer N of countries. If country i's efficiency producing any good j is Z_i

then the probability π_{ni} that country i is the lowest cost supplier to market n is:

$$\begin{aligned}
\Pr\left[\frac{w_i d_{ni}}{Z_i} \leq \min_{k \neq i}\left\{\frac{w_k d_{nk}}{Z_k}\right\}\right] &= \Pr\left[Z_i \geq w_i d_{ni} \max_{k \neq i}\left\{\frac{Z_k}{w_k d_{nk}}\right\}\right] \\
&= \prod_{k \neq i}^{N} \Pr\left[Z_k \leq \frac{w_k d_{nk} Z_i}{w_i d_{ni}}\right] \\
&= \int_0^\infty \prod_{k \neq i} \exp\left[-T_k\left(\frac{w_k d_{nk} z_i}{w_i d_{ni}}\right)^{-\theta}\right] \theta T_i z_i^{-\theta-1} \exp\left(-T_i z_i^{-\theta}\right) dz_i \\
&= \int_0^\infty \exp\left[-\Phi_n (w_i d_{ni})^{\theta} z_i^{-\theta}\right] \theta T_i z_i^{-\theta-1} dz_i \\
&= \frac{T_i (w_i d_{ni})^{-\theta}}{\Phi_n},
\end{aligned}$$

where now:

$$\Phi_n = \sum_{i=1}^{N} T_i (w_i d_{ni})^{-\theta}.$$

By the law of large numbers π_{ni} is also country i's share in country n's spending. This approach generalizes the two-country Ricardian model we considered above, and has the gravity form familiar from the Armington and monopolistic competition models.

B

3.1.4 A Summary for what Follows

How do these various approaches to international trade relate to the data discussed in the previous chapter and to the analysis in the rest of the book? The Armington model and monopolistic competition yield equations for bilateral trade very much in keeping with observations on gravity. Moreover, as general equilibrium systems they provide a means of connecting these observations with prices and welfare. But these two approaches have limitations.

One is that they have little to say about specialization in production, which has been a central issue in international trade. In Armington there is either no production at all or else countries are assumed, for exogenous reasons, to specialize in nonoverlapping commodities. In monopolistic competition every producer selects a different product from a menu of infinite length, so complete specialization is an endogenous outcome. In neither case is there direct competition between producers of the same or similar commodities. But such competition is at the heart, for example, of trade disputes involving particular industries, such as textiles or aircraft.

Another limitation is the absence of any connection between aggregate measures of international trade and observations on individual producers. While the firm does make an appearance in monopolistic competition, the basic framework cannot account for the heterogeneity we observe across individual producers described in the previous chapter.

In contrast, the Ricardian model does model international competition and specialization at the level of individual industries. But we are only beginning to see how it can grapple with the high dimensionality of the bilateral trade data. Moreover, it does no better at coming to terms with observations on individual producers.

In chapters 4 through 6 we develop a model technology, market structure, and international trade that encompasses both the Ricardian model and monopolistic competition. The model is able to account for observations on gravity among any number of countries while also accommodating the facts on producer heterogeneity in size, productivity, and export participation.

A

3.2 Economic Growth

The Ricardian and monopolistically competitive models of trade posit a given set of technologies available to different countries. How these technologies evolve over time is not addressed. During the 1980s papers by Romer (1986) and Lucas (1988), endogenizing technical change, spawned a large literature, some of which was aimed at understanding growth in a multicountry context. An important precursor to this literature does not endogenize the process of innovation itself, but shows how, together, the processes of innovation and diffusion can generate a common world growth rate, with countries remaining at different relative income levels.

B

3.2.1 A Product-Cycle Model

Krugman (1979b) provides a simple two-country formulation combining Dixit-Stiglitz preferences with Ricardian specialization. The measure of varieties available in each country i is fixed at any moment (as in Ricardo) but evolves over time. Competition is perfect and there are no transport costs. Following Krugman we call the two countries N and S.

At any moment there is a measure J of goods. Country N can produce all of them (with unit efficiency), but S can produce only a subset J_S (also with unit efficiency). In other words, country S has efficiency 0 in producing goods in the set $J_N = J - J_S$. Country i has L_i workers, constant over time. Competition is perfect.

With unit efficiency, a good produced in country i costs w_i. Since prices are proportional to wage costs, spending on a typical N good relative to an S good is:

$$\frac{x_N}{x_S} = \left(\frac{w_N}{w_S}\right)^{-(\sigma-1)}$$

where σ continues to represent the elasticity of substitution between products. Since the range of available products evolves over time, $\sigma > 1$, as in static monopolistic competition.

If N specializes in its exclusive goods then:

$$\frac{w_N L_N}{w_S L_S} = \frac{J_N}{J_S}\left(\frac{w_N}{w_S}\right)^{1-\sigma}.$$

The relative wage is thus:

$$\frac{w_N}{w_S} = \max\left\{\left(\frac{J_N/L_N}{J_S/L_S}\right)^{1/\sigma}, 1\right\}, \tag{3.30}$$

acknowledging that $w_N = w_S$ if N has to produce S goods. The wage in N is larger the smaller its labor force relative to S's and the larger the measure J_N of goods that are exclusive to it relative to the measure J_S that S can make as well.[30]

To this static formulation Krugman adds processes of innovation and imitation. Innovation is the development of new goods, which occurs according to the process:

$$\dot{J}(t) = \iota J(t) \tag{3.31}$$

where $J(t) = J_N(t) + J_S(t)$ is the total measure of goods existing at date t. The parameter ι represents the rate of innovation and corresponds to the growth rate in the total measure of goods. Imitation occurs as knowledge of how to make exclusively northern goods diffuses to S. A given N good faces a hazard ϵ of diffusing to S.[31] Hence:

$$\dot{J}_S(t) = \epsilon J_N(t). \tag{3.32}$$

Combining (3.31) and (3.32) implies that:

$$\dot{J}_N(t) = \iota J(t) - \epsilon J_N(t). \tag{3.33}$$

Krugman considers a balanced growth path in which $J_N(t)/J(t)$ is constant. Dividing (3.33) by $J_N(t)$ and insisting that $J_N(t)$ also grow at rate ι gives:

$$\frac{J_N(t)}{J(t)} = \frac{\iota}{\iota + \epsilon}.$$

[30] Note the parallel between this expression for the relative wage with expression (3.28) yielded by the parameterized version of the DFS model. Country i's range of goods J_i replaces the technology parameter T_i while the elasticity parameter σ replaces $\theta + 1$.

[31] Nelson and Phelps (1966) provide an earlier formulation of innovation and diffusion of this form.

Hence, on a balanced growth path, the wage ratio, in terms of the parameters of innovation and diffusion, is:

$$\frac{w_N}{w_S} = \max\left\{\left(\frac{\iota}{\epsilon}\right)^{1/\sigma}\left(\frac{L_N}{L_S}\right)^{-1/\sigma}, 1\right\}. \tag{3.34}$$

An important implication of the model is that, on a balanced growth path, N and S each grow at the same rate. The price index is:

$$P(t) = \left[J_N(t)w_N^{1-\sigma} + J_S(t)w_S^{1-\sigma}\right]^{1/(1-\sigma)} = J(t)^{1/(1-\sigma)}\left[\frac{\iota w_N^{1-\sigma}}{\iota+\epsilon} + \frac{\epsilon w_S^{1-\sigma}}{\iota+\epsilon}\right]^{1/(1-\sigma)}, \tag{3.35}$$

which is common to both countries since there are no transport costs.

The real wage in N is:

$$\frac{w_N}{P(t)} = J(t)^{1/(\sigma-1)}\left\{1 + \frac{\epsilon}{\iota+\epsilon}\left[\left(\frac{w_N}{w_S}\right)^{\sigma-1} - 1\right]\right\}^{1/(\sigma-1)}$$

while in S is:

$$\frac{w_S}{P(t)} = J(t)^{1/(\sigma-1)}\left\{1 + \frac{\iota}{\iota+\epsilon}\left[\left(\frac{w_N}{w_S}\right)^{-(\sigma-1)} - 1\right]\right\}^{1/(\sigma-1)}.$$

Since the number of products grows at rate ι, the real wage in each location grows at rate $\iota/(\sigma-1)$. Country N is perpetually ahead of S, however. How far ahead depends on the rate of innovation relative to the rate of diffusion and relative labor forces. Hence the model captures a first-order features of Figures 14 and 15 in the previous chapter by providing a simple explanation for why countries can continue to grow at very similar rates but at very different levels of income.

Obviously faster diffusion is good for S. More goods are available at the lower Southern price, and those that aren't are cheaper since faster diffusion means a lower relative wage in N. For N the effect is ambiguous. While faster diffusion means that more goods are available at the lower Southern price, it also means that this price is not as low. For ϵ near zero the first effect dominates and faster diffusion benefits the North. But at some point the effect is reversed. With enough diffusion N is brought back to where it would have been under autarky.

The model also points to an inverted U shaped response of trade to diffusion. With no diffusion there would be no S goods and nothing to trade. With only a small amount S is so small that the overall amount of trade would be miniscule. More diffusion at first means more trade but at some point S would know how to produce most goods itself, eliminating the gains from trade. Eaton and Kortum (2008) investigate these issues further.[32]

B

3.2.2 Endogenous Innovation: Monopolistic Competition

Krugman's framework does not try to model the process of innovation itself, which became an active research area subsequently. Romer (1990) and Grossman and Helpman (1991a, 1991b) endogenize the creation of new products in a dynamic version of

[32] With N incompletely specialized the amount of trade in S goods is indeterminate, but introducing a small trade cost would ensure that N never exported an S good.

monopolistic competition. We present their version as it applies to a closed economy.

As in Krugman (1979b) the measure of extant goods in period t is given at $J(t)$, but since the market structure is monopolistic competition the price of a good is $\overline{m}w$ (setting $z = 1$). From (3.35), the price index for a single economy is:

$$P(t) = J(t)^{1/(1-\sigma)}\overline{m}w(t).$$

Substituting into the expression for profit under monopolistic competition, (3.12) above, profit for a variety is then:

$$\pi(t) = \frac{X(t)}{\sigma J(t)}$$

where $X(t)$ is period t spending.

Posit a balanced growth path along which $X(t)$ grows at rate g_X and $J(t)$ grows at constant rate g_J (both to be determined). Setting $w(t) = w$, an implication is that the inflation rate is $-g_J/(\sigma - 1)$. Agents discount future profits at an exogenous rate ρ. The discounted value of profit at time t, taking into account future inflation, is thus:

$$\begin{aligned} V(t) &= \int_t^\infty e^{-\rho(s-t)}\frac{P(t)}{P(s)}\pi(s)ds \\ &= \frac{1}{\rho - g_X + [(\sigma-2)/(\sigma-1)]\, g_J}\frac{X(t)}{\sigma J(t)}, \end{aligned} \tag{3.36}$$

which corresponds to the value of developing a new good at time t. A higher growth in spending g_X means that ideas are more valuable since profits grow faster over time.

Growth in the number of varieties g_J has an ambiguous effect, as more varieties means both that a given level of profit can buy more but that more varieties compete for profits. If $\sigma < 2$ the first effect dominates while $\sigma > 2$ means the opposite.

In contrast with Krugman, innovation takes effort. One worker can innovate at rate $\alpha(t)$, so that:

$$\dot{J}(t) = \alpha(t)r(t)L(t), \tag{3.37}$$

where $L(t)$ are the number of workers at date t and $r(t)$ the fraction engaged in research. The reward to research activity is $\alpha(t)V(t)$ while a worker earns the wage w making goods. Labor-market equilibrium with an interior solution for $r(t)$ requires that:[33]

$$\alpha(t)V(t) = w. \tag{3.38}$$

Along a balanced growth path in which J grows at a constant rate g_J r is constant at r^*.

Total spending $X(t)$ consists of spending by production workers $[1-r(t)]wL(t)$ and profits (prior to paying researchers) $\Pi(t) = X(t)/\sigma$. Combining these terms:

$$X(t) = \overline{m}[1 - r(t)]wL(t)$$

To close the model we need to specify how $L(t)$ and $\alpha(t)$ evolve. Two different approaches appear in the literature.

[33] If $\alpha(t)V(t) < w$ then $r(t) = 0$ while if $\alpha(t)V(t) > w$ then $r(t) = 1$.

In the original endogenous growth models L is fixed while efficiency in producing ideas for goods grows with "knowledge capital," proxied by the stock of goods already developed. Thus we can set

$$\alpha(t) = \alpha J(t). \tag{3.39}$$

From (3.37):

$$g_J = \alpha r^* L, \tag{3.40}$$

while, since w and L are constant, $g_X = 0$. Substituting (3.36), (3.40), and (3.39) into (3.38) and solving for r^* gives:

$$r^* = \frac{1}{\sigma - 1} - \frac{\rho}{\alpha L}$$

so that the growth rate in the number of products is:

$$g_J = \frac{\alpha L}{\sigma - 1} - \rho.$$

The growth rate increases in proportion to the population adjusted for research productivity.[34]

[34]This solution requires parameter values such that $r^* \in [0, 1]$. If the discount rate is too high, for example, there is no research or growth. The assiduous reader may note that our expression differs slightly from Grossman and Helpman's (1991b, p. 61). The reason is that they assume logarithmic preferences, while our assumption of a fixed discount factor implies linear preferences. In their model the discount rate is equal to the exogenous rate of time preference ρ' plus the rate at which real

Jones (1995) provides an alternative formulation, treating research productivity α as fixed while letting L grow at an exogenous rate g_L. The ratio of goods $\varphi(t) = J(t)/L(t)$ evolves according to:

$$\dot{\varphi}(t) = \frac{\alpha r(t) L(t)}{L(t)} - \varphi(t) g_L.$$

On a balanced growth path with $r(t)$ constant at r^*:

$$\varphi(t) = \varphi^* = \frac{\alpha r^*}{g_L}.$$

Since $g_X = g_J = g_L$, the value of an idea is:

$$V = \frac{g_L}{\rho(\sigma - 1) - g_L} \frac{(1 - r^*)w}{\alpha r^*}.$$

For profit to be finite we require that $\rho > g_L/(\sigma - 1)$.

Substituting into the condition for an interior labor-market equilibrium implies:

$$r^* = \frac{g_L}{\rho(\sigma - 1)}.$$

More research is done the higher the population growth rate relative to the discount factor and the elasticity of substitution. More research no longer means a higher bal-

---

consumption grows, $g_J/(\sigma - 1)$. To derive their expression from ours, replace our discount factor ρ with $\rho' + g/(\sigma - 1)$, to obtain:

$$g_J = \frac{\alpha L}{\sigma} - \frac{\sigma - 1}{\sigma} \rho'.$$

Translating our notation into theirs, this expression is their (3.28).

anced growth rate but a higher φ^* and hence a higher level of income at any point along the path.

B

3.2.3 Endogenous Growth: Quality Ladders

Grossman and Helpman (1991a, 1991b) provide an alternative model of innovation and growth with elements much closer to DFS's formulation of the Ricardian model rather than to monopolistic competition. In their model, the most efficient (or highest quality) technology for each good j is the consequence of a sequence of innovations, each one raising efficiency (or quality) over the previous state of the art by a factor $\lambda > 1$. Hence the most efficient technology for making good j if it has experienced $m(j)$ innovations is $z_0\lambda^{m(j)}$, where z_0 is the efficiency level at date 0 (assumed constant across goods).[35]

As in DFS, preferences are Cobb-Douglas with equal share across the unit continuum of goods.

The state of the art technology for each good is proprietary. Potential producers of each good engage in Bertrand competition. The outcome is that only the most efficient technology is used for making each good. With Cobb-Douglas preferences, individual producers face unit elastic demand and charge the highest price that keeps the competition at bay. Hence a producer of a good j that has experienced $m(j)$ innovations charges a price $p(j) = w/(z_o\lambda^{m(j)-1})$, the unit cost using the previous state

[35] Aghion and Howitt (1992) provide a similar formulation.

of the art. Since its own unit cost is $c(j) = w/(z_o\lambda^m)$ its profit is:

$$\pi(j) = [p(j) - c(j)]\frac{X(j)}{p(j)} = \frac{\lambda - 1}{\lambda}X$$

where, since preferences are Cobb-Douglas and there are a unit continuum of goods, spending $X(j)$ on good j is the same as total spending X. Note that profit is independent of the state of technology in the sector.[36]

Since spending goes either to profits or to wages, $X(t) = \lambda wL(t)[1 - r(t)]$ so that $\pi = (\lambda - 1)wL(t)[1 - r(t)]$. Again, $r(t)$ represents the share of workers engaged in research, so that $L(t)[1 - r(t)]$ workers produce output. We continue to treat the wage w as fixed over time.

Innovations flow into the economy at rate $\iota(t)$ (to be derived later) and are equally likely to apply to each good j. Since there are a unit continuum of goods, for any particular good j innovations arrive according to a Poisson process with arrival rate $\iota(t)$.

With $r(t)L(t)$ workers engaged in research, ideas arrive at rate:

$$\iota(t) = \alpha r(t)L(t)$$

where again α is a parameter of research productivity. This model treats the labor force L as constant.

[36] In contrast, in the monopolistic competition framework above, lower cost producers earn a higher profit. The reason is, with CES preferences with the elasticity of substitution greater than one, lower unit cost, which translates into a lower price, means higher sales.

Consider a balanced growth path with r and ι constant. The expected number of innovations after a period of length t is ιt.

With Cobb-Douglas preferences the price index $P(t)$ is:

$$P(t) = \exp\left[\int_0^1 \ln[p(j)]dj\right] = \frac{w}{z_0\lambda}\exp\left[\ln\lambda\int_0^1 m(j)dj\right],$$

which along a balanced growth path is:

$$P(t) = w\lambda^{-\iota t} = w\exp[-(\iota\ln\lambda)t],$$

where, to simplify notation, we choose units so that $z_0 = \lambda$. The inflation rate in the economy is thus $-\iota\ln\lambda$. Since w and L are fixed, the real growth rate is $\iota\ln\lambda$.

The term ι is also the hazard with which the current state of the art for producing a good j is surpassed, at which point the owner of the surpassed invention no longer earns a profit. With a discount rate of ρ, taking into account inflation and the hazard of obsolescence, the value of a state of the art idea at time t is:

$$\begin{aligned}
V &= \int_t^\infty e^{-(\rho+\iota)(s-t)}\frac{P(t)}{P(s)}\pi ds \\
&= \frac{(\lambda-1)wL(1-r)}{\rho+\iota-\iota\ln\lambda} \\
&= \frac{(\lambda-1)wL(1-r)}{\rho+\alpha rL(1-\ln\lambda)}.
\end{aligned}$$

Note that a higher rate of innovation ι has a positive effect on the value of an idea by creating economic growth, which causes the real value of profit to rise over time. But it has a negative effect by increasing the hazard of obsolescence. For inventive steps $\lambda < e$, the negative obsolescence effect dominates.

Again, labor market equilibrium requires (**??**) above. At an interior solution the balanced growth path research share is:

$$r^* = \frac{\lambda - 1}{\Lambda} - \frac{\rho}{\alpha L \Lambda}$$

where $\Lambda = \lambda - \ln \lambda$. The implied rate of innovation is:

$$\iota = \frac{\lambda - 1}{\Lambda}\alpha L - \frac{\rho}{\Lambda}.$$

Again, growth increases with the labor force adjusted for research productivity.[37]

Grossman and Helpman (1991b) go on to develop two-country extensions of these dynamic models with technology diffusion and trade, examining the impact of various policies.

B

3.2.4 A Summary for What Follows

How successfully do these models of growth explain the features of the data described in the previous chapter? The models of innovation and diffusion can explain why, over

[37] Again, conditions on parameters need to be imposed to guarantee that $r^* \in [0, 1]$. To obtain Grossman and Helpman's (1991b, p. 96) result with logarithmic preferences, replace our fixed discount factor ρ with their pure rate of time preference ρ' plus the growth in real income $\iota \ln \lambda$. Translating our notation into theirs delivers their expression (4.18).

the long run, a common underlying process can generate very similar growth rates in different countries, while relative differences in income remain, much as we see in the data. But the models provide a parameterization of this phenomenon in only a two-country setting. As the data indicate, many countries both innovate and make use of the inventions of others.

In order to make their points as cleanly as possible, the models we have just discussed treat goods, producers, and inventions as identical or symmetric. A feature of the producer-level data, however, is the vast heterogeneity of producers in terms of size and where they sell. Data on cross-country patenting suggest that inventions also vary enormously in their importance and the geographic breadth of their applicability.

We have described two sets of models, one coming to grips with cross sectional observations of trade and another with growth facts. Could a single framework confront both sets of observations?

The next section of the book develops a framework for analyzing trade and growth in a multicountry world. It uses many of the elements of the models we just described in order to explain bilateral trade patterns and the phenomenon of parallel growth that we observe in the data. Additional features allow us to come to terms with producer-level heterogeneity and the complex patterns of producer-level participation in trade, and to understand patterns of innovation and diffusion in a multipolar world.

A

3.3 Further Reading

We've chosen to highlight a set of key results on technology in the global economy that set the stage for our own analysis. We have not provided an extensive survey of the literatures. The reader eager to expand her knowledge is fortunate to have a number of excellent surveys to turn to.

Grossman and Helpman (1995) provide an analytic overview of the general literature on trade and technology, as it stood in the mid 1990's. For a detailed discussion of the theoretical literature on monopolistic competition and its relationship to the econometrics of the gravity equation, we recommend Chapter 5 of Feenstra (2004). For a comprehensive review of efforts to measure bilateral trade costs we refer the reader to Anderson and van Wincoop (2004).

Various aspects of growth in a multi-country context are surveyed by Klenow and Rodriguez-Clare (2005) and Benhabib and Spiegel (2005). Keller (2004) provides a comprehensive survey of work on the theory and empirics of the diffusion of technologies across countries.

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CHAPTER 3 — MANUSCRIPT

Part II

Framework

The first Part of the book presented some basic facts about international trade, firm export behavior, research activity, patenting, and aggregate productivity. It then provided a brief review of the literatures that relate to these areas. Our approach builds on this work with three main objectives:

1. To develop a common framework to address questions about trade and innovation.
2. To capture key quantitative features of the data.
3. To connect observations at the aggregate and producer levels.

As we will see in Part III, the framework is readily quantifiable using data of the sort we discussed in Chapter 2. Finally, it makes a connection between observations about the trade behavior of individual producers and aggregate data.

We present the framework in four chapters. Chapter 4 presents the very simple representation of technologies that underlies all of our analysis, and derives the relevant results that this assumption delivers for unit costs of production. In Chapter 5 we complete the characterization of a closed economy by making the standard assumption of a constant elasticity of substitution aggregator, the aggregator used throughout the rest of the book. We then show what our cost structure implies for prices, income distribution, and the real wage under various market structures. Chapter 6 uses the framework to analyze international trade by introducing different locations with different technologies, with different factor markets, and with transport barriers between them. Finally, Chapter 7 introduces a process of innovation to analyze economic growth. How we connect this structure to the sorts of data described in Chapter 2, putting numbers on the various parameters and putting the theory to work on some policy questions, is the domain of Part III.

Chapter 4

Technology and Heterogeneous Costs

Here we derive the specification of technology and costs that underlies all the remaining analysis in the book. Our analysis in this chapter makes few assumptions about economic behavior. Its purpose is to characterize the distribution of unit costs that emerges from fundamental properties of technology. It provides a skeleton which we flesh out in the remaining chapters by adding specific assumptions about preferences, market structure, geography, and the production of knowledge.

The analysis here is dynamic in that it examines how the arrival of ideas over time gives rise to an evolving distribution of costs. We are interested in the dynamics of the process per se (to which we return in Chapter 7) but also in the cost distribution at any moment that this dynamic process engenders (which we exploit in Chapters 5 and 6).

The chapter is divided into four sections. The first sets out our basic assumption about ideas that underlies the remainder of the book. The second two provide a technical derivation of the properties of the distribution of costs implied by our assumption. The final section summarizes what we use of these properties in the ensuing chapters. The reader not interested in the probability theory behind the results should read the next section but can then safely skip to the last one, where we summarize what's relevant for the remainder of the book.

A

4.1 Ideas, Techniques, and Unit Costs

The fundamental atom of technology is an idea. An idea is a recipe to produce some good j with some efficiency q (which we call the quality of the idea) at some location i. Efficiency is simply the amount of output that can be produced with a unit of input. In this formulation, both output and input are measured in units of constant quality.

At any moment a location i is characterized by the ideas available to it for production, and an input cost w_i. (In Chapter 6, where we introduce trade among locations, their geography relative to one another becomes another important feature.) While inputs could involve a bundle of factors and intermediates, for simplicity of expression, except when pursuing these generalizations, we will refer to the input as labor and its reward as the wage. In this chapter we take w_i as given, and derive how the stock of ideas available at a location at any moment are determined by the history of their arrival.

Connecting an idea (for making a good j with efficiency q) with a location gives rise to a technique for producing the good there at unit cost w_i/q. For now we focus on ideas about a particular good j in a single location i, so suppress the indices j and i. Later we will make assumptions about the range of goods, which could be exogenous or endogenous, constant or growing over time.

The quality of an idea is the realization of a random variable Q drawn independently from the Pareto distribution with parameter $\theta > 1$, so that:

$$\Pr[Q > q] = \begin{cases} \left(q/\underline{q}\right)^{-\theta} & q \geq \underline{q} \\ 1 & q < \underline{q} \end{cases}$$

where $\underline{q} > 0$ is the minimum quality level.[1] A useful property of the Pareto distribution is that, conditional on an idea being better than q (for $q \geq \underline{q}$), the probability that the idea is better than q', for any $q' \geq q$, is:

$$\Pr[Q > q' | Q \geq q] = (q'/q)^{-\theta}. \tag{4.1}$$

That is, given that the idea is better than q, the probability distribution of its quality is Pareto with parameter θ and lower bound q.

[1] We denote random variables with capital letters and their realizations with the corresponding lower case.

Time is continuous. Ideas for good j arrive at date t according to a Poisson process with intensity $\overline{a}R(t)$. We can think of R as reflecting research effort and $\overline{a}$ (to be normalized shortly) as reflecting research productivity. Together these assumptions imply that for any $q > \underline{q}$, the arrival rate of ideas of efficiency $Q \geq q$ is:

$$\overline{a}R(t)\left(q/\underline{q}\right)^{-\theta}.$$

In this formulation there is no inherent distinction between $\overline{a}$ and the minimum quality of an idea $\underline{q}$. Hence we normalize $\overline{a}\underline{q}^{\theta} = 1$ so that the arrival rate of ideas of efficiency greater than q simplifies to $R(t)q^{-\theta}$. Taking the limit as $\underline{q} \to 0$ (and, hence, $\overline{a} \to \infty$ so that $\overline{a}\underline{q}^{\theta}$ remains at unity) allows us to consider ideas of all qualities in the domain $(0, \infty)$. In what follows we will always consider this limiting case so that $\overline{a}$ and $\underline{q}$ will not appear.

We assume that there is no forgetting: Once an idea has arrived it is available for production thereafter. The number of ideas available for producing good j thus reflects the history of the Poisson arrival of ideas about that good by date t. We summarize this history with the term $T(t)$ given by the integral:

$$T(t) = \int_{-\infty}^{t} R(\tau)d\tau.$$

Our assumptions imply that the number of ideas about good j with quality $Q > q$ is distributed Poisson with parameter $T(t)q^{-\theta}$. The distribution of quality among these ideas is given by (4.1).

Since a bundle of inputs costs w, the unit cost of producing good j with a technique of efficiency Q is $C = w/Q$. We now turn to the distribution of the random variable C. The key parameter for this distribution is:

$$\Phi(t) = T(t)w^{-\theta}, \tag{4.2}$$

which combines the history of the arrival of ideas together with input costs. The following proposition characterizing properties of the set of techniques with unit cost $C \leq c$ is immediate:

Proposition 1 *Given $\Phi(t)$: (i) The number of techniques providing unit cost less than c is distributed Poisson with parameter $\Phi(t)c^{\theta}$. (ii) The conditional distribution of unit costs using these techniques is:*

$$\Pr[C \leq c' | C \leq c] = \Pr\left[Q \geq \frac{w}{c'} | Q \geq \frac{w}{c}\right] = (c'/c)^{\theta} \quad c' \leq c, \tag{4.3}$$

which is invariant to input costs w and the technology parameter T.

As ideas arrive at a location over time, there will be many available recipes for producing good j. At any time t we can rank techniques according to their implied unit costs $C^{(1)} \leq C^{(2)} \leq C^{(3)} \leq \ldots$. For the time being our analysis does not depend on time, so we suppress t, reintroducing j, i, and t when they become relevant.

The next two sections present some basic properties of the joint distribution of the order statistics $C^{(k)}$, $k = 1, 2, 3, \ldots$, for given Φ. Section 4.4 summarizes what is needed of these results for the subsequent analysis. The reader not interested in the probability theory behind them can skip ahead.

A

4.2 The Basic Theorem

Our ensuing analysis is based on the many layers of costs for a good, starting with the lowest unit cost $C^{(1)}$, then the second-lowest $C^{(2)}$, and working up from there (with the economics of any particular application telling us how many layers we need to go up). The following theorem characterizes the joint distribution of these layers of unit costs for a particular good, where $C^{(k)}$ denotes the k'th lowest unit cost technology for producing it. This theorem on the distribution of costs serves as the basis for many of our subsequent results.

Theorem 1 *The joint density of $C^{(k)}$ and $C^{(k+1)}$ is:*

$$g_{k,k+1}(c_k, c_{k+1}) = \frac{\theta^2}{(k-1)!}\Phi^{k+1} c_k^{\theta k-1} c_{k+1}^{\theta-1} \exp[-\Phi c_{k+1}^{\theta}]$$

for $0 < c_k \leq c_{k+1} < \infty$ while the marginal density of $C^{(k)}$ is:

$$g_k(c) = \frac{\theta}{(k-1)!}\Phi^k c^{\theta k-1} \exp[-\Phi c^{\theta}].$$

for $0 < c < \infty$.

G *Proof.* We first focus on the distribution of the order statistics for techniques with cost less than $\overline{c}$. From Proposition 4.1, the distribution of C given that $C \leq \overline{c}$ is:

$$F(c|\overline{c}) = \begin{cases} \left(\frac{c}{\overline{c}}\right)^{\theta} & c \leq \overline{c} \\ 1 & c > \overline{c} \end{cases}.$$

The probability that a cost is less than c_k is $F(c_k|\overline{c})$ while the probability that it is more than c_{k+1} is $1 - F(c_{k+1}|\overline{c})$. Hence, if there are n techniques with unit cost less than $\overline{c}$, where $c_k \leq c_{k+1} \leq \overline{c}$, the probability that k are less than c_k while the remaining $n-k$ are greater than c_{k+1} is, from the multinomial:

$$\Pr[C^{(k)} \leq c_k, C^{(k+1)} \geq c_{k+1}|n] = \binom{n}{k} F(c_k|\overline{c})^k [1 - F(c_{k+1}|\overline{c})]^{n-k}.$$

This object is closely related to the joint c.d.f. of $C^{(k)}$ and $C^{(k+1)}$, the only difference being that one inequality is greater than or equal. Taking the negative (to account for this reversal) of the cross derivative of this expression with respect to c_k and c_{k+1} gives the joint density of $C^{(k)}$, $C^{(k+1)}$:

$$g_{k,k+1}(c_k, c_{k+1}|\overline{c}, n) = \frac{n!\,[F(c_k|\overline{c})]^{k-1}\,[1 - F(c_{k+1}|\overline{c})]^{n-k-1}\,F'(c_k|\overline{c})F'(c_{k+1}|\overline{c})}{(k-1)!(n-k-1)!},$$

for $c_{k+1} \geq c_k$ and $n \geq k+1$.[2] For $n < k+1$ we can set $g_{k,k+1}(c_k, c_{k+1}|\overline{c}, n) = 0$. Since, from Proposition 4.1, the number of techniques is drawn from the Poisson distribution

[2] See section 4.6 of Hogg and Craig (1995) for generalizations of this result.

with parameter $\Phi\overline{c}^{\theta}$, the expectation of this joint distribution unconditional on n is:

$$\begin{aligned}
g_{k,k+1}(c_k, c_{k+1}|\overline{c}) &= \sum_{n=0}^{\infty} \frac{\exp(-\Phi\overline{c}^{\theta})\left(\Phi\overline{c}^{\theta}\right)^n}{n!} g_{k,k+1}(c_k, c_{k+1}|\overline{c}, n) \\
&= \frac{[F(c_k|\overline{c})]^{k-1}(\Phi\overline{c}^{\theta})^{k+1}\exp[-\Phi\overline{c}^{\theta}F(c_{k+1}|\overline{c})]F'(c_k|\overline{c})F'(c_{k+1}|\overline{c})}{(k-1)!} \\
&\quad \sum_{n=k+1}^{\infty} \frac{e^{-\Phi\overline{c}^{\theta}[1-F(c_{k+1}|\overline{c})]}\left\{\Phi\overline{c}^{\theta}\left[1-F(c_{k+1}|\overline{c})\right]\right\}^{n-k-1}}{(n-k-1)!} \\
&= \frac{[F(c_k|\overline{c})]^{k-1}(\Phi\overline{c}^{\theta})^{k+1}\exp[-\Phi\overline{c}^{\theta}F(c_{k+1}|\overline{c})]F'(c_k|\overline{c})F'(c_{k+1}|\overline{c})}{(k-1)!} \\
&\quad \sum_{m=0}^{\infty} \frac{e^{-\Phi\overline{c}^{\theta}[1-F(c_{k+1}|\overline{c})]}\left\{\Phi\overline{c}^{\theta}\left[1-F(c_{k+1}|\overline{c})\right]\right\}^{m}}{m!} \\
&= \frac{[F(c_k|\overline{c})]^{k-1}(\Phi\overline{c}^{\theta})^{k+1}\exp[-\Phi\overline{c}^{\theta}F(c_{k+1}|\overline{c})]F'(c_k|\overline{c})F'(c_{k+1}|\overline{c})}{(k-1)!}.
\end{aligned}$$

The last result follows since the summation is over the domain of the Poisson distribution with parameter $\Phi\overline{c}^{\theta}\left[1-F(c_{k+1}|\overline{c})\right]$. Substituting our expression for $F(c|\overline{c})$ we get:

$$g_{k,k+1}(c_k, c_{k+1}|\overline{c}) = \frac{\theta^2}{(k-1)!}\Phi^{k+1}c_k^{\theta k-1}c_{k+1}^{\theta-1}\exp[-\Phi c_{k+1}^{\theta}].$$

By letting $\overline{c} \to \infty$ this joint density is defined for all $c_k > 0$, delivering the joint density of the Theorem. To get the marginal density we calculate:

$$g_k(c) = \int_c^{\infty} g_{k,k+1}(c, c_{k+1})dc_{k+1}.$$

■

Theorem 1 characterizes the joint distribution of each pair of adjacent order statistics. By induction these distributions are sufficient to characterize the full distribution across any number of ordered unit costs, i.e., $C^{(1)}, C^{(2)}, C^{(3)}, \ldots, C^{(k)}$ for any finite integer k. (See Karlin and Taylor, Chapter 13, 1981.) Note that the distributions depend only on the two parameters θ and Φ ($= Tw^{-\theta}$). Hence the parameter Φ summarizes all we need to know for the distribution of costs. The theorem thus provides a connection between the history of the arrival of ideas T and input costs w to the distribution of the unit cost of making good j at date t.

A

4.3 Probabilistic Implications

With this central result in hand we are able to show a number of features about the cost distribution that we apply repeatedly in the following chapters. The first two lemmata give the distribution of the k'th lowest cost and its moments.

Lemma 1 *The distribution of the k'th lowest cost $C^{(k)}$ is:*

$$\Pr[C^{(k)} \leq c] = F_k(c) = 1 - \sum_{i=0}^{k-1} \frac{\left(\Phi c^{\theta}\right)^i}{i!} \exp\left[-\Phi c^{\theta}\right],$$

G *Proof.* As is necessary for any cumulative distribution function, $F_k(c)$ approaches 1 as $c \to \infty$. Furthermore, from Theorem 1, $F_k'(c) = g_k(c)$ as required. ■

Of particular interest for what follows is the distribution of the lowest cost $C^{(1)}$. Setting $k = 1$ gives the Type 3 extreme value (or Weibull) distribution:[3]

$$F_1(c) = 1 - \exp(-\Phi c^{\theta}).$$

In our applications below, whether we need to probe into further layers depends on our assumptions about market structure and the ownership of technology.

Say that a large number of potential producers have access to the lowest-cost technology and compete perfectly with each other to produce a homogeneous good j at cost $C^{(1)}$. In this case only the distribution of $C^{(1)}$ is of interest, since it applies to both cost and price.

Say, instead, that only a single producer has access to the lowest cost technology (due, for example, to patent protection or trade secrecy), while at least one other producer has access to the second-lowest cost technology to produce a homogenous

[3] If $C^{(1)} = w/Q^{(1)}$ is Type 3 then $Q^{(1)}$, the most efficient idea, is Type 2 (Fréchet):

$$\begin{aligned}
\Pr[Q^{(1)} &\leq q] = \Pr\left[w/C^{(1)} \leq q\right] \\
&= \Pr\left[C^{(1)} \geq w/q\right] \\
&= \exp\left[-\Phi(w/q)^{\theta}\right] \\
&= \exp\left[-Tw^{-\theta}(w/q)^{\theta}\right] \\
&= \exp\left(-Tq^{-\theta}\right).
\end{aligned}$$

good. Under Bertrand competition the cost distribution is also given by the frontier ($k = 1$) but prices are related to the distribution of the second lowest cost ($k = 2$).

Say that each technology is available to only a single potential producer, and each produces a differentiated version of good j. Then higher values of k will be relevant. The following Chapter explores different forms of competition in greater depth.

The second lemma is useful in calculating price indices:

Lemma 2 *For each order k, the b'th moment ($b > -\theta k$) is:*

$$E[\left(C^{(k)}\right)^b] = \left(\Phi^{-1/\theta}\right)^b \frac{\Gamma\left[(\theta k + b)/\theta\right]}{(k-1)!}$$

where $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy$ is the gamma function.[4]

G *Proof.* First consider $k = 1$:

$$\begin{aligned} E\left[\left(C^{(1)}\right)^b\right] &= \int_0^\infty c^b g_1(c)dc \\ &= \int_0^\infty \Phi\theta c^{\theta+b-1}\exp[-\Phi c^\theta]dc. \end{aligned}$$

[4]The gamma function will appear numerous times throughout the book. While it is not defined for $\alpha = 0$ and has other poles for negative arguments, we wiill only consider the positive domain. In this domain it approaches ∞ for α near zero as well as for large α. It decreases in α up to $\alpha_{\min} = 1.4616$...where it achieves a minimum value $\Gamma(\alpha_{\min}) = 0.8856$... and increases thereafter. Integrating by parts

$$\Gamma(\alpha + 1) = \alpha\Gamma(\alpha).$$

Since simple integration yields $\Gamma(1) = 1$, it follows that, for integer n, $\Gamma(n + 1) = n!$.

Changing the variable of integration to $v = \Phi c^{\theta}$ and applying the definition of the gamma function, we get:

$$E\left[\left(C^{(1)}\right)^{b}\right] = \int_{0}^{\infty} (v/\Phi)^{b/\theta} e^{-v} dv = \left(\Phi^{-1/\theta}\right)^{b} \Gamma\left[\frac{\theta + b}{\theta}\right],$$

which is well defined for $\theta + b > 0$. In general, using the fact that,

$$g_k(c) = \frac{1}{(k-1)!} \Phi^{k-1} c^{\theta(k-1)} g_1(c)$$

$$\begin{aligned} E\left[\left(C^{(k)}\right)^{b}\right] &= \int_{0}^{\infty} c^{b} g_k(c) dc \\ &= \frac{\Phi^{k-1}}{(k-1)!} \int_{0}^{\infty} c^{b+\theta(k-1)} g_1(c) dc \\ &= \frac{\Phi^{k-1}}{(k-1)!} E\left[\left(C^{(1)}\right)^{b+\theta(k-1)}\right] \end{aligned}$$

Calculating the $b + \theta(k-1)$ moment of $C^{(1)}$ (which can done as long as $b + \theta k > 0$) gives the general result. ■

This lemma provides a link between the state of technology and wages, as reflected in Φ, and moments of costs at various tiers k. The homogeneity of prices with respect to costs then implies a link between technology and wages, on one hand, and the price index, on the other.[5]

[5] In the following chapters we assume a CES aggregator across goods, with elasticity of substitution σ. Under perfect competition, the price index is:

$$P = \left\{E\left[\left(C^{(1)}\right)^{-(\sigma-1)}\right]\right\}^{-1/(\sigma-1)}.$$

We have now characterized the various layers of the cost distribution. We will also be using results on the distribution of one layer conditional on the realization of an adjacent one. The next two lemmata concern the distribution of the $k+1$'st lowest unit cost given the realization of the k'th.

Lemma 3 *The distribution of $C^{(k+1)}$ conditional on $C^{(k)} = c_k$ is:*

$$\Pr[C^{(k+1)} \leq c_{k+1} | C^{(k)} = c_k] = 1 - \exp\left[-\Phi(c_{k+1}^{\theta} - c_k^{\theta})\right] \qquad c_{k+1} \geq c_k \geq 0$$

Setting $b = 1 - \sigma$ in Lemma 2 implies that the price index is homogeneous of degree 1 in the wage and of degree $-1/\theta$ in the state of technology T. That is, given the wage an increase in T lowers the price index with an elasticity $-1/\theta$. This result holds with Cobb-Douglas preferences ($\sigma = 1$) as well, although setting $b = 0$ in Lemma 2 won't work. In this case

$$P = \exp\left\{E\left[\ln C^{(1)}\right]\right\}.$$

We can then calculate

$$\begin{aligned} E\left[\ln C^{(1)}\right] &= \int_0^{\infty} \ln(c) g_1(c) dc \\ &= \int_0^{\infty} \ln(c)\theta\Phi c^{\theta-1} e^{-\Phi c^{\theta}} dc \\ &= \int_0^{\infty} \ln((v/\Phi)^{1/\theta}) e^{-v} dv \\ &= \frac{1}{\theta}\int_0^{\infty} \ln(v) e^{-v} dv - \frac{1}{\theta}\ln\Phi \int_0^{\infty} e^{-v} dv \\ &= \frac{-\gamma}{\theta} - \frac{1}{\theta}\ln\Phi, \end{aligned}$$

where $\gamma = 0.5772...$ is Euler's constant.

G *Proof.* We solve:

$$\begin{aligned}\Pr[C^{(k+1)} &\leq c_{k+1}|C^{(k)} = c_k] = \int_{c_k}^{c_{k+1}} \frac{g_{k,k+1}(c_k, c)}{g_k(c_k)} dc \\ &= \int_{c_k}^{c_{k+1}} \theta\Phi c^{\theta-1} \exp[-\Phi c^\theta + \Phi c_k^\theta] dc,\end{aligned}$$

delivering the result. ■

Lemma 4 *The distribution of the ratio of $C^{(k+1)}$ to $C^{(k)}$ conditional on $C^{(k)} = c_k$ is:*

$$\Pr\left[\frac{C^{(k+1)}}{C^{(k)}} \leq m|C^{(k)} = c_k\right] = 1 - \exp\left[-\Phi c_k^\theta (m^\theta - 1)\right].$$

G *Proof.* Since

$$\Pr\left[\frac{C^{(k+1)}}{C^{(k)}} \leq m|C^{(k)} = c_k\right] = \Pr\left[C^{(k+1)} \leq mc_k|C^{(k)} = c_k\right]$$

the result follows from Lemma 2. ■

Reversing the conditioning order of the previous two lemmata, the next two concern the distribution of the k'th layer given the realization of the $k+1$'st.

Lemma 5 *The distribution of $C^{(k)}$ conditional on $C^{(k+1)} = c_{k+1}$ is:*

$$\Pr[C^{(k)} \leq c_k|C^{(k+1)} = c_{k+1}] = \left(\frac{c_k}{c_{k+1}}\right)^{\theta k} \qquad c_{k+1} \geq c_k \geq 0.$$

G *Proof.* We evaluate:

$$\begin{aligned}\Pr[C^{(k)} &\leq c_k|C^{(k+1)} = c_{k+1}] = \int_0^{c_k} \frac{g_{k,k+1}(c, c_{k+1})}{g_{k+1}(c_{k+1})} dc \\ &= \int_0^{c_k} \theta k \frac{c^{\theta k-1}}{c_{k+1}^{\theta k}} dc\end{aligned}$$

which upon integrating delivers the result. ■

Lemma 6 *The distribution of the ratio of* $C^{(k+1)}$ *to* $C^{(k)}$ *conditional on* $C^{(k+1)} = c_{k+1}$ *is:*

$$\Pr\left[\frac{C^{(k+1)}}{C^{(k)}} \leq m | C^{(k+1)} = c_{k+1}\right] = 1 - m^{-\theta k}.$$

(Hence, $C^{(k+1)}/C^{(k)}$ *is independent of* $C^{(k+1)}$*.)*

G *Proof.* We rewrite:

$$\begin{aligned}\Pr\left[\frac{C^{(k+1)}}{C^{(k)}} \leq m | C^{(k+1)} = c_{k+1}\right] &= \Pr\left[C^{(k)} \geq \frac{c_{k+1}}{m} | C^{(k+1)} = c_{k+1}\right] \\ &= 1 - \Pr[C^{(k)} \leq \frac{c_{k+1}}{m} | C^{(k+1)} = c_{k+1}].\end{aligned}$$

Applying the previous lemma for $c_k = c_{k+1}/m$ delivers the result. ■

This result will prove useful in describing the distribution of the markup of price over cost under Bertrand competition.

For some purposes it is convenient to work with transformed costs defined as $U^{(k)} = \Phi\left(C^{(k)}\right)^{\theta}$ for $k = 1, 2, 3, ...$ The following lemma characterizes the very simple joint unit exponential distribution of the $U^{(k)}$'s:

Lemma 7 *The distribution of* $U^{(1)}$ *is the unit exponential distribution:*

$$\Pr[U^{(1)} \leq u] = 1 - \exp(-u), \tag{4.4}$$

while the distribution of $U^{(k+1)}$ *conditional on* $U^{(k)} = u_k$ *is:*

$$\Pr[U^{(k+1)} \leq u_{k+1} | U^{(k)} = u_k] = 1 - \exp\left[-(u_{k+1} - u_k)\right]. \tag{4.5}$$

Proof. Using the definition of $U^{(1)}$ and Lemma 1:

$$\begin{aligned}
\Pr[U^{(1)} &\leq u] = \Pr[\Phi\left(C^{(1)}\right)^{\theta} \leq u] = \Pr\left[C^{(1)} \leq \left(\frac{u}{\Phi}\right)^{1/\theta}\right] \\
&= 1 - \exp(-u),
\end{aligned}$$

establishing (4.4). Using the definition of $U^{(k)}$ and $U^{(k+1)}$ and Lemma 3:

$$\begin{aligned}
\Pr[U^{(k+1)} &\leq u_{k+1} | U^{(k)} = u_k] = \Pr\left[\Phi\left(C^{(k+1)}\right)^{\theta} \leq u_{k+1} | \Phi\left(C^{(k)}\right)^{\theta} = u_k\right] \\
&= \Pr\left[C^{(k+1)} \leq \left(\frac{u_{k+1}}{\Phi}\right)^{1/\theta} | C^{(k)} = \left(\frac{u_k}{\Phi}\right)^{1/\theta}\right] \\
&= 1 - \exp\left[-(u_{k+1} - u_k)\right],
\end{aligned}$$

establishing (4.5). ■

We can reformulate (4.5) for any integer k' as:

$$\Pr[U^{(k'+1)} - U^{(k')} \leq x] = 1 - e^{-x}$$

independent of $U^{(k')}$. An implication is that $U^{(k)}$ is the sum of k independent draws from the unit exponential distribution, which is gamma with parameters k and 1, which has the density function:

$$f(x) = \frac{1}{(k-1)!} x^{k-1} e^{-x}.$$

This result allows us to draw a series of transformed costs, starting with the lowest cost and working up, from the unit exponential distribution. (Hence, no parameter values are needed!). The costs themselves can then be recovered by applying the inverse transformation, $C^{(k)} = \left(U^{(k)}/\Phi\right)^{1/\theta}$, which depends on the two parameters, Φ and θ. The process is analogous to building up a general multivariate normal distribution from independent standard normal distributions. This technique is directly applicable in simulation, where it is advantageous to isolate the parameters of the model from the stochastic elements of the model.

This result also leads to the following lemma about any function $H(C^{(1)}, C^{(2)}, C^{(3)}, ...)$ homogenous of degree one in ordered unit costs costs.[6]

Lemma 8 *A function $H(C^{(1)}, C^{(2)}, C^{(3)}, ...)$ that is homogeneous of degree one in ordered unit costs can be written*

$$H(C^{(1)}, C^{(2)}, C^{(3)}, ...) = \Phi^{-1/\theta} H\left[\left(U^{(1)}\right)^{1/\theta}, \left(U^{(2)}\right)^{1/\theta}, \left(U^{(3)}\right)^{1/\theta}, ...\right]$$

where the joint distribution of the $U^{(k)}$'s are given by (4.4) and (4.5) above.

The result follows immediately from the relationship $C^{(k)} = \Phi^{-1/\theta}\left(U^{(k)}\right)^{1/\theta}$. Since $\Phi^{-1/\theta} = T^{-1/\theta}w$, any linear homogeneous function of unit costs is proportional to the cost w of an input bundle. We use this result to derive general properties of the price index in the next chapter.

[6] The function need not actually depend on all the ordered costs. For example, the function $H(C^{(1)}, C^{(2)}, C^{(3)}, ...) = \alpha C^{(1)}$ for $\alpha > 0$ has the required linear homogeneity property.

A

4.4 Aggregate Implications

So far we have dealt with the unit cost of producing some particular good j. We now integrate these results into a model of the aggregate economy with multiple goods. Following Ricardian tradition, we assume that inputs are mobile across the production of different goods in the economy, and that the production of any good uses inputs in the same combination. Hence producing any good entails paying the same input cost w.

As in the quality ladders literature, we assume that as ideas arrive, they pertain to each good j with equal likelihood. The outcome for any individual good is random, but the stochastic processes that govern the outcome are the same for all goods. Specifically, all goods share: (i) the same process of arrival and (ii) the same distribution of quality of the ideas that have arrived. The randomness is independent across goods.

If we think of the aggregate economy as a finite collection J of goods, with $j = 1, 2, .., J$, then, as long as J is finite, the aggregate outcome will inherit the randomness associated with individual goods. Even though this randomness declines as J becomes large, the uncertainty that the aggregate economy inherits from the specific outcomes for individual goods can be inconvenient for general equilibrium analysis.

A useful alternative is to regard the space of goods as a continuum. As in the Ricardian model with a continuum of goods and the quality ladders model (discussed in Chapter 3) we can assume that there are a unit measure of goods, so that $j \in [0,1]$. For most of what follows we adopt this formulation.

We specify the aggregate flow of ideas at date τ with quality better than q as $R(\tau)q^{-\theta}$. Since these ideas fall randomly across the continuum, the number applicable to good j is distributed Poisson with parameter $R(\tau)q^{-\theta}$.[7]

We can summarize the history of the arrival of ideas by time t with the term:

$$T(t) = \int_{-\infty}^{t} R(\tau)d\tau.$$

Hence the $T(t)$ and $\Phi(t)$ that apply to any single good also apply in the aggregate.

Due to the independence of the efficiency draws across j, the probability distribution of the efficiency for any particular good j also describes the distribution of efficiency draws across goods. Since our focus is now on the distribution of costs at some moment t we can safely suppress the t argument for the rest of this chapter, as well as for the next two.

From Section 4.1, the number of ideas that deliver a unit cost less than or equal to c for an individual good is distributed Poisson with parameter Φc^{θ}. An immediate implication is that, across the range of goods, the measure of techniques with unit cost less than c is:

$$H(c) = \Phi c^{\theta}. \tag{4.6}$$

This result will prove useful in applying this framework to monopolistic competition in

[7] See Feller (1968, Chapter VI) as applied in Grossman and Helpman (1991).

the next chapter.

We go on to use the probabilistic results from the previous section to make statements that apply across goods. Here we summarize and interpret those results one by one:

1. From Lemma 1, the distribution of the lowest cost $C^{(1)}$ for producing a good is:

$$F_1(c) = \Pr[C^{(1)} \leq c] = 1 - \exp\left[-\Phi c^{\theta}\right] \tag{4.7}$$

   This result gives the distribution of costs delivered by the best, or frontier, ideas. The previous section derived $F_1(c)$ as the probability that a particular good j can be produced at a cost less than c using the best technology. Our aggregate assumptions then imply that $F_1(c)$ is the fraction of goods that can be produced at cost less than c, using best practice. Since T reflects how advanced the state of technology is, we can think of $\Phi = Tw^{-\theta}$ as translating more advanced technology into lower (on average) unit costs, as tempered by the cost of inputs w. The parameter θ reflects the variability of costs, with larger values of θ implying less variability. Under both perfect and Bertrand competition producing a homogeneous good j, the best ideas are the only ones in use. Moreover, under perfect competition this distribution also corresponds to the distribution of prices, whose moments are given by the next result.

2. From Lemma 2, the moments of $C^{(1)}$ are given by (for $\theta + b > 0$):

$$E\left[\left(C^{(1)}\right)^b\right]^{1/b} = \left[\Gamma\left(\frac{\theta+b}{\theta}\right)\right]^{1/b} \Phi^{-1/\theta}. \tag{4.8}$$

   Under our aggregate assumptions this result yields cross-sectional moments of lowest cost, which are decreasing in Φ. A moment that will be of particular interest, requiring a particular value of b, is the CES price index under perfect competition.

3. From Lemma 2, the moments of $C^{(2)}$ are given by (for $2\theta + b > 0$):

$$E\left[\left(C^{(2)}\right)^b\right]^{1/b} = \Gamma\left(\frac{2\theta+b}{\theta}\right)^{1/b} \Phi^{-1/\theta}. \tag{4.9}$$

   Under Bertrand competition producing a homogeneous good, even though only $C^{(1)}$ is in use, $C^{(2)}$ is often the price. Hence this result is useful in constructing the CES price index under Bertrand competition.

4. From Lemma 6, the ratio $M = C^{(2)}/C^{(1)}$ is independent of $C^{(2)}$ and is distributed:

$$F_{2/1}(m) = \Pr\left[M \leq m\right] = 1 - m^{-\theta}. \tag{4.10}$$

   Under Bertrand competition M is often the markup of price over unit cost. An important consequence of this result for what follows is that markups are unrelated to any features embodied in Φ, such as the history of technology or input costs, a feature it shares with the fixed markup of monopolistic competition and quality ladders.

5. From Lemma 3, conditional on $C^{(1)} = c_1$, the distribution of $C^{(2)}$ is:

$$\Pr[C^{(2)} \leq c_2 | C^{(1)} = c_1] = 1 - \exp\left[-\Phi(c_2^\theta - c_1^\theta)\right]. \tag{4.11}$$

   The lower c_1, the more likely a low $C^{(2)}$. Under Bertrand competition, then, low-cost producers are more likely to charge a lower price, since $C^{(2)}$ is often the price.

6. From Lemma 3, the distribution of the ratio $M = C^{(2)}/C^{(1)}$ given $C^{(1)} = c_1$ is:

$$\Pr\left[M \leq m | C^{(1)} = c_1\right] = 1 - \exp\left[-\Phi c_1^\theta (m^\theta - 1)\right]. \tag{4.12}$$

   The lower c_1, the more likely a high markup. Under Bertrand competition, then, low-cost producers are more likely to charge a higher markup.

7. From Lemma 8, any linear homogeneous function of unit costs is homogeneous of degree $-1/\theta$ in Φ.

We next turn to how these results can be combined with various assumptions about preferences and market structure to deliver general equilibrium results.

CHAPTER 4 — MANUSCRIPT

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Chapter 5

Preferences and Market Structure

In the previous chapter we considered a world in which ideas for producing a good arrive over time. We begin by summarizing the elements of the analysis there that feed into this chapter.

The basic unit of analysis is an idea about how to make a good. The key feature of an idea is its efficiency, how much of the good it can produce with a unit bundle of inputs. Efficiencies are drawn from a Pareto distribution, so that the probability that efficiency Q exceeds some level q', given that Q exceeds some lower bound $q > 0$, is:

$$\Pr[Q \geq q' | Q \geq q] = (q'/q)^{-\theta}$$

where $\theta > 1$. The higher θ the more similar are the techniques in terms of their efficiency.

Associated with each idea's efficiency Q is a unit cost $C = w/Q$, where w is the cost of a bundle of inputs. Here we assume that using any technology requires paying the same w.

As ideas arrive, different techniques for producing the same good build up. We can order the set of techniques for producing any good j according to their unit costs $C^{(1)}(j) \leq C^{(2)}(j) \leq ... \leq C^{(k)}(j) \leq ....$ The joint distribution of the $C^{(k)}(j)$'s depends on only two parameters, the Pareto parameter θ and the state variable:

$$\Phi = Tw^{-\theta},$$

where T summarizes the history of arrival of ideas and w is the cost of a bundle of inputs. A higher Φ means unit costs tend to be lower and a higher θ means unit costs tend to be closer together. The previous chapter provides a set of results characterizing features of the joint distribution of the $C^{(k)}(j)$'s which we put to use below.

We now embed this cost structure into a static general equilibrium framework, drawing out its implications under various assumptions about market structure. We begin by specifying how consumers value what can be produced with these techniques.

A

5.1 Preferences

It might seem natural to assume that consumers regard the output produced by different techniques for making the same good as identical. In many situations we will. But it turns out that there is much of interest to say if we imagine that each technique produces a different *variety* of a good, and that consumers might distinguish among different varieties of the same good. Sticking with the constant elasticity of substitution (CES) preference structure introduced in Chapter 3, we assume that the utility a consumer derives from the different varieties of good j is:

$$Y(j) = \left[\sum_{k=1}^{\infty} Y^{(k)}(j)^{(\sigma'-1)/\sigma'}\right]^{\sigma'/(\sigma'-1)}. \tag{5.1}$$

where $Y^{(k)}(j)$ is the amount consumed of variety k of good j and σ' is the elasticity of substitution among varieties. We will interpret $Y(j)$ as a measure of the consumption of good j. In the limiting case $\sigma' \to \infty$ consumers regard all the varieties as identical.

As in the Dornbusch, Fischer, and Samuelson (1977) and Grossman and Helpman (1991a,1991b) models discussed in Chapter 3, we consider an economy with a unit continuum of goods. Hence overall utility is:

$$U = \left[\int_0^1 Y(j)^{(\sigma-1)/\sigma} dj\right]^{\sigma/(\sigma-1)}, \tag{5.2}$$

where $\sigma \geq 0$ is the elasticity of substitution across goods.[1]

Consider a consumer facing prices for varieties and goods $P^{(k)}(j), k = 1, 2, 3, ...; j \in [0, 1]$. With a total amount $X(j) = P(j)Y(j)$ spent on good j, the amount $X^{(k)}(j) =$

[1]With $\sigma = 1$ this expression can be written more conveniently as:

$$U = \exp\left(\int_0^1 \ln Y(j) dj\right).$$

$P^{(k)}(j)Y^{(k)}(j)$ spent on variety k of good j is:

$$X^{(k)}(j) = X(j)\left(\frac{P^{(k)}(j)}{P(j)}\right)^{-(\sigma'-1)}, \tag{5.3}$$

where the good j price index is:

$$P(j) = \left[\sum_{k=1}^{\infty}\left[P^{(k)}(j)\right]^{-(\sigma'-1)}\right]^{-1/(\sigma'-1)}. \tag{5.4}$$

Note that as $\sigma' \to \infty$ we get $P(j) = \min_k\{P^{(k)}(j)\}$.[2]

If a consumer is spending a total amount X, spending on all varieties of good j is:

$$X(j) = X\left(\frac{P(j)}{P}\right)^{-(\sigma-1)}, \tag{5.5}$$

where:

$$P = \left[\int_0^1 P(j)^{-(\sigma-1)}dj\right]^{-1/(\sigma-1)} \tag{5.6}$$

is the overall price index.[3] The resulting utility is X/P.

How prices $P^k(j)$ relate to unit costs $C^{(k)}(j)$ depends on particular assumptions about market structure. We pursue several variants below which deliver a closed

[2] The derivation of (5.3) is as in footnote 5 in Chapter 3 (with all $\alpha_i = 1$).

[3] The derivation is as in the case of monopolistic competition. See footnote 14 of Chapter 3. With $\sigma = 1$ the price index can be written more conveniently as:

$$P = \exp\left(\int_0^1 \ln P(j)dj\right).$$

form solution for the price index P. But we can say quite a bit in general, imposing only a reasonable restriction which holds under most assumptions about market structure, that no variety is available at a price below its unit cost, i.e. $P^{(k)}(j) \geq C^{(k)}(j)$ for all k and j. Even with this mild condition on prices, in order for P to be well defined requires restrictions on σ, σ', and the availability of varieties.

In order for utility not to explode to infinity requires that the price index P be bounded away from zero. Two problems can emerge.

First, if the elasticity of substitution σ is very high and if the distribution of prices has a fat lower tail, consumers can attain infinite utility by concentrating their expenditure on goods with prices arbitrarily close to zero. To avoid this outcome we need to impose restrictions on σ relative to the parameter θ governing the distribution of unit costs.

Second, if σ' is very low and there is no restriction on the number of varieties of good j that are available, the consumer's utility from good j can explode toward infinity due to the plethora of varieties.

The following theorem states sufficient conditions for a strictly positive price index P, ruling out these two possibilities:

Theorem 2 *As long as prices weakly exceed unit costs, the price index P is bounded strictly above zero if (A) $\sigma < \theta + 1 < \sigma'$ or (B) $1 < \sigma < \theta + 1$, $\sigma \leq \sigma'$, and an upper bound $\bar{c}$ on unit costs, so that any variety k of good j is unavailable for $C^{(k)}(j) > \bar{c}$.*

The long and intricate proof is relegated to the Appendix.[4]

In considering various market structures we may need to impose the restrictions of the theorem to obtain a well-defined price index. For example, with perfect competition we will always assume $\sigma' > \theta + 1$ to satisfy condition (A) of the theorem. Often we will go to the extreme of $\sigma' \to \infty$, and not care about the availability of inferior varieties. With monopolistic competition we set $1 < \sigma' = \sigma < \theta + 1$, requiring us to make assumptions that limit the availability of varieties in order to satisfy (B).[5]

5.2 Unit Costs, the Price Index, and Welfare

Under the restrictions just discussed, we can write the price index P as:

$$P = \left(E\left[P(j)^{-(\sigma-1)}\right]\right)^{-1/(\sigma-1)}.$$

In all of the market structures we consider the price index $P(j)$ is linear homogeneous in unit costs. Doubling all unit costs, for example, doubles prices. For some homogeneous function H, then, we can write:

[4] We also need to avoid the terrible situation in which the price index P goes to infinity. When $\sigma \leq 1$, utility becomes infinitely negative if there is any good with no available variety. We thus have to ensure that at least one variety of each good is available at a finite price. With $\sigma > 1$ we have no problem if no variety of a good is available as long as a measure of goods have at least one variety.

[5] In our discussion in this section we treat goods and varieties as entering utility directly. Following Ethier (1982), we could also think of them as intermediates entering a production function.

$$P(j) = H(C^{(1)}(j), C^{(2)}(j), ...),$$

where H is homogeneous of degree one, with its exact form depending on how the market structure translates unit cost into market prices. With $\sigma' \to \infty$ and perfect competition, for example, $H(C^{(1)}(j), C^{(2)}(j), ...) = C^{(1)}(j)$.

We exploit the result from Lemma 7 of the previous chapter, that:

$$C^{(k)}(j) = (U^{(k)}(j)/\Phi)^{1/\theta},$$

where $U^{(k)}(j)$ is the sum of k independent unit exponential random variables, to write:

$$\begin{aligned}
P &= \left(E\left[P(j)^{-(\sigma-1)}\right]\right)^{-1/(\sigma-1)} = \left(E\left[\left(H(C^{(1)}(j), C^{(2)}(j), ...)\right)^{-(\sigma-1)}\right]\right)^{-1/(\sigma-1)} \\
&= \left(E\left[\left(H(\left[U^{(1)}(j)/\Phi\right]^{1/\theta}, \left[U^{(2)}(j)/\Phi\right]^{1/\theta}, ...)\right)^{-(\sigma-1)}\right]\right)^{-1/(\sigma-1)} \\
&= \Phi^{-1/\theta}\left(E\left[\left(H(\left[U^{(1)}(j)\right]^{1/\theta}, \left[U^{(2)}(j)\right]^{1/\theta}, ...)\right)^{-(\sigma-1)}\right]\right)^{-1/(\sigma-1)}.
\end{aligned}$$

We can thus express any price index P as:

$$P = \Gamma\Phi^{-1/\theta} = \Gamma T^{-1/\theta} w \tag{5.7}$$

where Γ varies according to σ', market structure, and possibly market size and overhead costs, but not directly on either the state of technology T or the cost of inputs w.

Treating labor as the only input we can immediately obtain an expression for the real wage:

$$\frac{w}{P} = \Gamma^{-1} T^{1/\theta} \tag{5.8}$$

showing how advances in technology raise welfare. The larger θ the more similar are the ideas that arrive, and the less likely that one of them constitutes a major advance.

What is in Γ will depend on market structure, to which we now turn.

A

5.3 Market Structure

Our specifications of technology and preferences can accommodate a wide variety of assumptions about market structure. A basic dichotomy is between environments in which technologies are freely available to a large number of potential producers and those in which technologies are proprietary. The first case gives rise to perfect competition and the second case to imperfect (e.g., Bertrand, Cournot, and monopolistic) competition. We take up each case in turn.

If technologies are freely available, there is no reason why a consumer couldn't buy any variety of a good. Hence to accommodate Theorem 2 we restrict our analysis to situations in which varieties are highly substitutable, i.e., assuming $\sigma' > \theta + 1$. With proprietary technologies we also consider this case. In addition, we allow for the opposite case of $\sigma' < \theta + 1$ by introducing a fixed overhead cost $E > 0$ of producing any variety. Since only a relatively efficient potential producer can earn enough in variable profit to cover the fixed cost, the fixed cost implies an upper bound on the unit cost of any active producer.

While we provide general characterizations when possible, we emphasize three special cases that (1) deliver relatively simple closed-form solutions and (2) relate to existing analysis and applications:

1. Assuming $\sigma' \to \infty$, $E = 0$, and perfect competition delivers the Ricardian competitive model with a continuum of goods analyzed in Eaton and Kortum (2002) and Alvarez and Lucas (2007).

2. Assuming $\sigma' \to \infty$, $E = 0$, and Bertrand competition among proprietary owners of each technique delivers quality ladders. Kortum (1997), Eaton and Kortum (1999), and BEJK (2003), and Atkeson and Burstein (2007) consider this case.

3. Assuming $\sigma' = \sigma$, $E > 0$, and monopolistic competition delivers a variant of the models developed in Helpman, Melitz, and Yeaple (2004), Chaney (2005), Helpman, Melitz, and Rubinstein (2005) Baldwin and Robert-Nicaud (2006), and Eaton, Kortum, and Kramarz (2010).

We can characterize the full equilibrium for these three cases but we can say a fair bit more generally.

B

5.3.1 Freely Available Technology

With technology freely available and no fixed cost of production, competition among potential producers makes any variety of a good available at its unit cost of production. Hence $P^{(k)}(j) = C^{(k)}(j)$. Since there is no constraint on the number of available varieties, we assume $\sigma' > \theta + 1$ to keep the price index strictly positive.

If total spending on good j is $X(j)$, spending on variety k is

$$X^{(k)}(j) = X(j) \frac{\left[C^{(k)}(j)\right]^{-(\sigma'-1)}}{\sum_{l=1}^{\infty} \left[C^{(l)}(j)\right]^{-(\sigma'-1)}}.$$

Since $\sigma' > 1$ revenue is decreasing in k, but how dominant is the lowest cost ($k = 1$) variety? The following proposition addresses this question.

Proposition 2 *The expected value of the overall market for good j relative to the market for the low-cost variety of good j is:*

$$E\left[\frac{X(j)}{X^{(1)}(j)}\right] = \frac{\sigma' - 1}{\sigma' - (\theta + 1)}.$$

This measure must exceed 1, but will be close to 1 if the low cost variety dominates the market. Remember that we are assuming $\sigma' > \theta + 1$. As σ' approaches $\theta + 1$ from above, this expectation becomes arbitrarily large. In this case, while the low cost variety is still the biggest seller, its share of the market becomes infinitesimally small. As σ' increases from there, the size of the market shrinks relative to the sales of the low cost variety. As $\sigma' \to \infty$ the ratio approaches 1. As expected, the low cost firm takes over the market when varieties are perfect substitutes. We now turn to a complete characterization of that case.

Varieties perfect substitutes. As $\sigma' \to \infty$, consumers regard all varieties of each good j as equivalent. Under perfect competition only the lowest unit cost version of the good will be purchased, with price $P(j)$ equal to the lowest unit cost $C^{(1)}(j)$. The distribution of prices will thus correspond to the distribution of lowest costs given in (4.7). We now demonstrate:

Proposition 3 *Under perfect competition, given $\sigma > \theta + 1$ and $\sigma' \to \infty$ the CES price index is:*

$$P = \gamma^{PC} \Phi^{-1/\theta} \tag{5.9}$$

where

$$\gamma^{PC} = \left[\Gamma \left(\frac{\theta - (\sigma - 1)}{\theta} \right) \right]^{-1/(\sigma-1)}$$

and $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$ is the gamma function.

The Chapter Appendix provides the proof of this proposition and of Propositions 5 through 8.

Note that the term Γ in the general price index (5.7) and in the expression for the real wage (5.8) reduces to γ^{PC}, which involves only the parameters, θ and σ, but not the state of technology T.

We conclude with one further result for perfect competition which anticipates results that follow:

Remark 1 *With $\sigma > 1$, revenues $X(j)$ are greater for a good with a lower realization of cost $C^{(1)}(j)$.*

The result follows immediately from substituting $C^{(1)}(j)$ for $P(j)$ in (5.5). Hence goods with lower costs occupy a larger share of expenditure.

Since there are no profits, aggregate expenditure is simply $X = wL$ and welfare per worker is:

$$W = \frac{w}{P} = \left(\gamma^{PC}\right)^{-1} T^{1/\theta}.$$

Since γ^{PC} depends only on the parameters θ and σ, the relationship between welfare and technology is very tight. Being particularly stark, perfect competition is a good baseline for the embellishments that follow.

B

5.3.2 Proprietary Technologies

We now turn to situations in which each technology is associated with a particular producer who has a monopoly on its use. We call this producer a firm and assume that its owner makes production choices that maximize profit. We allow for the possibility that the same firm owns the technologies for multiple varieties of the same good, but exclude the possibility that a firm owns varieties across a positive measure of the continuum of goods, so that any firm takes the overall price level P and level of spending

X as given. But individual producers of varieties of good j can affect the price index for that good $P(j)$ given in (5.4). Hence we need to make specific assumptions about how the producers of different varieties of the same good interact.

We follow the two standard approaches in treating each firm as making its production decision taking (1) the prices of each other variety as given (Bertrand competition) and (2) the outputs of each other variety as given (Cournot competition).

With Bertrand and Cournot competition we will consider both the possibility of a high elasticity of substitution $\sigma' > \theta + 1$ and the possibility of a fixed cost of production, which keeps inefficient producers out of the market. In the case of a fixed cost there is a possibility of multiple equilibria. We resolve this multiplicity by picking the most efficient outcome, assuming that in equilibrium any entrant has lower unit cost than any non-entrant. In other respects, the basic set up of the problem looks the same whether or not all firms enter.

Atkeson and Burstein (2008) provide expressions for a firm's price under each type of competition in terms of its own unit cost, the elasticities of substitution σ and σ', and the market share $S^f(j)$ within its good, where f labels the firm. While this last term depends on the prices the firms are charging, the expressions are nevertheless very illuminating.

C Bertrand Competition

The profit of firm f producing a set $\vartheta^f(j)$ of varieties of good j is

$$\begin{aligned}\pi^f(j) &= \sum_{k\in\vartheta^f(j)} \left[P^{(k)}(j) - C^{(k)}(j)\right] Y^{(k)}(j) \\ &= \sum_{k\in\vartheta^f(j)} \left[1 - \frac{C^{(k)}(j)}{P^{(k)}(j)}\right] X^{(k)}(j).\end{aligned}$$

Combining (5.3) and (5.5) to express the producer's revenue in terms of prices and aggregate spending, profit can be written as:

$$\pi^f(j) = \sum_{k\in\vartheta^f(j)} \left[1 - \frac{C^{(k)}(j)}{P^{(k)}(j)}\right] \left[P^{(k)}(j)\right]^{-(\sigma'-1)} \left[P(j)\right]^{(\sigma'-\sigma)} P^{(\sigma-1)}X.$$

Given the prices $P^{(m)}(j)$, $m \notin \vartheta^f(j)$, chosen by all other firms producing varieties of good j, firm f chooses its prices $P^{(k)}(j)$, $k \in \vartheta^f(j)$ to maximize $\pi^f(j)$. Taking $P^{(\sigma-1)}X$ as given, but realizing that a change in the price of any one variety alters $P(j)$ and hence profits on all the others that it owns, the firm's first-order conditions for a maximum (for each $k \in \vartheta^f(j)$) are:

$$(\sigma'-\sigma)\sum_{l\in\vartheta^f(j)} \left[1 - \frac{C^{(l)}(j)}{P^{(l)}(j)}\right] \left(\frac{P^{(l)}(j)}{P(j)}\right)^{-(\sigma'-1)} - (\sigma'-1)\left[1 - \frac{C^{(k)}(j)}{P^{(k)}(j)}\right] + \frac{C^{(k)}(j)}{P^{(k)}(j)} = 0.$$

Note that the first term in this expression is the same for all $k \in \vartheta^f(j)$. An implication is that the firm charges the same markup $m^f = P^{(k)}(j)/C^{(k)}(j)$ on all its varieties. Let $S^f(j) = \sum_{l\in\vartheta^f(j)} X^{(l)}(j)/X(j)$ be firm f's share of the market for good j. From (5.3):

$$S^f(j) = \sum_{l\in\vartheta^f(j)} \left(\frac{P^{(l)}(j)}{P(j)}\right)^{-(\sigma'-1)},$$

which we can substitute into the first-order condition to get:

$$m^f = \frac{\varepsilon^f_{BC}(j)}{\varepsilon^f_{BC}(j) - 1} \tag{5.10}$$

where:

$$\varepsilon^f_{BC}(j) = \sigma S^f(j) + \sigma'(1 - S^f(j)). \tag{5.11}$$

Hence the markup can be expressed in terms of a weighted average of the two elasticities of substitution, where the weight on the upper tier elasticity of substitution σ is firm f's share of spending on good j.[6] In the case in which the firm dominates a good ($S^f(j) \to 1$) the price converges to the Dixit-Stiglitz markup $\sigma/(\sigma - 1)$ across **goods** while as it becomes negligible ($S^f(j) \to 0$) it falls to the Dixit-Stiglitz markup $\sigma'/(\sigma'-1)$ across **varieties**. The problem is well defined as long as $S^f(j) < (\sigma' - 1)/(\sigma' - \sigma)$ for any firm f selling varieties of good j. For $\sigma > 1$ this constraint is never binding, but for $\sigma \leq 1$ we need to place restrictions on the ownership of the technologies for producing good j so that one firm does not become too dominant. (The restrictions imposed by Theorem 2 rule out $\sigma' \leq 1$.)

[6] Expression (5.10) might suggest that in the limit as $\sigma' \to 1$ the firm owning the most efficient technology would charge the markup $\overline{m} = \sigma/(\sigma - 1)$. But if $\overline{m}C^{(1)} > C^{(2)}$ then the firm owning the technique associated with $C^{(2)}$, if different, would charge a price just below, so that the low cost firm's share would be zero. But the low cost firm would respond with an even lower price, etc. The equilibrium in this case is for the low cost firm to charge a price just below $C^{(2)}$, capturing the entire market. We turn to this limiting case shortly.

In particular, for $\sigma \leq 1$ we rule out the case in which one firm owns all the technologies for producing varieties of good j. For $\sigma > 1$, a firm with a monopoly on all varieties of good j will have a market share $S^f(j) = 1$, hence its markup on each variety will be $\sigma/(\sigma-1)$. Its profit on variety k is therefore $X^{(k)}(j)/\sigma$.

Recall from Theorem 2 that, for the case $\sigma' \leq \theta + 1$ we impose an upper bound on the unit cost of available varieties. As we show below, a fixed cost $E > 0$ of establishing a variety implies such a bound.

We can go no further in deriving general results. Instead, we turn to the special case in which $\sigma' \to \infty$ and each potential variety has a different owner, which yields simple closed-form solutions for the distribution of the markup, the price index, and the profit share.

Varieties perfect substitutes. In this case, as with perfect competition with $\sigma' \to \infty$, only the lowest unit cost variety is sold, so that $S^{(1)}(j) = 1$. With each variety owned by a single producer, this supplier will charge a price equal to the lesser of the Dixit-Stiglitz markup:

$$\overline{m} = \begin{cases} \frac{\sigma}{\sigma-1} & \sigma > 1 \\ \infty & \sigma \leq 1 \end{cases}.$$

and the unit cost of the second lowest cost supplier $C^{(2)}(j)$:

$$P(j) = \min\left\{\overline{m}C^{(1)}(j), C^{(2)}(j)\right\}.$$

Any potential variety of a good with unit cost ranked third or more is irrelevant to the market equilibrium.[7] The implied markup, then, is:

$$M(j) = \frac{P(j)}{C^{(1)}(j)} = \min\left\{\frac{C^{(2)}(j)}{C^{(1)}(j)}, \overline{m}\right\}. \tag{5.12}$$

Unlike the case of monopolistic competition, which we turn to below, the markup varies depending on the unit costs of the first and second lowest cost supplier, and will vary stochastically across goods. Applying (4.12) to (5.12) delivers:

Proposition 4 *Under Bertrand competition with all varieties perfect substitutes the distribution of the markup $M(j)$ is:*

$$\Pr\left[M(j) \leq m\right] = F_M(m) = 1 - m^{-\theta}$$

for $m \leq \overline{m}$. With probability $\overline{m}^{-\theta}$ the markup is $\overline{m}$. The markup is independent of $C^{(2)}(j)$.

We now establish the following result on the price index and on the profit share of the economy:

Proposition 5 *Under Bertrand competition with all varieties perfect substitutes the price index is:*

$$P = \gamma^{BC}\Phi^{-1/\theta} \tag{5.13}$$

[7] We have no analytical demonstration that (5.10) converges to this price as $\sigma' \to \infty$, but in a large number of numerical simulations it did so nicely.

where:

$$\gamma^{BC} = \left[\left(1 + \frac{(\sigma-1)\overline{m}^{-\theta}}{\theta-(\sigma-1)} \right) \Gamma \left(\frac{2\theta-(\sigma-1)}{\theta} \right) \right]^{-1/(\sigma-1)}.$$

As with perfect competition, the term Γ in the general price index and real wage term reduces to a constant γ^{BC} depending only on the parameters θ and σ.

Under perfect competition there were, of course, no profits. That is not the case under Bertrand competition. With $\sigma' \to \infty$ the owner of the lowest cost idea earns a rent equal to:

$$\pi(j) = \left[1 - \frac{C^{(1)}(j)}{P(j)}\right] X(j) = \left[1 - \frac{1}{M(j)}\right] X(j). \tag{5.14}$$

For any individual producer this rent depends not only on the realization of her own cost, but that of the second lowest-cost producer as well. However, averaging across all active producers, the profit share in the economy turns out to have a simple form. We now establish:

Proposition 6 *Under Bertrand competition with all varieties perfect substitutes aggregate profit is:*

$$\Pi = \delta^{BC} X$$

where

$$\delta^{BC} = \frac{1}{1+\theta}$$

and

$$X = \frac{1+\theta}{\theta} wL$$

is total spending.

It might come as a surprise that, even though the markup is capped at $\overline{m} = \sigma/(\sigma-1)$, the share of profit in the economy is independent of σ. The explanation is that while a higher value of σ limits the markup that any producer will charge, it also implies greater sales and hence higher profit to low cost sellers who are more likely to be constrained by $\overline{m}$, with the two effects cancelling out.

This result on the profit share of the economy is very useful in Chapter 7, where we endogenize the production of ideas in the quality ladders model. It implies a simple expression for the expected discounted value of an idea, and hence the return to innovative activity.

We conclude with a result on the relationship between cost, price, sales, and the markup.

Remark 2 *A lower unit cost $C^{(1)}(j)$ is associated with: (i) a lower price, (ii) with $\sigma > 1$ larger sales, and (iii) a higher markup.*

The first result holds for two reasons. First, if $\overline{m}C^{(1)}(j) < C^{(2)}(j)$ then the result is immediate. But, if not, conditioning on a lower $C^{(1)}(j)$ in the distribution (4.11) yields a distribution of $C^{(2)}(j)$ that is worse (in the sense of stochastic dominance). Either way, a firm with a lower unit cost is likely to charge a lower price. The second result follows from a lower price leading to higher sales if $\sigma > 1$. The third result follows from (4.12): A lower $C^{(1)}(j)$ implies a distribution of $M(j)$ that is better (in the sense of stochastic dominance).

In contrast with perfect competition and monopolistic competition (taken up below), a producer with a lower unit cost will, on average, charge a price that is not proportionately lower, thus charging a higher markup. The result implies a correlation between size and markups. A seller with lower cost is more likely both to sell more and to earn a higher profit per sale.

Taking profits into account, welfare per worker is:

$$W = \frac{1+\theta}{\theta}\frac{w}{P} = \frac{1+\theta}{\theta}\left(\gamma^{BC}\right)^{-1} T^{1/\theta}.$$

As with perfect competition, γ^{BC} depends only on the parameters θ and σ, delivering a tight relationship between welfare and technology. In the absence of technological heterogeneity ($\theta \to \infty$) the expression for welfare reduces to the same as that for perfect competition.

C Cournot Competition

This case is more complicated, and yields less in terms of closed-form solutions, but some work yields an interesting expression for the markup. To start out we rearrange the equations for demand to express prices in terms of quantities and aggregates. From (5.3) we have:

$$\frac{P^{(k)}(j)}{P(j)} = \left(\frac{Y^{(k)}(j)}{Y(j)}\right)^{-1/\sigma'} \tag{5.15}$$

and from (5.5) we have:

$$P(j) = Y(j)^{-1/\sigma} P^{(\sigma-1)/\sigma} X^{1/\sigma}.$$

Combining these two:

$$P^{(k)}(j) = \left[Y^{(k)}(j)\right]^{-1/\sigma'} Y(j)^{-\Omega} P^{(\sigma-1)/\sigma} X^{1/\sigma}, \tag{5.16}$$

where $\Omega = (1/\sigma) - (1/\sigma')$. Multiplying both sides of (5.15) by $Y^{(k)}(j)/Y(j)$, the market share of variety k in the market for good j is:

$$S^{(k)}(j) = \left(\frac{Y^{(k)}(j)}{Y(j)}\right)^{(\sigma'-1)/\sigma'}. \tag{5.17}$$

We have now been able to express $P^{(k)}(j)$ and $S^{(k)}(j)$ in terms of $Y^{(k)}(j)$, $Y(j)$, X, and P. Because there are a continuum of goods, X and P are not affected by decisions made by producers of varieties of good j.

We now turn to the profit-maximizing decisions of these producers. Employing (5.16), the variable profit from variety k is:

$$\begin{aligned}
\pi^{(k)}(j) &= P^{(k)}(j) Y^k(j) - C^{(k)}(j) Y^k(j) \\
&= \left[Y^{(k)}(j)\right]^{(\sigma'-1)/\sigma'} Y(j)^{-\Omega} P^{(\sigma-1)/\sigma} X^{1/\sigma} - C^{(k)}(j) Y^k(j).
\end{aligned}$$

Using (5.1), we can rewrite this expression as:

$$\pi^{(k)}(j) = \left[Y^{(k)}(j)\right]^{(\sigma'-1)/\sigma'} \left[\sum_{k'=1}^{\infty} \left[Y^{(k')}(j)\right]^{(\sigma'-1)/\sigma'}\right]^{-\Omega\sigma'/(\sigma'-1)} P^{(\sigma-1)/\sigma} X^{1/\sigma} - C^{(k)}(j) Y^k(j)$$

Given the output of all other firms, firm f producing varieties $k \in \vartheta^f(j)$ chooses $Y^{(k)}(j)$ for $k \in \vartheta^f(j)$ to maximize:

$$\begin{aligned}\pi^f(j) \quad &= \quad \sum_{k\in\vartheta^f(j)} \left[Y^{(k)}(j)\right]^{(\sigma'-1)/\sigma'} \left[\sum_{k'\in\vartheta^f(j)} \left[Y^{(k')}(j)\right]^{(\sigma'-1)/\sigma'} + V^{\tilde{}f}\right]^{-\Omega\sigma'/(\sigma'-1)} P^{(\sigma-1)/\sigma} X^{1/\sigma} \\ &\quad - \sum_{k\in\vartheta^f(j)} C^{(k)}(j) Y^k(j).\end{aligned}$$

where:

$$V^{\tilde{}f} = \sum_{l\notin\vartheta^f(j)} \left[Y^{(l)}(j)\right]^{(\sigma'-1)/\sigma'}.$$

pertains to varieties produced by firms other than f.

The first-order conditions for a maximum are:

$$\left[\frac{(\sigma'-1)}{\sigma'} - \Omega \sum_{k'\in\vartheta^f(j)} \left[Y^{(k')}(j)\right]^{(\sigma'-1)/\sigma'} Y(j)^{-(\sigma'-1)/\sigma'}\right] \left[Y^{(k)}(j)\right]^{-1/\sigma'} Y(j)^{-\Omega} P^{(\sigma-1)/\sigma} X^{1/\sigma} = C^{(k)}(j).$$

Using (5.16) and (5.17) this expression simplifies to:

$$\left[\frac{(\sigma'-1)}{\sigma'} - \Omega S^f(j)\right] P^{(k)}(j) = C^{(k)}(j).$$

where:

$$S^f(j) = \sum_{k'\in\vartheta^f(j)} S^{(k')}(j)$$

is firm f's market share in product j. Rearrangement delivers:

$$P^{(k)}(j) = \left[\frac{\varepsilon^{(k)}_{CC}(j)}{\varepsilon^{(k)}_{CC}(j) - 1}\right] C^{(k)}(j) \tag{5.18}$$

where now:

$$\varepsilon^f_{CC}(j) = \left[\frac{1}{\sigma} S^f(j) + \frac{1}{\sigma'}(1 - S^f(j))\right]^{-1}.$$

As with Bertrand competition, if a variety dominates a good ($S^f(j) \to 1$) the price converges to the Dixit-Stiglitz markup across **goods** while if it becomes negligible ($S^f(j) \to 0$) it goes to the Dixit-Stiglitz markup across **varieties**. Since we assume $\sigma' \geq \sigma$ more efficient firms charge a higher markup but a lower price.[8] Unlike Bertrand competition, however, even as $\sigma' \to \infty$ multiple varieties of a good can coexist. This limit no longer yields closed-form results.[9]

Since expressions (5.10) and (5.18) involve the term $S^f(j)$ they are not fully-reduced form expressions. They come in very handy, however, in computing a solution numerically. Moreover, they hold regardless of whether $\sigma' > \theta + 1$, in which case we don't need to impose any restrictions on entry of inefficient varieties, or if there is a fixed cost of entry.

C Monopolistic Competition

Consider either (5.10) or (5.18) with $\sigma' = \sigma$, so that buyers regard different varieties of the same product as distinct from each other as from varieties of different products.

[8] Assuming the contrary delivers the same contradiction as in the Bertrand case.

[9] In the case $\sigma' \to \infty$ multi-variety firms will at most sell only their lowest-cost version. All varieties sold in strictly positive amounts will have a common price $P(j)$. The firm with unit cost $C^{(k)}(j)$ will have a market share:

$$S^{(k)}(j) = \sigma \left[1 - \frac{C^{(k)}(j)}{P(j)} \right]$$

for all k such that $S^{(k)}(j) > 0$.

Both Bertrand and Cournot competition reduce to the familiar monopolistic competition model with producers of a variety setting a markup $\overline{m} = \sigma/(\sigma - 1)$ over unit cost. Since the distinction between variety and product disappears there is no longer any distinction between j, indexing products, and k, indexing varieties of a product.

Since now $\sigma' = \sigma \leq \theta + 1$, to assure a positive price index we introduce an overhead cost $E > 0$ that the owner must incur to serve the market. This fixed cost ensures that for each good j there is some threshold $\bar{c} < \infty$ such that it is unprofitable for a firm with a unit cost greater than $\bar{c}$ to enter, thus satisfying Part B of Theorem 2. We treat this fixed cost as involving hiring overhead labor.

The number of active sellers is thus endogenous. As before, labor is the only input, but now we need to distinguish between production labor and overhead labor.

From (4.6), the distribution of costs is the same as if each good were produced with a level of efficiency drawn from a Pareto distribution, as in the model of monopolistic competition of Helpman, Melitz, and Yeaple (2004), Chaney (2005), and Helpman, Melitz, and Rubinstein (2005). Hence this section provides a bridge between their work and ours.

We establish the following two results:

Proposition 7 *Under monopolistic competition the price index in a market with total sales X is:*

$$P = \gamma^{MC} \left(\frac{X}{\sigma E} \right)^{-[\theta - (\sigma - 1)]/[(\sigma - 1)\theta]} \Phi^{-1/\theta} \tag{5.19}$$

where

$$\gamma^{MC} = \overline{m}\left[\frac{\theta}{\theta-(\sigma-1)}\right]^{-1/\theta}.$$

Entry is profitable only for producers with cost $c \leq \overline{c}$ given by:

$$\begin{aligned}\overline{c} &= \left(\frac{X}{\sigma E}\right)^{1/(\sigma-1)}\frac{P}{\overline{m}} \\ &= \left(\frac{\theta-(\sigma-1)}{\theta}\frac{X}{\sigma E}\right)^{1/\theta}\Phi^{-1/\theta}\end{aligned}$$

and the measure of active sellers H is:

$$H = \frac{X}{\sigma E}\frac{\theta-(\sigma-1)}{\theta}. \tag{5.20}$$

The term Γ in the general price index (5.7) and real wage (5.8) is now:

$$\Gamma = \gamma^{MC}\left(\frac{X}{\sigma E}\right)^{-[\theta-(\sigma-1)]/[(\sigma-1)\theta]}$$

Not only does it depend on the term γ^{MC}, a function of σ and θ, it also depends on the size of the market X relative to the overhead cost E. In contrast to perfect and Bertrand competition with no overhead cost, with monopolistic competition a larger market attracts a greater variety of sellers. A larger market thus provides more variety.

Because of the presence of overhead costs, we need to distinguish variable profit, without overhead costs netted out, from profit itself, revenues less both production and overhead costs.

Having derived the price index and cutoff cost in terms of E,we now establish results on profits:

Proposition 8 *Under Monopolistic Competition: (i) aggregate variable profit is:*

$$\Pi^V = \frac{X}{\sigma};$$

(ii) aggregate profit is:

$$\Pi = \delta^{MC} X \tag{5.21}$$

where:

$$\delta^{MC} = \frac{\sigma - 1}{\theta\sigma} = \frac{1}{\theta\overline{m}}$$

with:

$$X = \frac{\theta\overline{m}}{\theta\overline{m} - 1} wL;$$

(iii) average profit per producer is:

$$\frac{\sigma - 1}{\theta - (\sigma - 1)} E. \tag{5.22}$$

Here L includes both production and overhead workers.

Remark 3 *A lower unit cost $C^{(k)}(j)$ is associated with: (i) a lower price, (ii) larger sales, but (iii) an invariant markup.*

We now derive the price level P, the cutoff cost level $\bar{c}$, and the measure of active sellers H. An issue is how the required number of overhead workers varies with the state of technology T. We introduce the possibility that advances in T reduce overhead costs by specifying:

$$E = wFT^{-\eta}$$

where $FT^{-\eta}$ is the number of overhead workers required for a product. With $\eta = 0$, the overhead requirement is independent of technology while $\eta > 0$ means that advances in technology reduce it. Using this specification we get the following:

1. The price index:

$$P = \gamma^{MC} \left(\frac{LT^{\eta}}{(1-\delta^{MC})\sigma F} \right)^{-[\theta-(\sigma-1)]/[(\sigma-1)\theta]} \Phi^{-1/\theta}. \tag{5.23}$$

The price index differs from perfect and Bertrand competition in that market size reduces the price index by allowing for more variety. Moreover, for $\eta > 0$, more advanced technology, by reducing overhead costs, has more impact than just its effect through Φ.For the same reason, advances in technology have a more potent effect on the real wage in case 2.

2. The entry cutoff:

$$\bar{c} = \left(\frac{\theta - (\sigma - 1)}{\theta} \frac{LT^{\eta}}{(1-\delta^{MC})\sigma F} \right)^{1/\theta} T^{-1/\theta} w.$$

Note that the effect of T on the cutoff is ambiguous. For $\eta = 0$ the effect is negative since high cost producers find it harder to survive as more low cost ones appear. For $\eta > 0$ this effect is mitigated by the fact that advances in technology reduce overhead costs. In the case $\eta = 1$ the two effects exactly offset each other and $\bar{c}$ is independent of T.

3. The measure of active producers:

$$H = \frac{LT^{\eta}}{(1-\delta^{MC})\sigma F}\frac{\theta-(\sigma-1)}{\theta}$$

The measure of active producers is proportional to the ratio of total workers to the overhead requirement, unlike the market structures we considered before. In the case $\eta = 0$ the measure is independent of T. Advances in technology, meaning that there are more low cost producers, lower the entry cutoff to the point that the number of active firms is constant.

Summarizing these last two results, under case 1 advances in technology keep the measure of firms unchanged, but weed out the high cost ones. In case 2, advances in technology increase variety, but have no effect on the average cost of what is sold. While these two cases yield particularly simple outcomes, one can imagine intermediate cases in which advances in technology contribute at both margins, allowing for some expansion in variety while pruning the economy of high cost firms.[10]

In either of our cases we can solve for the fraction of the labor force engaged in overhead production. In case 1 the overhead requirement per firm F is invariant to T, as is the measure H of active firms, which is proportional to L. Hence the share of

[10] One specification that yields this intermediate results posits an overhead cost in terms of a bundle of goods. The overhead cost would then rise with P and hence decline with Φ with an elasticity of $1/\theta < 1$. An added complication is that overhead costs also would fall with market size.

overhead workers in the labor force is:

$$\frac{HF}{L} = \frac{\theta - (\sigma - 1)}{\theta\sigma - (\sigma - 1)}.$$

In case 2 the overhead requirement falls with T, while the measure of active firms rises with T. The two forces cancel, leaving the share as above.

***Welfare:

The real wage:

Case 1:

$$\frac{w}{P} = \frac{1}{\gamma^{MC}} \left(\frac{L}{(1 - \delta^{MC})\sigma F} \right)^{[\theta - (\sigma-1)]/[(\sigma-1)\theta]} T^{1/\theta}.$$

Case 2:

$$\frac{w}{P} = \frac{1}{\gamma^{MC}} \left(\frac{L}{(1 - \delta^{MC})\sigma F} \right)^{[\theta - (\sigma-1)]/[(\sigma-1)\theta]} T^{1/(\sigma-1)}.$$

***

The price index properly takes into account the range of goods sold as well as what they cost. If we were suppress the effect of variety on the price index, considering only the average price of what is actually sold, the price index P^S would be:

$$\begin{aligned} P^S &= \left[\frac{1}{H} \int_0^{\overline{c}} (\overline{m}c)^{1-\sigma} \, dH(c) \right]^{1/(1-\sigma)} \\ &= \overline{m} \left[\frac{\theta}{\theta - (\sigma - 1)} \right]^{1/(1-\sigma)} \overline{c}. \end{aligned}$$

which increases in $\overline{c}$ and hence in X/E. A larger market attracts more entrants, but the marginal entrants have higher unit costs. Hence the average price of a good sold in a larger market is higher, even though the true price index is lower.[11]

The critical difference between this formulation and the standard model of monopolistic competition described in Chapter 3 is producer heterogeneity. As described in Chapter 4, producers differ in the quality of their techniques for production, with efficiency drawn from the Pareto distribution with parameter θ. In the limit as $\theta \to \infty$, all producers are the same. Note that, taking this limit, the analysis above reduces to the closed economy version of monopolistic competition presented in Chapter 3. In particular, profits net of fixed costs, go to zero as all producers are at the margin of entry. With finite θ, there are rents associated with better techniques.

With monopolistic competition the fixed cost E assures that the price index P is bounded above zero since it is unprofitable for a producer with unit cost above some cutoff $\bar{c}$ to enter. What about the more general case Bertrand or Cournot competition with $1 < \sigma \leq \sigma' < \theta + 1$? The following lemma guarantees that for these parameter values a fixed cost of entry will imply an even lower cutoff unit cost, to the satisfaction of part 2 of Theorem 2.

Lemma 9 *Consider imperfect competition with* $1 < \sigma \leq \sigma' < \theta + 1$ *and an overhead cost* E. *As long as markups under imperfect competition* $M_{IC}^{(k)}(j)$ *are bounded above by the markup under monopolistic competition* $\overline{m} = \sigma/(\sigma - 1)$, *the entry cutoff for any*

[11] Ghironi and Melitz (2004) make this point in a related model.

good j under imperfect competition $\overline{c}(j)$ is bounded above by the entry cutoff $\overline{c}$ under monopolistic competition.

See the appendix for the proof. Remember that with Bertrand and Cournot competition markups lie between $\overline{m}\prime = \sigma'/(\sigma' - 1)$ and $\overline{m}$, satisfying the condition of the lemma. Hence in these cases a fixed entry cost ensures a well-behaved price index.

A

5.4 Conclusion

In summary, our assumptions about ideas provide the flexibility to explore a wide range of market structures. The three that we have explored in no way exhaust the possibilities. With Cournot competition, for example, multiple varieties of the same good (with different costs) could compete against each other, even with $\sigma' = 0$, thus potentially bringing in producers with higher costs ($k > 2$). Alternatively, one could admit intermediate values of σ' (some finite value strictly greater than σ), again making $k > 2$ relevant. One could also examine economies with mixed market structures, for example, one in which some goods are supplied monopolistically because of patent protection or trade secrets, while others are supplied competitively.

Two particular aspects of our analysis are key for the following two chapters. One is the determination of profits in Bertrand and monopolistic competition, which will serve as the driving force of innovation in Chapter 7. The other is the price index under each of the different market structures that we consider, which differs only in the constant term. The relationship between the state of the economy Φ and the price level is invariant to the form of competition. This invariance allows us to investigate a large number of issues in international trade, the topic of the next chapter, without taking a stand on market structure.

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A

5.5 Appendix

This Appendix provides proofs of the Theorem and Propositions 1 and 3 through 6 stated in the Chapter. We begin with a lemma that is used in several of the proofs. It is based on the representation of ordered costs in terms of normalized random variables $U^{(k)}(j)$, for $k = 1, 2, ...$ described in Lemma 7 of Chapter 4. We now establish that expectations of a function of these normalized random variables constitute a convergent series.

Lemma 10 *Define:*

$$U(\xi) = \sum_{k=1}^{\infty} E\left[\left(U^{(k)}(j)\right)^{-\xi} | U^{(1)}(j) = u\right].$$

For $\xi > 1$:

$$U(\xi) = u^{-\xi}\left[1 + \frac{1}{\xi - 1}u\right]. \tag{5.24}$$

G *Proof.* We begin by separating $U^{(1)}(j)^{-\xi} = u^{-\xi}$, which we condition on, from the series:

$$U(\xi) = u^{-\xi} + \sum_{k=2}^{\infty} E\left[\left(U^{(k)}(j)\right)^{-\xi} | U^{(1)}(j) = u\right].$$

From Lemma 7 of the Chapter 4, $U^{(k)}(j)$ is distributed gamma with parameters k and 1. (which is the unit exponential for $k = 1$) and $U^{(k+1)}(j) - U^{(k)}(j)$ is unit exponential, independent of $U^{(k)}(j)$. Hence, given $U^{(1)}(j) = u$, we can write $U^{(k)}(j)$ for $k > 1$ as the sum of u and a random variable $V^{(k-1)}(j)$ which is distributed gamma with parameters

$k-1$ and 1:

$$\begin{aligned} U(\xi) &= u^{-\xi} + \sum_{l=1}^{\infty} E\left[\left(u + V^{(l)}(j)\right)^{-\xi}\right] \\ &= u^{-\xi} + \sum_{l=1}^{\infty} \int_0^{\infty} (u+x)^{-\xi} \frac{x^{l-1} e^{-x}}{(l-1)!} dx \end{aligned}$$

Changing variables to $y = u + x$:

$$\begin{aligned} U(\xi) &= u^{-\xi} + \sum_{l=1}^{\infty} \int_u^{\infty} y^{-\xi} \frac{(y-u)^{l-1} e^{-(y-u)}}{(l-1)!} dy \\ &= u^{-\xi} + \int_u^{\infty} y^{-\xi} e^{-(y-u)} \left[\sum_{l=1}^{\infty} \frac{(y-u)^{l-1}}{(l-1)!}\right] dy \\ &= u^{-\xi} + \int_u^{\infty} y^{-\xi} dy \end{aligned}$$

where the last step follows from the fact that:

$$\sum_{l=1}^{\infty} \frac{x^{l-1}}{(l-1)!}$$

is the Taylor series for e^x. Simple integration delivers the result. ■

This result will deliver the implication that, even though an infinite chain of varieties might be available, under some parameter restrictions limiting the relevance of high cost varieties, the price index remains bounded.

B

5.5.1 Proof of Theorem 2

G *Proof.* We proceed as follows. In Section I of the proof we consider the case $\sigma > 1$ for both Parts A and B of the Theorem. In Sections II and III we turn to the cases $\sigma < 1$

and $\sigma = 1$ respectively, which are relevant, of course, only for Part A of the Theorem.

I. The aggregate price index can be written:

$$P = (E[P(j)^{-(\sigma-1)}])^{-1/(\sigma-1)}.$$

For $\sigma > 1$, $P > 0$ if and only if $E[P(j)^{-(\sigma-1)}] < \infty$. We now consider separately Parts A and B of the Theorem.

A. The price index for good j is:

$$P(j) = \left[\sum_{k=1}^{\infty} \left[P^{(k)}(j)\right]^{-(\sigma'-1)}\right]^{-1/(\sigma'-1)},$$

with $\sigma' > \theta + 1$. In the previous chapter we showed that we can write $C^{(1)}(j) = \left(U^{(1)}(j)/\Phi\right)^{1/\theta}$ where $U^{(1)}(j)$ is unit exponential, i.e. $\Pr[U^{(1)} \leq u] = 1 - e^{-u}$. We thus have:

$$E[P(j)^{-(\sigma-1)}] = \int_0^{\infty} E[P(j)^{-(\sigma-1)}|U^{(1)}(j) = u]e^{-u}du.$$

Consider the conditional expectation in this integral:

$$
\begin{aligned}
E[P(j)^{-(\sigma-1)}|U^{(1)}(j) &= u] \leq \left\{E\left[\left(P(j)^{-(\sigma-1)}\right)^{(\sigma'-1)/(\sigma-1)}|U^{(1)}(j)=u\right]\right\}^{(\sigma-1)/(\sigma'-1)} \\
&= \left\{E\left[\sum_{k=1}^{\infty}\left(P^{(k)}(j)\right)^{-(\sigma'-1)}|U^{(1)}(j)=u\right]\right\}^{(\sigma-1)/(\sigma'-1)} \\
&\leq \left\{E\left[\sum_{k=1}^{\infty}\left(C^{(k)}(j)\right)^{-(\sigma'-1)}|U^{(1)}(j)=u\right]\right\}^{(\sigma-1)/(\sigma'-1)} \\
&= \left\{\sum_{k=1}^{\infty}E\left[\left(\Phi^{-1/\theta}\left[U^{(k)}(j)\right]^{1/\theta}\right)^{-(\sigma'-1)}|U^{(1)}(j)=u\right]\right\}^{(\sigma-1)/(\sigma'-1)} \\
&= \Phi^{(\sigma-1)/\theta}\left\{\sum_{k=1}^{\infty}E\left[\left(U^{(k)}(j)\right)^{-\phi'}|U^{(1)}(j)=u\right]\right\}^{(\sigma-1)/(\sigma'-1)} ,(5.25)
\end{aligned}
$$

where $\phi' = (\sigma'-1)/\theta > 1$. The first inequality follows from our restriction $\sigma' \geq \sigma$ and the fact that, for any random variable Y, $\left(E\left[Y^a\right]\right)^{1/a} \geq E[Y]$ for $a \geq 1$, in this case with $a = (\sigma'-1)/(\sigma-1)$. The second follows from our assumption that $P^{(k)}(j) \geq C^{(k)}(j)$ for all k and j, so that $P^{(k)}(j)^{-(\sigma'-1)} \leq C^{(k)}(j)^{-(\sigma'-1)}$. Combining the results so far and employing Lemma 10, setting $\xi = \phi' > 1$:

$$
\begin{aligned}
E[P(j)^{-(\sigma-1)}] &\leq \int_0^{\infty}\Phi^{(\sigma-1)/\theta}\left\{\sum_{k=1}^{\infty}E\left[\left(U^{(k)}(j)\right)^{-\phi'}|U^{(1)}(j)=u\right]\right\}^{(\sigma-1)/(\sigma'-1)} e^{-u}du \\
&= \Phi^{(\sigma-1)/\theta}\int_0^{\infty}u^{-(\sigma-1)/\theta}\left[1+\frac{1}{\phi'-1}u\right]^{(\sigma-1)/(\sigma'-1)} e^{-u}du \\
&\leq \Phi^{(\sigma-1)/\theta}\int_0^{\infty}u^{-(\sigma-1)/\theta}\left[1+\frac{1}{\phi'-1}u\right]e^{-u}du \\
&= \Phi^{(\sigma-1)/\theta}\left[\Gamma\left(1-(\sigma-1)/\theta\right)+\frac{1}{\phi'-1}\Gamma\left(2-(\sigma-1)/\theta\right)\right],
\end{aligned}
$$

where the first inequality is from (5.25) and last inequality uses the fact that $u > 0$ and $(\sigma - 1)/(\sigma' - 1) < 1$. Our requirement that $\sigma < \theta + 1$ guarantees that the arguments of the gamma functions are strictly positive, delivering finite, positive values. Thus:

$$P \geq \Phi^{-1/\theta} \left[\Gamma\left(1 - (\sigma - 1)/\theta\right) + \frac{\theta}{\sigma' - 1 - \theta} \Gamma\left(2 - (\sigma - 1)/\theta\right) \right]^{-1/(\sigma-1)} .$$

where the right-hand side is strictly positive. The bottom line is that the price index P is bounded away from 0 for the case $1 < \sigma < \theta + 1 < \sigma'$.

B. Now consider a finite upper bound $\bar{c}$ on costs. Define

$$K(j) = \max\left\{k : C^{(k)}(j) \leq \bar{c}\right\},$$

so that only varieties $k = 1, ..., K(j)$ of good j are available. As above, we seek an upper bound on $E[P(j)^{-(\sigma-1)}]$. For this part we have $1 < \sigma < \theta + 1$ and $\sigma' \geq \sigma$. The price index for good j is

$$P(j) = \left[\sum_{k=1}^{K(j)} \left(P^{(k)}(j)\right)^{-(\sigma'-1)} \right]^{-1/(\sigma'-1)} .$$

Without loss of generality, take $P^{(1)}(j)$ to be the low-price variety so that $P^{(k)}(j)/P^{(1)}(j) \geq$

1. Factoring out $P^{(1)}(j)$ and then replacing σ' with σ:

$$
\begin{aligned}
P(j) &= P^{(1)}(j)\left[1+\sum_{k=2}^{K(j)}\left(\frac{P^{(k)}(j)}{P^{(1)}(j)}\right)^{-(\sigma'-1)}\right]^{-1/(\sigma'-1)} \\
&\geq P^{(1)}(j)\left[1+\sum_{k=2}^{K(j)}\left(\frac{P^{(k)}(j)}{P^{(1)}(j)}\right)^{-(\sigma-1)}\right]^{-1/(\sigma'-1)} \\
&\geq P^{(1)}(j)\left[1+\sum_{k=2}^{K(j)}\left(\frac{P^{(k)}(j)}{P^{(1)}(j)}\right)^{-(\sigma-1)}\right]^{-1/(\sigma-1)} \\
&= \left[\sum_{k=1}^{K(j)}\left(P^{(k)}(j)\right)^{-(\sigma-1)}\right]^{-1/(\sigma-1)} .
\end{aligned}
$$

Raising each side to the power $-(\sigma-1)<1$ and taking expectations:

$$
\begin{aligned}
E[P(j)^{-(\sigma-1)}] &\leq E\left[\sum_{k=1}^{K(j)}\left(P^{(k)}(j)\right)^{-(\sigma-1)}\right] \\
&\leq E\left[\sum_{k=1}^{K(j)}\left(C^{(k)}(j)\right)^{-(\sigma-1)}\right]
\end{aligned}
$$

The second inequality follows from our assumption that $P^{(k)}(j) \geq C^{(k)}(j)$ for all k and j, so that $P^{(k)}(j)^{-(\sigma-1)} \leq C^{(k)}(j)^{-(\sigma-1)}$. Since each cost draw is independent, we can ignore their ordering and restate this last expression as simply the product of the expected number of cost draws below $\bar{c}$ and the expected cost $E[C^{-(\sigma-1)}|C \leq \bar{c}]$ for any one of them. Using Proposition 1 of Chapter 4 we can thus write:

$$\begin{aligned} E\left[\sum_{k=1}^{K(j)}\left(C^{(k)}(j)\right)^{-(\sigma-1)}\right] &= \Phi\bar{c}^{\theta}\int_0^{\bar{c}} c^{-(\sigma-1)}\theta c^{\theta-1}\left(\bar{c}\right)^{-\theta} dc \\ &= \int_0^{\bar{c}} c^{-(\sigma-\theta)}\theta\Phi dc \\ &= \frac{\theta\Phi}{\theta-(\sigma-1)}\bar{c}^{\theta-(\sigma-1)}, \end{aligned}$$

The last expression is bounded from above for finite $\bar{c}$. Raising each side to the power $-1/(\sigma-1) < 0$:

$$P = \left(E[P(j)^{-(\sigma-1)}]\right)^{-1/(\sigma-1)} \geq \left[\frac{\theta\Phi}{\theta-(\sigma-1)}\bar{c}^{\theta-(\sigma-1)}\right]^{-1/(\sigma-1)}.$$

The bottom line is that the price index P is bounded away from 0 for the case $1 < \sigma < \theta + 1,\ \sigma \leq \sigma'$, and unit costs not above $\bar{c}$.

II. We now turn to $\sigma < 1$ with all varieties available and $\sigma' > \theta + 1$. We can write:

$$P = (E[P(j)^{1-\sigma}])^{1/(1-\sigma)},$$

so that $P > 0$ now requires $E[P(j)^{1-\sigma}] > 0$. We can write:

$$E[P(j)^{1-\sigma}] = \int_0^{\infty} E[P(j)^{1-\sigma}|U^{(1)}(j) = u]e^{-u}du \tag{5.26}$$

Working with the expectation inside the integral:

$$
\begin{aligned}
E[P(j)^{1-\sigma}|U^{(1)}(j) &= u] \geq \left\{E[\left(P(j)^{1-\sigma}\right)^{-(\sigma'-1)/(1-\sigma)}|U^{(1)}(j)=u]\right\}^{-(1-\sigma)/(\sigma'-1)} \\
&= \left\{E\left[\sum_{k=1}^{\infty}\left(P^{(k)}(j)\right)^{-(\sigma'-1)}|U^{(1)}(j)=u\right]\right\}^{-(1-\sigma)/(\sigma'-1)} \\
&\geq \left\{E\left[\sum_{k=1}^{\infty}\left(C^{(k)}(j)\right)^{-(\sigma'-1)}|U^{(1)}(j)=u\right]\right\}^{-(1-\sigma)/(\sigma'-1)} \\
&= \left\{\sum_{k=1}^{\infty}E\left[\left(\Phi^{-1/\theta}\left[U^{(k)}(j)\right]^{1/\theta}\right)^{-(\sigma'-1)}|U^{(1)}(j)=u\right]\right\}^{-(1-\sigma)/(\sigma'-1)} \\
&= \Phi^{-(1-\sigma)/\theta}\left\{\sum_{k=1}^{\infty}E\left[\left[U^{(k)}(j)\right]^{-\phi'}|U^{(1)}(j)=u\right]\right\}^{-(1-\sigma)/(\sigma'-1)} \\
&= \Phi^{-(1-\sigma)/\theta}\left[u^{-\phi'}+\frac{1}{\phi'-1}u^{1-\phi'}\right]^{-(1-\sigma)/(\sigma'-1)} \qquad (5.27)
\end{aligned}
$$

where the first inequality follows from the fact that $\left[E\left(Y^{a}\right)\right]^{1/a} \leq E\left[Y\right]$ for $a \leq 0$ and the second from our assumption that $P^{(k)}(j) \geq C^{(k)}(j)$. The last step comes from

Lemma 10, again setting $\xi = \phi' = (\sigma' - 1)/\theta > 1$. Inserting (5.27) into (5.26) we get:

$$\begin{aligned}
E[P(j)^{1-\sigma}] &\geq \Phi^{-(1-\sigma)/\theta} \int_0^\infty \left[u^{-\phi'} + \frac{1}{\phi' - 1} u^{1-\phi'}\right]^{-(1-\sigma)/(\sigma'-1)} e^{-u} du \\
&\geq \Phi^{-(1-\sigma)/\theta} \left(\int_0^\infty \left[u^{-\phi'} + \frac{1}{\phi' - 1} u^{1-\phi'}\right]^{1/(\sigma'-1)} e^{-u} du\right)^{-(1-\sigma)} \\
&\geq \Phi^{-(1-\sigma)/\theta} \left(\int_0^\infty \left[u^{-\phi'/(\sigma'-1)} + \left(\frac{1}{\phi' - 1}\right)^{1/(\sigma'-1)} u^{(1-\phi')/(\sigma'-1)}\right] e^{-u} du\right)^{-(1-\sigma)} \\
&= \Phi^{-(1-\sigma)/\theta} \left[\Gamma\left(\frac{\theta-1}{\theta}\right) + \left(\frac{\theta}{\sigma' - 1 - \theta}\right)^{1/(\sigma'-1)} \Gamma\left(\frac{\sigma'(\theta-1)+1}{(\sigma'-1)\theta}\right)\right]^{-(1-\sigma)}
\end{aligned}$$

which is strictly positive given our restriction that $\sigma' \geq \theta + 1$ and $\theta > 1$. Here the first inequality is inherited from the set of equations (5.27). The second follows from the fact that, for $a \leq 0$, $E(Y^a) \geq [E(Y)]^a$. The third from the fact that, for $1 \geq b \geq 0$ and x_1, x_2 positive, $x_1^b + x_2^b \geq (x_1 + x_2)^b$, since $1 \geq 1/(\sigma' - 1) \geq 0$.[12] Thus we have:

$$P \geq \Phi^{-1/\theta} \left[\Gamma\left(\frac{\theta-1}{\theta}\right) + \left(\frac{\theta}{\sigma' - 1 - \theta}\right)^{1/(\sigma'-1)} \Gamma\left(\frac{\sigma'(\theta-1)+1}{(\sigma'-1)\theta}\right)\right]^{-1}.$$

The right-hand side is strictly positive. The bottom line is that the price index P is bounded away from 0 for $0 < \sigma < 1 < \theta + 1 < \sigma'$.

[12] Without loss of generality, let $x_1 \leq x_2$. It follows that:

$$\begin{aligned}
(x_1^b + x_2^b)^{1/b} &= x_2 \left((x_1/x_2)^b + 1\right)^{1/b} \geq x_2 \left((x_1/x_2) + 1\right)^{1/b} \\
&\geq x_2 \left((x_1/x_2) + 1\right) = x_1 + x_2.
\end{aligned}$$

III. For $\sigma = 1$ we write

$$P = \exp\left\{E\left[\ln P(j)\right]\right\}$$

so that $P > 0$ if and only if $E\left[\ln P(j)\right] > -\infty$. We seek a lower bound on $E[\ln P(j)]$. After conditioning on $U^{(1)}(j) = u$ we have

$$\begin{aligned}
E[\ln P(j)|U^{(1)}(j) &= u] = \frac{-1}{\sigma'-1}E\left[\ln\left(\sum_{k=1}^{\infty}\left(P^{(k)}(j)\right)^{-(\sigma'-1)}\right)|U^{(1)}(j)=u\right] \\
&\geq \frac{-1}{\sigma'-1}\ln\left(E\left[\sum_{k=1}^{\infty}\left(P^{(k)}(j)\right)^{-(\sigma'-1)}|U^{(1)}(j)=u\right]\right) \\
&\geq \frac{-1}{\theta}\ln\Phi - \frac{1}{\sigma'-1}\ln\left(\sum_{k=1}^{\infty}E\left[\left(U^{(k)}(j)\right)^{-\phi'}|U^{(1)}(j)=u\right]\right) \\
&= \frac{-1}{\theta}\ln\Phi - \frac{1}{\sigma'-1}\ln\left(u^{-\phi'}\left[1+\frac{1}{\phi'-1}u\right]\right) \\
&\geq \frac{-1}{\theta}\ln\Phi - \frac{1}{\sigma'-1}\left[\ln\left(u^{-\phi'}\right)+\frac{1}{\phi'-1}u\right]
\end{aligned}$$

The first inequality follows from the concavity of the logarithm and the second from the assumption that prices exceed unit costs. The final equality invokes Lemma 10.

The final inequality follows from the fact that $\ln(1+x) \leq x$ for $x \geq 0$.[13] Marching on:

$$\begin{aligned}
E[\ln P(j)] &= \int_0^\infty E[\ln P(j)|U^{(1)}(j) = u]e^{-u}du \\
&\geq \frac{-1}{\theta}\ln\Phi - \frac{1}{\sigma'-1}\int_0^\infty \ln\left(u^{-\phi'}\right)e^{-u}du - \frac{1}{\sigma'-1}\int_0^\infty \left(\frac{1}{\phi'-1}u\right)e^{-u}du \\
&= \frac{-1}{\theta}\ln\Phi + \frac{1}{\theta}\int_0^\infty \ln(u)\,e^{-u}du - \frac{1}{\sigma'-1}\frac{1}{\phi'-1}\int_0^\infty ue^{-u}du \\
&= \frac{-1}{\theta}\ln\Phi - \frac{\gamma}{\theta} - \frac{1}{\sigma'-1}\left(\frac{1}{\phi'-1}\right)
\end{aligned}$$

where $\gamma = 0.57721...$ is Euler's constant. Taking the antilog:

$$P \geq \Phi^{-1/\theta}\exp\left\{-\left[\frac{\gamma}{\theta} + \frac{1}{\sigma'-1}\frac{\theta}{(\sigma'-1)-\theta}\right]\right\}.$$

The right-hand side is strictly positive. The bottom line is that the price index P is bounded away from 0 for $\sigma = 1 < \theta + 1 < \sigma'$. ■

[13] Note that there is equality at $x = 0$ and that the derivative of $\ln(1+x)$ below one.

B

5.5.2 Proof of Proposition 2

G

Proof. Conditioning on the stochastic term $U^{(1)}$ underlying the cost of the low-cost producer, and letting $\phi' = (\sigma' - 1)/\theta$:

$$
\begin{aligned}
E\left[\frac{X(j)}{X^{(1)}(j)}|U^{(1)}(j) = u\right] &= E\left[\left(\frac{P(j)}{P^{(1)}(j)}\right)^{-(\sigma'-1)}|U^{(1)}(j) = u\right] \\
&= E\left[\frac{\sum_{k=1}^{\infty}\left[U^{(k)}(j)\right]^{-\phi'}}{\left[U^{(1)}(j)\right]^{-\phi'}}|U^{(1)}(j) = u\right] \\
&= u^{\phi'}\sum_{k=1}^{\infty}E\left[\left(U^{(k)}(j)\right)^{-\phi'}|U^{(1)}(j) = u\right] \\
&= 1 + \frac{1}{\phi' - 1}u,
\end{aligned}
$$

where the second to the last equality employs Lemma 10. Integrating this conditional expectation over the density of $U^{(1)}$ gives the result:

$$
\begin{aligned}
E\left[\frac{X(j)}{X^{(1)}(j)}\right] &= \int_0^{\infty}\left(1 + \frac{1}{\phi' - 1}u\right)e^{-u}du \\
&= \frac{\sigma' - 1}{\sigma' - (\theta + 1)}.
\end{aligned}
$$

■

B

5.5.3 Proof of Proposition 3

G

Proof. Since $P(j) = C^{(1)}(j)$, if we were to integrate across goods we would calculate:

$$
P = \left[\int_0^1 C^{(1)}(j)^{-(\sigma-1)}dj\right]^{-1/(\sigma-1)}.
$$

For technical reasons we prefer to integrate across costs, which yields:

$$
\begin{aligned}
P &= \left[\int_0^\infty c_1^{-(\sigma-1)} dF_1(c_1)\right]^{-1/(\sigma-1)} \\
&= E\left[\left(C^{(1)}\right)^{-(\sigma-1)}\right]^{-1/(\sigma-1)}.
\end{aligned}
$$

Setting $b = -(\sigma - 1)$ in (4.8) delivers the result. ■

B

5.5.4 Proof of Proposition 5

G

Proof. Since:

$$P^{-(\sigma-1)} = \int_0^1 P(j)^{-(\sigma-1)} dj,$$

we integrate across the ratio $M'(j) = C^{(2)}(j)/C^{(1)}(j)$ to get:

$$P^{-(\sigma-1)} = \int_1^{\overline{m}} E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right] dF_{2/1}(m') + \int_{\overline{m}}^\infty E\left[\left(\overline{m}C^{(2)}(j)/m'\right)^{-(\sigma-1)} | C^{(2)}(j)/C^{(1)}(j) = m'\right] dF_{2/1}(m$$

From (4.12) the distribution of $M'(j)$ is independent of $C^{(2)}(j)$, so we can write:

$$
\begin{aligned}
P^{-(\sigma-1)} &= E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right](1 - \overline{m}^{-\theta}) + E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right]\overline{m}^{-(\sigma-1)} \int_{\overline{m}}^\infty (m')^{(\sigma-1)} dF_{2/1}(m') \\
&= E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right]\left[1 + \frac{\sigma-1}{\theta-(\sigma-1)}\overline{m}^{-\theta}\right].
\end{aligned}
$$

Hence:

$$P = \left\{E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right]\right\}^{-1/(\sigma-1)}\left[1 + \frac{\sigma-1}{\theta-(\sigma-1)}\overline{m}^{-\theta}\right]^{-1/(\sigma-1)}. \tag{5.28}$$

The result follows from applying (4.9). ■

5.5.5 Proof of Proposition 6

Proof. We rewrite expression (5.14) as:

$$\pi(j) = \left(1 - M(j)^{-1}\right)\left(\frac{P(j)}{P}\right)^{1-\sigma} X$$

Integrating across j, and dividing by total spending, we get:

$$\frac{\Pi}{X} = 1 - \frac{1}{P^{-(\sigma-1)}}\int_0^1 M(j)^{-1}P(j)^{-(\sigma-1)}dj.$$

Following closely the proof of Proposition 3:

$$\begin{aligned}
& \int_0^1 M(j)^{-1}P(j)^{-(\sigma-1)}dj \\
= \;& \int_1^{\overline{m}} E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right](m')^{-1}dF_{2/1}(m') + \overline{m}^{-1}\int_{\overline{m}}^{\infty} E\left[\left(\overline{m}C^{(2)}(j)/m'\right)^{-(\sigma-1)}\right]dF_{2/1}(m') \\
= \;& E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right]\left[\frac{\theta}{1+\theta}(1-\overline{m}^{-\theta-1}) + \frac{\theta}{\theta-(\sigma-1)}\overline{m}^{-\theta-1}\right] \\
= \;& E\left[\left(C^{(2)}(j)\right)^{-(\sigma-1)}\right]\frac{\theta}{1+\theta}\left[1 + \frac{\sigma-1}{\theta-(\sigma-1)}\overline{m}^{-\theta}\right]
\end{aligned}$$

Dividing by $P^{1-\sigma}$, from (5.28) in the previous proof, gives the result. ■

5.5.6 Proof of Proposition 7

Proof. The variable profit of a firm with cost c and charging price p is:

$$\Pi^V(c) = (p-c)X(j)/p.$$

As is easy to verify, our cost structure preserves a basic result from monopolistic competition, that profit is at a maximum at:

$$p = \overline{m}c$$

so that variable profit is:

$$\Pi^V(c) = \frac{X(j)}{\sigma} = \frac{X}{\sigma}\left(\frac{\overline{m}c}{P}\right)^{-(\sigma-1)}, \tag{5.29}$$

which decreases in cost c. Hence entry is profitable only for producers with cost $c \leq \overline{c}$ given by:

$$\overline{c} = \left(\frac{X}{\sigma E}\right)^{1/(\sigma-1)} \frac{P}{\overline{m}}. \tag{5.30}$$

For this case we can rewrite the price index as the integral over the prices charged by sellers with different costs c in the range $[0, \overline{c}]$ weighted by the measure of suppliers with that cost. This Pareto measure is the derivative of the function (4.6) with respect to c. The price index is consequently:

$$\begin{aligned} P &= \left[\int_0^{\overline{c}} (\overline{m}c)^{-(\sigma-1} \, dH(c)\right]^{-1/(\sigma-1)} \qquad (5.31) \\ &= \overline{m}\left[\Phi \int_0^{\overline{c}} \theta c^{\theta-\sigma} dc\right]^{-1/(\sigma-1)} \\ &= \overline{m}\left[\frac{\theta\Phi}{\theta-(\sigma-1)}\overline{c}^{\theta-(\sigma-1)}\right]^{-1/(\sigma-1)}. \end{aligned}$$

Equations (5.30) and (5.31) each involve the price index P and the maximum cost for entry $\overline{c}$. Solving for each we get a cut-off cost:

$$\overline{c} = \left(\frac{\theta-(\sigma-1)}{\theta\Phi}\frac{X}{\sigma E}\right)^{1/\theta}$$

Substituting $\overline{c}$ into (5.31) and (4.6) establishes the proposition. ■

B

5.5.7 Proof of Proposition 8

G

Proof. Using (5.29) above, total variable profit is:

$$\begin{aligned}\Pi^V &= \frac{X}{\sigma}\left(\frac{\overline{m}}{P}\right)^{-(\sigma-1)}\int_0^{\overline{c}} c^{-(\sigma-1)}dH(c)\\ &= \frac{X}{\sigma}\left(\frac{\overline{m}}{P}\right)^{1-\sigma}\frac{\theta\Phi}{\theta-(\sigma-1)}\overline{c}^{\theta-(\sigma-1)}\end{aligned}$$

Substituting in the price index as it appears in (5.31) yields (i). Total overhead cost is the individual overhead cost E multiplied by the measure of entrants (5.20). Subtracting total overhead cost HE from Π^V delivers (ii). Dividing Π by H in (5.20) gives (iii).

■

B

5.5.8 Proof of Lemma 9

G

Proof. Fix σ and the set of varieties of good j available under monopolistic competition, $k = 1, 2, ..., K(j)$. Consider the profit of the firm producing the highest unit cost variety, $K(j) = K$. Assume first that this firm produces only this variety of good j.

Its profit as a function of its price $P^{(K)}(j)$, the set of prices of other varieties of that good $P^{\{-K\}}(j) = \left\{P^{(k)}(j) : k < K\right\}$, the overall price index P, and the elasticity of substitution across varieties σ' is:

$$\begin{aligned}
&\pi(P^{(K)}(j), P^{\{-K\}}(j), P, \sigma') \\
= \quad &\left(1 - \frac{C^{(K)}(j)}{P^{(K)}(j)}\right) \left(P^{(K)}(j)\right)^{-(\sigma'-1)} \left(\left(P^{(K)}(j)\right)^{-(\sigma'-1)} + \sum_{k=1}^{K-1} \left(P^{(k)}(j)\right)^{-(\sigma'-1)}\right)^{-(\sigma'-\sigma)/(\sigma'-1)} P^{\sigma-1} X.
\end{aligned}$$

Evaluated at $\sigma' = \sigma$, as in monopolistic competition, this expression simplifies to

$$\pi(P^{(K)}(j), P^{\{-K\}}(j), P, \sigma) = \left(1 - \frac{C^{(K)}(j)}{P^{(K)}(j)}\right) \left(P^{(K)}(j)\right)^{-(\sigma-1)} P^{\sigma-1} X$$

and, given P, $P^{\{-K\}}(j)$ becomes irrelevant. Denote by $P_{MC}^{(k)}(j)$ the equilibrium prices of the varieties of good j and by P_{MC} the overall price index under monopolistic competition with $\sigma' = \sigma$. Denote by $P_{IC}^{(k)}(j)$ and P_{IC} the corresponding magnitudes under imperfect competition with $\sigma' \geq \sigma$. (Remember that we are holding the set of active varieties fixed to those under monopolistic competition.) Since under monopolistic competition the firm producing variety K is choosing its price optimally:

$$\pi(P_{MC}^{(K)}(j), P_{MC}^{\{-K\}}(j), P_{MC}, \sigma) \geq \pi(P_{IC}^{(K)}(j), P_{MC}^{\{-K\}}(j), P_{MC}, \sigma).$$

Because of the lower markups $P_{IC} \leq P_{MC}$ and each element of $P_{IC}^{\{-K\}}(j)$ is lower than the corresponding element of $P_{MC}^{\{-K\}}(j)$. As is evident from the expression for π, a lower price of another variety of good j or a lower price index implies a lower profit. Hence:

$$\pi(P_{IC}^{(K)}(j), P_{MC}^{\{-K\}}(j), P_{MC}, \sigma) \geq \pi(P_{IC}^{(K)}(j), P_{IC}^{\{-K\}}(j), P_{IC}, \sigma).$$

Factoring out $P^{(K)}$ from $P(j)$ we can write:

$$
\begin{aligned}
& \pi(P_{IC}^{(K)}(j), P_{IC}^{\{-K\}}(j), P_{IC}, \sigma') \\
= \;& \left(1 - \frac{C^{(K)}(j)}{P_{IC}^{(K)}(j)}\right) \left(P_{IC}^{(K)}(j)\right)^{-(\sigma-1)} \left(1 + \sum_{k=1}^{K-1} \left(\frac{P_{IC}^{(k)}(j)}{P_{IC}^{(K)}}\right)^{-(\sigma'-1)}\right)^{-(\sigma'-\sigma)/(\sigma'-1)} P_{IC}^{\sigma-1} X \\
\leq \;& \left(1 - \frac{C^{(K)}(j)}{P_{IC}^{(K)}(j)}\right) \left(P_{IC}^{(K)}(j)\right)^{-(\sigma-1)} P_{IC}^{\sigma-1} X \\
= \;& \pi(P_{IC}^{(K)}(j), P_{IC}^{\{-K\}}(j), P_{IC}, \sigma),
\end{aligned}
$$

where the inequality follows because the large term within backets is greater than one and is raised to a negative power. Combining inequalities we have

$$\pi(P_{IC}^{(K)}(j), P_{IC}^{\{-K\}}(j), P_{IC}, \sigma') \leq \pi(P_{MC}^{(K)}(j), P_{MC}^{\{-K\}}(j), P_{MC}, \sigma).$$

Hence the profit of the least profitable variety under monopolistic competition with $\sigma' = \sigma$ is an upper bound on the profit from that same least profitable variety under imperfect competition with $\sigma' > \sigma$. What if the firm producing the least profitable variety under monopolistic competition also produced other varieties of the same good? Under monopolistic competition with $\sigma' = \sigma$ the production of variety K would not affect profit on the other varieties while with $\sigma' > \sigma$ it would lower those profits. Hence the incentive to drop variety K would be even greater for a multivariety firm. Either way the profitability of the K'th variety is lower under imperfect competition, implying a lower entry cutoff. Hence monopolistic competition allows for the greatest possible number of varieties given the fixed entry cost E. ■

CN

Chapter 6

CT

Trade

We now show how the framework developed in the previous two chapters extends very naturally into a model of international trade. The model encompasses and generalizes the models of international trade based on Ricardo and monopolistic competition presented in Chapter 3. It delivers a specification for bilateral trade flows consistent with gravity analysis, and provides a connection between these flows and what goes on at the level of individual producers.

The previous two chapters concerned techniques at a single location that are used to produce goods for local consumption. Introducing the notion that different locations have different techniques and can exchange goods that they produce using these techniques delivers our model of trade.

Most of our analysis treats locations as Ricardian countries and thus trade as international. Techniques for producing any good differ across countries while inputs are mobile for use across available techniques within, but not between, countries. While all goods are in principle tradable across countries we allow for trade costs (and in some of the analysis overhead costs as well), so that some goods turn out not to be traded.

The framework allows for an arbitrary counting number N of countries. A country will both export and import. We follow our convention in Chapter 3 of indexing countries in their role as producers by $i = 1, ..., N$ and in their role as consumers by $n = 1, ..., N$. Hence, for example, we denote expenditure by country n as X_n, production by country i as Y_i, and imports of n from i as X_{ni}.

Extending the analysis of Chapters 4 and 5 to multiple countries, the relevant features of a country are the following:

1. Each country i has an endowment L_i of factors. We bundle factors into a single entity which, in the Ricardian tradition, we call labor.[1]

2. Each country i has a state of technology T_i reflecting the number of ideas that have arrived there. In this chapter we assume both that the arrival process is

[1] We can easily allow for multiple factors as long as production does not vary in factor intensity across goods. While this generalization of the analysis to multiple factors is analytically trivial, it's useful in connecting the model to data. It's possible to introduce multiple factors in a deeper way, but doing so requires additional modeling assumptions that are beyond the scope of what we do here. Shikher (20XX), Costinaud (20XX), Chor (20XX), and Bustein and Vogel (20XX) pursue various approaches to incorporating factor-intensity differences into this framework.

independent across countries and that the quality of each idea drawn independently from the Pareto distribution with parameter θ (treated as common across countries). We turn to the evolution of T_i over time in Chapter 7. Chapter 8 considers some implications of relaxing these strong independence assumptions.

3. Selling in any country n may entail hiring an amount F_n of local labor. We assume that this requirement is the same for potential producers from any country. Hence, while F_n can vary across destinations, in a given destination n it is the same for sellers from any source i.[2]

4. Delivering a unit of any variety of any good to n requires shipping $d_{ni} \geq 1$ units from country i, the standard iceberg assumption. We normalize $d_{ii} = 1$. We do not require symmetry in that we allow for $d_{ni} \neq d_{in}$ but impose the triangle inequality that it is cheaper to ship directly from i to n rather than going through some third country h: $d_{ni} \leq d_{nh}d_{hi}$.[3]

Having made these assumptions about technology, we can solve for the gen-

[2] Note the distinction between our formulation, in which the overhead cost applies to each market entered, and trade models with monopolistic competition discussed in Chapter 3, in which firms face a fixed cost of setting up production, but not of entering individual markets. Chaney (2008) shows how to allow for differences in entry costs that vary according to the country of origin as well as destination.

[3] Arbitrage would eliminate violations of the triangle inequality since h would emerge as an entrepot, so that $d_{ni} = d_{nh}d_{hi}$. Chapter 3 discusses the analytic convenience, as well as the limitations, of the iceberg specification of transport costs.

eral equilibrium of the world economy under the different assumptions about market structure and preferences in Chapter 5. As in Chapter 4, however, it's useful to begin the analysis conditioning on the wage w_i in each country.[4]

A

6.1 Cost Distributions in the Open Economy

Condition on the w_i's we can incorporate international trade into the analysis in Chapter 4 very seamlessly. The key step is to reformulate Proposition 1 of this Chapter to allow for imports as well as domestic production.

Consider techniques that provide country n with some good j at unit cost less than c. From Proposition 1 itself, the number of *local* techniques that can do so is distributed Poisson with parameter $\Phi_{nn}c^{\theta}$ where $\Phi_{nn} = T_n w_n^{-\theta}$. In that chapter, since n can't import, using these techniques is the only way for it to get good j.

Now consider some other country i with its own techniques for making good j. Just as above, the number of country i's techniques that can produce good j for delivery to itself at unit cost less than c is distributed Poisson with parameter.$\Phi_{ii}c^{\theta}$ where $\Phi_{ii} = T_i w_i^{-\theta}$. Say that country n can import good j from country i, but, because of the transport cost, importing one unit requires producing $d_{ni} \geq 1$ units. From the perspective of country n, then, country i's input cost is not w_i, but rather $w_i d_{ni}$. Hence

[4] In the closed economy, the wage could serve as numeraire. In a multicountry world, relative wages are determined by the general equilibrium of the global economy, to which we turn later in the chapter.

the number of i's techniques for making good j available to n at unit cost less than c is distributed Poisson with parameter $\Phi_{ni}c^{\theta}$ where $\Phi_{ni} = T_i \left(w_i d_{ni}\right)^{-\theta}$.

Taking into account country n's ability to import from i, as well as to produce locally, the number of techniques for making good j available in country n is the sum of domestic and imported techniques. The sum of independent draws from two Poisson distributions A and B with parameters λ_A and λ_B is Poisson with parameter $\lambda_A + \lambda_B$.[5] If i and n are the only countries that n can buy from, the number of techniques for making good j in country n at unit cost less than c is distributed Poisson with parameter $(\Phi_{nn} + \Phi_{ni})c^{\theta} = \left[T_n w_n^{-\theta} + T_i \left(w_i d_{ni}\right)^{-\theta}\right] c^{\theta}$.

Having drawn a total of $k > 0$ from two Poisson distributions A and B with parameters λ_A and λ_B, the probability that any one of the k was drawn from distribution

[5] To see this result note that the probability that the two random variables K_A and K_B, when drawn independently from these distributions, sum to k can be written as:

$$\begin{aligned}
\Pr[K_A + K_B &= k] = \sum_{x=0}^{k} \Pr[K_A = x] \Pr[K_B = k - x] \\
&= \sum_{x=0}^{k} \frac{e^{-\lambda_A} \lambda_A^x}{x!} \frac{e^{-\lambda_B} \lambda_B^{k-x}}{(k-x)!} \\
&= \frac{e^{-(\lambda_A + \lambda_B)}}{k!} \sum_{x=0}^{k} \frac{k!}{(k-x)!x!} \lambda_A^x \lambda_B^{k-x} \\
&= \frac{e^{-(\lambda_A + \lambda_B)} (\lambda_A + \lambda_B)^k}{k!}.
\end{aligned}$$

A is $\lambda_A/(\lambda_A+\lambda_B)$ irrespective of k.[6] Hence the probability π_{ni} that a technique available in country n with unit less than c is from country i is:

$$\pi_{ni} = \frac{\Phi_{ni}c^{\theta}}{\Phi_{nn}c^{\theta} + \Phi_{ni}c^{\theta}} = \frac{T_i\left(w_i d_{ni}\right)^{-\theta}}{T_n w_n^{-\theta} + T_i\left(w_i d_{ni}\right)^{-\theta}}.$$

Note that the probability does not depend on c: The probability that a technique is foreign is the same regardless of the associated unit cost.

Extending this reasoning to a world of N countries we define:

$$\Phi_n = \sum_{i=1}^{N}\Phi_{ni} = \sum_{i=1}^{N} T_i\left(w_i d_{ni}\right)^{-\theta}. \tag{6.1}$$

This expression summarizes what the history of the arrival of ideas around the world, along with input costs and trade costs, implies for the distribution of unit costs in any location n. We use it to provide an open-economy version of Proposition 1.

Proposition 9 *Given Φ_n: (i) The number of techniques providing good j at unit cost less than c for country n is distributed Poisson with parameter $\Phi_n c^{\theta}$. (ii) The probability*

[6] To see this result, conditional on a total of $K = K_A + K_B = k$, the probability that $K_A = k_A$ and hence $K_B = k - k_A$ is:

$$\begin{aligned}\Pr[K_A &= k_A, K_B = k - k_A | K = k] = \frac{\frac{\exp(-\lambda_A)\lambda_A^{k_A}}{k_A!}\frac{\exp(-\lambda_B)\lambda_B^{k-k_A}}{(k-k_A)!}}{\frac{\exp[-(\lambda_A+\lambda_B)](\lambda_A+\lambda_B)^k}{k!}} \\ &= \frac{k!}{k_A!\,(k-k_A)!}\left(\frac{\lambda_A}{\lambda_A+\lambda_B}\right)^{k_A}\left(\frac{\lambda_B}{\lambda_A+\lambda_B}\right)^{k-k_A}.\end{aligned}$$

Note that this last expression is the binomial expression for the likelihood of k_A "successes" in k trials with probability $\lambda_A/(\lambda_A+\lambda_B)$ of a "success" on any trial.

π_{ni} that a technique providing unit cost less than c is from country i is:

$$\pi_{ni} = \frac{T_i \left(w_i d_{ni}\right)^{-\theta}}{\Phi_n}. \tag{6.2}$$

which is independent of c. (iii) The conditional distribution of unit costs provided by techniques from country i in country n is:

$$\Pr[C \leq c' | C \leq c] = \Pr\left[Q \geq \frac{w_i d_{ni}}{c'} | Q \geq \frac{w_i d_{ni}}{c}\right] = (c'/c)^{\theta} \quad c' \leq c. \tag{6.3}$$

Parts (i) and (ii) follow by induction from our arguments above for two countries. Part (iii) falls out exactly as in Proposition 1 of Chapter 4.

Note from (6.3) that the conditional distribution of costs depends only on the parameter θ, and not on any parameter specific to country i or n. In particular, conditional on a technique delivering a unit cost to market n less than c, the distribution of the unit cost does not depend on the source country i.

Since all techniques available to a location, through local production or imports, provide the same conditional distribution of unit cost, what differs across locations is simply their number, as reflected in the term Φ_n, and their origin, as reflected in π_{ni}. The term Φ_n defined in (6.1) is the open economy version of (4.2) of Chapter 4. In the open economy Φ_n reflects not only country n's own state of technology T_n, but the states around the world, tempered by input and trade costs. The more remote is country n (as implied by higher d_{ni}'s) the lower its Φ_n.

Because the conditional distribution (6.3) of unit cost is the same as (4.3) for the closed economy, all our results from Chapter 4 survive for each country n, with each country n having its own Φ_n governing the joint distribution of the ordered costs $C_n^{(1)} \leq C_n^{(2)} \leq C_n^{(3)} \leq \ldots$ of each good j there. Since π_{ni} is the probability that country i is the source of a technique and since costs are drawn from (6.3) independently of i, π_{ni} also applies to all rankings of costs k; i.e., π_{ni} is the probability that country i can deliver some good j at the lowest unit cost, second lowest cost, etc.

We can now talk about international trade in terms of unit cost draws from different sources i in country n, as governed by Φ_n and π_{ni}. From (6.2), the expected share of techniques from i in n is higher the larger country i's stock of technology T_i, the lower its wage w_i, and the lower the cost d_{ni} of shipping from i to n (relative to these magnitudes in n from other sources, as reflected in Φ_n).

With the same w_i, d_{ni}, and T_i applying across a unit continuum of goods, as in Chaper 5, Φ_n governs the distribution of unit costs across goods and π_{ni} the share that originate in i. Of course equilibrium in the labor markets, to which we turn later, will link w_i to T_i and to labor supplies L_i. To make this link we need to complete the description of the global economy.

A

6.2 Preferences and Market Structure

Having thus redefined Φ_n for the open economy, can we proceed exactly as in Chapter 5 above, only with country n importing from other countries as well as buying domestically? We can if no feature of any source i is relevant for its participation in market n other than its appearance in Φ_n in expression (6.1). We call this property *neutrality*. What does neutrality rule out? It doesn't allow, for instance, preferences in which utility depends directly on a good's provenance, as under the Armington assumption discussed in Chapter 3 or with home-bias as in Trefler (1995). It prohibits entry costs that vary according to source, as in Chaney (2008). It forbids a seller to base her pricing decision in a market on conditions in other markets in which she participates.

Under neutrality the term π_{ni} derived above is both: (i) the likelihood that a version of good j bought by country n comes from country i and (ii) the expected share of country n's expenditure on good j bought from country i. From (6.3), the conditional distribution of unit costs doesn't depend on the nationality of the source. Combining this result with any anonymous market structure yields the result. In the case of perfect or Bertrand competition only the low cost supplier is active, and π_{ni} is the probability that a producer from country i is the low-cost supplier of good j to country n. In the case of monopolistic competition with a common overhead cost, π_{ni} is the probability that any variety with unit cost below the common threshold $\overline{c}_n$ comes from country i.

A

6.3 Aggregate Implications

Having characterized the implications of the model for a particular good j we now integrate across goods to explore the aggregate implications of our model. As before, we treat w_i as pertaining to all goods j that might be produced in source country i. In addition we treat the trade cost parameters d_{ni} as common across any good shipped from i to n. An immediate implication is that Φ_n defined in (6.1), as well as the π_{ni} defined in (6.2), are also common across all goods. As before, the probability distribution of the efficiency for any particular good j is also the distribution of efficiency draws across goods.

Our results for the closed economy in the previous chapter apply, with Φ_n redefined as (6.1). In particular, the price index P_n in country n remains:

$$P_n = \Gamma_n \Phi_n^{-1/\theta} \tag{6.4}$$

where Γ_n can be derived explicitly for the various market structures considered in the previous chapter. In the open economy Φ_n reflects not only the country's own state of technology T_n, but the states around the world, tempered by input and trade costs. The more remote country n is (as implied by higher d_{ni}'s) the lower its Φ_n and, hence, the higher its P_n.

As derived in Chapter 5, with perfect and Bertrand competition, since the range of goods is fixed, Γ_n is the same across countries and depend only on the parameters σ and θ.

While the parameter Φ_n summarizes all that the parameters of the model imply for price differences across countries, the π_{ni} indicate the direction of trade. In particular, since π_{ni} is the probability that a purchase by country n is from i, π_{ni} becomes the fraction of purchases that n makes from i. Since π_{ni} is country i's expected share in country n's spending on any particular good, it is the fraction of n's total spending that is spent on goods from i.

The result that π_{ni} is the fraction of goods bought from i follows immediately from Part (iii) of Proposition 6.1, since it is the probability that any single purchase from i. We can thus divide the measure of goods supplied in country n into the range supplied by each source country i. By Part (iv) of the Proposition, conditional on a country supplying a particular good, its cost is drawn from the same distribution as a supplier from any other source. Moreover, under anonymity, conditional on the realization of its cost, the distribution of its price is the same. Since the price distribution doesn't depend on source, the fraction of spending going to i is the same as the fraction of goods bought from i.

This result provides a link between π_{ni} and trade shares, that:

$$\pi_{ni} = \frac{X_{ni}}{X_n}$$

where X_n is total spending by n and X_{ni} is the value of imports from i (including domestic production when $i = n$). We exploit this simple and direct link between the theory and data in several of our applications below.

Filling in the determinants of π_{ni} gives us an expression for bilateral trade shares:

$$\frac{X_{ni}}{X_n} = \frac{T_i(w_i d_{ni})^{-\theta}}{\sum_{h=1}^{N} T_h(w_h d_{nh})^{-\theta}}. \tag{6.5}$$

This expression for trade shares resembles those for Armington (3.3) and for monopolistic competition (3.18). There are two important differences. First, the scale measure for country i is no longer its share in preferences or its labor force, but its state of technology T_i, reflecting the history of ideas that have arrived in the country. Second, the elasticity parameter is no longer the elasticity of substitution in preferences but the parameter θ of the Pareto distribution for the quality of ideas, reflecting their heterogeneity. A greater value of θ means that ideas are more similar, so that comparative unit costs differ less from good to good. Hence, with a greater θ, a given increase in trade costs d_{ni} will cause country n to switch its sourcing of more goods away from country i. Unlike Armington or monopolistic competition, adjustment is not at the extensive margin, how much of each good is purchased, but at the intensive margin, the range of goods purchased.

A

6.4 Gravity

Having drawn the analogy with Armington and monopolistic competition, we can put expression (6.5) through similar paces to obtain various gravity-like expressions. First, we can write total sales Y_i of country i as:

$$Y_i = \sum_{n=1}^{N} X_{ni} = T_i w_i^{-\theta} \sum_{n=1}^{N} \frac{d_{ni}^{-\theta} X_n}{\Phi_n} = T_i w_i^{-\theta} \Xi_i \tag{6.6}$$

where:

$$\Xi_i = \sum_{m=1}^{N} \frac{d_{mi}^{-\theta} X_m}{\Phi_m} \tag{6.7}$$

reflects country i's market potential, similar to the expressions Ξ_i derived for Armington and monopolistic competition in Chapter 3.

Solving (6.6) for $T_i w_i^{-\theta}$ and substituting this expression and the definition of Φ_n (6.1) into (6.5) gives:

$$X_{ni} = \frac{Y_i X_n}{\Xi_i} \frac{{d_{ni}}^{-\theta}}{\Phi_n} \tag{6.8}$$

an expression much like the gravity equation derived from Armington (3.7) and for monopolistic competition (3.19). The difference is that the term Φ_n enters in place of $P_n^{-\theta}$ both indirectly through Ξ_i and directly. In perfect and Bertrand competition these terms are interchangeable (since γ^{PC} and γ^{MC} cancel) so we have yet again the identical equation.

With monopolistic competition, there is a substantive difference, however, since the price level P_n depends not only on technology and input costs but on market size relative to overhead cost. Substituting the price index for monopolistic competition (5.19) into (6.8) and Ξ_i gives us:

$$X_{ni} = \frac{Y_i X_n}{\Xi_i} \left(\frac{d_{ni}}{P_n} \right)^{-\theta} \left(\frac{X_n}{\sigma E_n} \right)^{[\theta-(\sigma-1)]/(\sigma-1)}.$$

Given its price level, a large country, imports more than in proportion to its size. Low prices due to variety, rather than due to low cost competitors, are not a deterrent to sales there.

A

6.5 The Gains from Trade

In this section and the one that follows we will take labor to be the only input. Hence w_i is the wage in country i. In the last section of this chapter we generalize the analysis to allow for intermediates.

The model provides an immediate expression for the gains from trade, in the form of higher real wages, as a function of trade shares. Using the price index (6.4) we can rewrite equation (6.2) for $n = i$ as:

$$\frac{w_i}{P_i} = \frac{1}{\Gamma_i} \left(\frac{T_i}{\pi_{ii}} \right)^{1/\theta}$$

where π_{ii} is the fraction of spending that i does at home. Under autarky, $\pi_{ii} = 1$ and we have our expression for the real wage in the closed economy as in the previous chapter. Trade augments a country's effective technology by a factor of $1/\pi_{ii}$. Country's that trade more, gain more. Taking a value of $\theta = 8$ (close to one of our estimates below), a country that has an import share of 0.2 would suffer a 2.8 percent decline in its real wage from a move to autarky. The reasoning here is analogous to price indices constructed to account for the introduction of new goods over time. Such price indices adjust the price index for goods available in all periods by the fraction of goods each period that are available in all periods (see Feenstra, 1994).

With perfect and Bertrand competition Γ_i is just a constant. Local technology and openness are the only determinants of cross-country differences in real wages. With monopolistic competition and overhead costs, we get:

$$\frac{w_i}{P_i} = \frac{1}{\gamma^{MC}} \left(\frac{X_i}{\sigma E_i} \right)^{[\theta-(\sigma-1)]/[\theta(\sigma-1)]} \left(\frac{T_i}{\pi_{ii}} \right)^{1/\theta}$$

An additional factor is market size relative to the overhead cost. A larger market can sustain greater variety, raising welfare. To give some sense of magnitudes, combine our value of $\theta = 8$ with an elasticity of substitution $\sigma = 5$. The elasticity of the price level with respect to X_i/E_i is then 1/8. Note that technology, trade, and market size affect the real wage multiplicatively, allowing for a clean decomposition of their effects.

This analysis takes π_{ii}, X_i and E_i as given, so has not dug down to fundamentals. To perform this task we turn to markets for inputs into production.

A

6.6 Labor-Market Equilibrium

Simplest is to make the standard Ricardian assumption that labor is the only input. Consider the condition for labor-market equilibrium in each country, choosing one country's wage as numeraire. We provide a stripped-down analysis here. Alvarez and Lucas (2004) tackle a more general set up and also provide conditions for uniqueness of the equilibrium wage vector.

Let L_i denote the number of workers available for production (or, with overhead costs, for overhead as well) in country i. Total spending on labor in country i is:

$$w_i L_i = (1-\delta) \sum_{n=1}^{n} \frac{(w_i d_{ni})^{-\theta} T_i}{\Phi_n} X_n \quad i = 1, ..., N$$

where δ is the profit share. In the case of perfect competition $\delta = 1$, while the previous chapter derived expressions for δ in the cases of Bertrand and monopolistic competition. With balanced trade, spending X is equal to labor income plus profit, so that:

$$X_n = \frac{1}{1-\delta} w_n L_n.$$

Hence we can write our labor-market equilibrium conditions as:

$$w_i L_i = w_i^{-\theta} T_i \sum_{n=1}^{N} \frac{d_{ni}^{-\theta} w_n L_n}{\sum_{k=1}^{N} (w_k d_{nk})^{-\theta} T_k} \quad i = 1, ..., N \tag{6.9}$$

(Magically, the profit share has disappeared.) In equilibrium the wages w satisfy this set of equations. In general there is no closed-form solution, but a numerical solution is easy to obtain even for a realistically large N. Note that the conditions for labor market equilibrium are the same across market structures.

Note that we can use our definition of market potential to reformulate this expression as:

$$w_i = \frac{\Xi_i}{L_i}, \quad i = 1, ..., N$$

the wage is equal to market potential divided by the labor force. Since market potential depends on wages everywhere, this expression does not constitute a closed-form solution.

A special case provides insight into what relative wages depend on. Consider the case of "frictionless" trade in which $d_{ni} = 1$ for all i and n. The summation term in expression (6.9) is then the same for all countries i. Taking the ratio of the wages in two countries i and k gives:

$$\frac{w_i}{w_k} = \left(\frac{T_i/L_i}{T_k/L_k} \right)^{1/(1+\theta)}.$$

With all $d_{ni} = 1$ price levels are the same everywhere without overhead costs. Hence this ratio is also the ratio of real wages for the cases of perfect and Bertrand competition. (In the case of monopolistic competition the price levels will still differ across markets of different size, since larger markets attract more sellers.)

Note that without trade costs relative wages depend on the state of technology relative to the labor force, with an elasticity $1/(1+\theta)$. In comparison, from the previous chapter, the ratio in the case of a closed world ($d_{ni} \to \infty,\ n \neq i$), is:

$$\frac{w_i}{w_k} = \left(\frac{T_i}{T_k}\right)^{1/\theta}.$$

Since trade allows workers to specialize in a narrow range of goods, T/L rather than T is what matters for the relative wage. Moreover, in the open economy the benefit of an increase in a particular country's T is shared by others through lower prices, so that the elasticity of the relative wage with respect to the relative T is lower.

A

6.7 Intermediates

The economic geography literature has emphasized the role of location not only for market potential, but also for production costs. We can do so in our framework by incorporating intermediate goods into the analysis. Assume that inputs combine labor and intermediate goods, with labor having a share β, and that intermediates are representative of goods generally, and that the same CES aggregator applies. The cost w_i of a bundle of inputs in country i is then proportional to $v_i^{\beta} P_i^{1-\beta}$ where now v_i is the wage in country i. Using the expression for the price index, a condition relating prices around the world, given wages v, is then:

$$P_n^{-\theta} = \varepsilon \sum_{i=1}^{N} T_i \left(v_i^{\beta} P_i^{1-\beta} d_{ni}\right)^{-\theta} \quad n = 1, ..., N \tag{6.10}$$

where $\varepsilon = \beta^{-\beta}(1-\beta)^{-(1-\beta)}$. This expression shows how higher prices in one country spill-over to others through input costs.

The condition for labor-market equilibrium becomes:

$$v_i L_i^P = \left(v_i^\beta P_i^{1-\beta}\right)^{-\theta} T_i \sum_{n=1}^{N} \frac{d_{ni}^{-\theta} w_n L_n^P}{\sum_{k=1}^{N} \left(v_k^\beta P_k^{1-\beta} d_{nk}\right)^{-\theta} T_k} \quad i = 1, ..., N \tag{6.11}$$

An equilibrium is a set of price indices P_i and wages v_i that solve (6.10) and (6.11). Again, while there is no closed-form solution for the general case, a numerical one is easy to obtain for a realistic number of countries.

This formulation delivers the result that more remote locations suffer not only from lack of access to foreign markets, but from higher input prices.

CHAPTER 6 — MANUSCRIPT

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CN

Chapter 7

CT

Growth

Chapter 4 showed how the process of the arrival of ideas gives rise to distribution of unit costs. Chapter 5 characterized the static equilibrium of an economy with those costs under different assumptions about market structure. Neither addressed the process behind the arrival of ideas, the incentive to innovate, and the allocation of resources between inventive and productive activity. In this chapter we complete the circle by introducing economic incentives to undertake research, thereby endogenizing the arrival of ideas (taken as exogenous in Chapter 4).[1]

We treat labor as the fundamental input into the creation of ideas, as well as into the production of goods. We denote the quantity of labor engaged in research by

[1] The reader should note the close connections between the analysis in Sections 7.1, 7.2, and 7.3 and in Chapters 3 and 4 in Grossman and Helpman (1991).

$L^R(t)$ and the quantity engaged in production as $L^P(t)$, with $L(t) = L^R(t) + L^P(t)$. The total labor supply $L(t)$ is exogenous but how workers divide themselves between the two activities is the outcome of market forces.

A basic premise of perfect competition is that competing sellers have access to the same technology, so there is no natural mechanism for the market to reward an inventor for her effort. For an inventor to benefit from her idea she must have some ownership rights. Our analysis in Chapter 5 considered two market structures that could potentially generate profits for the owners of ideas, Bertrand competition and monopolistic competition. Here we make the strong assumption that the creator of an idea can appropriate all the profits that her invention generates. An extension, which is left as an exercise for the end, is to examine the implications of a hazard of imitation.

We now see how these market structures create incentives to undertake research, and derive the consequences for economic growth.

We first consider growth in a single economy, and then in a multicountry world in which various countries do research, and ideas flow among them. In both the single economy and in the multicountry world, we proceed in four steps. We first specify how research effort translates into the production of ideas. We then derive the value of an idea under each form of market structure. Combining the two, we characterize the market allocation of labor between research and production. We then solve for the balanced growth equilibrium.

A 7.1 The Single Economy

We begin with an isolated economy. Ideas never cross borders. Hence growth must rely entirely on home-grown ideas while inventors earn returns from innovation only from their home market.

B 7.1.1 The Creation of Ideas

The output of research activity is the creation of ideas, as described in Chapter 4. We now relate this output to labor allocated to research. A production function for ideas relates the arrival of ideas at date t $R(t)$ to research effort:

$$\dot{T}(t) = R(t) = \alpha(t) r(t)^{\beta} L(t) \tag{7.1}$$

where $\alpha(t)$ is research productivity, $L(t)$ is the total number of workers, and $r(t) = L^R(t)/L(t)$ is the share engaged in research, all at time t. Finally, $\beta \in [0, 1]$ is a parameter reflecting the extent of diminishing returns to putting a larger share of workers into research. This specification has the property of homogeneity of degree one the total labor force given the fraction doing research. With $\beta = 0$, knowledge accumulates exogenously regardless of research effort, so that research effort makes no contribution to growth. With $\beta = 1$ there are constant returns to scale in doing research, the assumption in Grossman and Helpman (1991). Phelps (1966) motivates diminishing returns positing underlying heterogeneity among workers in their research talent.

B

7.1.2 The Value of an Idea

Under our assumptions, all ideas are drawn from the same distribution regardless of when they arrived. Not conditioning on quality, then, at any moment t the expected profit $h(t)$ of an idea that arrived up to that point is simply the total profit generated in an economy $\Pi(t)$ relative to the measure of ideas that have arrived so far, including ideas that are no longer in use. Thus at any date t we can write:

$$h(t) = \frac{\Pi(t)}{T(t)} = \frac{\delta X(t)}{T(t)},$$

where δ is the profit share.

In Chapter 5 we derived expressions for δ for Bertrand and monopolistic competition. With Bertrand competition:

$$\delta^{BC} = \frac{1}{1+\theta}$$

while, with monopolistic competition:

$$\delta^{MC} = \frac{\sigma - 1}{\theta\sigma}.$$

It is useful to express profits relative to the income of production workers. We continue to take the wage w as our numeraire, so its value remains constant over time. As before we find it instructive to keep w in our expressions rather than setting it to one. Income of production workers is:

$$wL^P(t) = (1-\delta)X(t).$$

We can thus rewrite the flow of profit at period t as:

$$h(t) = \frac{\delta}{1-\delta}\frac{wL^P(t)}{T(t)}. \tag{7.2}$$

In the case of monopolistic competition, overhead workers are included in $L^P(t)$.

Having collected the relevant pieces from Chapter 5 about profit at any date s, we can assemble them into an expression for the value of an idea at some date t looking forward in time. This calculation requires discounting future profit flows. First, we need to take into account that the purchasing power of profit flows varies inversely with the price index, so we adjust by a factor $P(t)/P(s)$, translating nominal flows to comparable utility flows We also assume a constant discount rate $\rho > 0$, which we treat as the reflection of pure time preference.[2]

Combining these elements, the expected discounted value at date t of an existing idea, again **not** conditioning on the idea's quality, is:

$$V(t) = \int_t^\infty e^{-\rho(s-t)}\frac{P(t)}{P(s)}h(s)ds. \tag{7.3}$$

[2] Instead of introducing a pure time discount rate, Melitz (2003) assumes that ideas "die" with a hazard rate $\rho > 0$ irrespective of their quality. Their death provides room for new ideas to enter and make money. But since the new ideas are, on average, no better than the dead ones they replace, the economy does not advance. It runs just to stay in place like a hamster on an exercise wheel.

Finally, we need to consider how the ratio of the $P(t)/P(s)$ evolves over time, which varies across our three cases, Bertrand competition, monopolistic competition with a fixed overhead requirement, entailing an overhead cost wF, and monopolistic competition with a declining overhead cost $wF/T(t)$. Consider the price indices for the three cases, reported as expressions (5.13), (5.23), and (**??**) in Chapter 5. A general expression for the ratio of prices at two dates is:

$$\frac{P(t)}{P(s)} = \left(\frac{T(t)}{T(s)}\right)^{-1/\eta} \left(\frac{L^P(t)}{L^P(s)}\right)^{-\nu} \tag{7.4}$$

We set $\eta = \theta$ for Bertrand competition and the first case of monopolistic competition and $\eta = \sigma - 1$ for the second case of monopolistic competition. We set $\nu = 0$ for Bertrand competition and $\nu = [\theta - (\sigma - 1)]/[\theta(\sigma - 1)]$ for either case of monopolistic competition. This notation allows us to explore features common across the three cases, avoiding a repetitive taxonomy.

Under monopolistic competition, unlike Bertrand competition, growth in the production labor force attracts entry, so lowers the price index. Hence our different values of ν. With declining overhead costs, growth in $T(t)$ not only lowers costs, but increases variety, so it has a magnified effect on the price index. Hence our two values of η.

Substituting (7.2) and (7.4) into (7.3) gives us:

$$V(t) = \frac{\delta}{1-\delta} w \int_t^\infty e^{-\rho(s-t)} \left(\frac{L^P(t)}{L^P(s)}\right)^{-\nu} \left(\frac{T(t)}{T(s)}\right)^{-1/\eta} \frac{L^P(s)}{T(s)} ds. \tag{7.5}$$

To reiterate, in interpreting this equation:

Bertrand competition:

$$\delta = \frac{1}{1+\theta}, \quad \eta = \theta, \quad \nu = 0;$$

monopolistic competition, case 1:

$$\delta = \frac{\sigma - 1}{\theta\sigma}, \quad \eta = \theta, \quad \nu = \frac{\theta - (\sigma - 1)}{\theta(\sigma - 1)};$$

monopolistic competition, case 2:

$$\delta = \frac{\sigma - 1}{\theta\sigma}, \quad \eta = \sigma - 1, \quad \nu = \frac{\theta - (\sigma - 1)}{\theta(\sigma - 1)}.$$

Having derived an expression for the value of an idea, we can combine it with our production function for ideas to solve for the allocation of labor to research.

B

7.1.3 Equilibrium Research Effort

Working in the production sector yields a wage w while the value of an idea and the chance of getting one drive the return to research. Workers engaged in research don't know how good their ideas will be before they are invented. Since each idea is worth $V(t)$ in expectation, the total value of research output at time t is $\alpha(t)r(t)^\beta L(t)V(t)$.

The marginal product of an additional researcher is $\beta\alpha(t)V(t)r(t)^{\beta-1}$. If research workers earn their marginal product labor-market equilibrium entails the complementary slackness conditions:

$$\beta\alpha(t)V(t)r(t)^{\beta-1} = w \quad r(t) \in [0,1] \tag{7.6}$$

$$\beta\alpha(t)V(t)r(t)^{\beta-1} < w \quad r(t) = 0 \tag{7.7}$$

$$\beta\alpha(t)V(t)r(t)^{\beta-1} > w \quad r(t) = 1 \tag{7.8}$$

Given an initial state of technology $T(t)$ and labor force $L(t)$ a dynamic equilibrium is a value of $r(s)$ for each $s \geq t$ that satisfies either (**??**) or (**??**) and (7.6), with $T(s)$ evolving according to (7.1).

B

7.1.4 Balanced Growth

In general, we can relate the growth rate $g_T(t)$ of ideas to research effort $r(t)$ using expression (7.1):

$$g_T(t) = \alpha(t)r(t)^{\beta}\frac{L(t)}{T(t)}.$$

We now assume that the labor force grows at a constant rate $g_L \geq 0$.

We define a balanced growth path as a dynamic equilibrium entailing a constant share of the labor force $r(t) = r$ engaged in research and a constant growth rate of ideas $g_T(t) = g_T$. We admit only parameter values for which $r < 1$. Under these conditions the growth rate of ideas reduces to:

$$g_T = r^{\beta}\Upsilon(t).$$

where:

$$\Upsilon(t) = \alpha(t)\frac{L(t)}{T(t)}$$

To achieve balanced growth, we need assumptions that ensure the constancy of Υ over time. We consider two different sets of assumptions about the labor force and research productivity that do the trick.

C Endogenous Growth

We first consider a case, often called "endogenous growth," in which the long-run growth rate is the outcome of the interaction of research productivity, scale, and preferences. A balanced growth path requires a constant labor force, $L(t) = L$, determining the scale of the economy. Since both L and r are constant, so is L^P. Following Romer (1990) we also allow the stock of ideas to enhance research productivity in proportion, setting $\alpha(t) = \alpha T(t)$, where α is a positive parameter. In this case:

$$g_T = \alpha r^{\beta} L.$$

With $\beta = 0$ we are back in a world of exogenous growth, as in Krugman (1979).[3]

[3] As the reader will see, $\beta = 0$ does indeed imply that $r = 0$. A question is what happens to the rents associated with the new ideas that arrive on their own. Perhaps it's most natural to think that they

Consider the expression for the value of an idea (7.5). Since r and L are constant, so is $L^P = (1-r)L$; the term T grows at the constant rate g_T given just above. Integrating we get:

$$V(t) = \frac{\delta\eta}{1-\delta}\frac{w(1-r)L}{T(t)}\frac{1}{\rho\eta + (\eta-1)\alpha r^{\beta} L}.$$

Substituting $V(t)$ into the condition for an interior labor market equilibrium (7.6) gives:

$$1 = \frac{\delta\eta}{1-\delta}\frac{(1-r)L}{\rho\eta + (\eta-1)\alpha r^{\beta} L}\alpha\beta r^{\beta-1}$$

which can be rearranged to become:

$$\beta\frac{\delta\eta}{1-\delta} - \eta\Lambda r^{1-\beta} = \left(\beta\frac{\delta\eta}{1-\delta} + \eta - 1\right) r \tag{7.9}$$

where:

$$\Lambda = \frac{\rho}{\alpha L}.$$

While there is no closed-form solution for the general case, inspection of this expression gives us a result on the determinants of research intensity and growth:

Proposition 10 *Under endogenous growth, research effort and growth are increasing in the size of the labor force adjusted for research productivity αL and decreasing in the discount factor ρ.*

become common knowledge so that the economy is then perfectly competitive, as in Krugman (1979). Another story, not modeled here, is that research effort is a struggle among competing interests to lay claim to the rents associated with ideas, but does nothing to hasten their arrival.

G *Proof.* The proof is a simple geometric one. The left-hand side of (7.9) is continuously and monotonically decreasing in r while the right-hand side is increasing linearly in r. An increase in Λ shifts the right-hand side down, so that at the crossing point r falls.

■

Note that indeed $r = 0$ if $\beta = 0$ and that $r > 0$ if $\beta < 1$. The reason for this last result is that $\beta < 1$ ensures that the marginal productivity of research approaches infinity as $r \to 0$.

In the case of constant returns to scale ($\beta = 1$) we can get a closed-form solution. At an interior:

$$r = \frac{\frac{\delta\eta}{1-\delta} - \Lambda\eta}{\frac{\delta\eta}{1-\delta} + \eta - 1} \tag{7.10}$$

so that:

$$g_T = \frac{\frac{\delta\eta}{1-\delta}\alpha L - \rho\eta}{\frac{\delta\eta}{1-\delta} + \eta - 1} \tag{7.11}$$

If parameter values are such that the right-hand side of (7.10) (or, equivalently, of (7.11)) is negative, certainly a permissible outcome, then $r = g_T = 0$.

Substituting the appropriate values for Bertrand competition ($\delta = 1/(1+\theta)$ and $\eta = \theta$) yields particularly stark solutions:

$$r = \frac{1}{\theta} - \frac{\rho}{\alpha L}$$

and:

$$g_T = \frac{\alpha L}{\theta} - \rho.$$

The two cases of monopolistic competition yield more cumbersome expressions which we leave as exercises.

C Semi-Endogenous Growth

We now consider a case, which Jones (199X) calls "semi-endogenous growth," in which the long-run growth rate is the growth rate of the labor force, with research productivity, scale, and preferences determining the level of technology.

We now assume that the labor force grows at rate $g_L > 0$ while making research productivity constant at $\alpha(t) = \alpha$. To guarantee that discounted utility is bounded we restrict:

$$\frac{\rho}{g_L} > \frac{1}{\eta} + \nu. \tag{7.12}$$

In this case:

$$g_T = \alpha r^\beta / \tau(t),$$

where $\tau(t) = T(t)/L(t)$. The dynamics of $\tau(t)$ are captured by

$$\dot{\tau}(t) = \alpha r^\beta - \tau(t) g_L.$$

On a balanced-growth path $g_T - g_L$ is constant, but from the dynamics of $\tau(t)$, this condition requires a constant $\tau(t)$; hence $g_T = g_L$ and

$$\tau = \frac{\alpha r^\beta}{g_L}.$$

Incorporating these ingredients into our expression for the value of an idea (7.5) and integrating gives us:

$$V = \frac{\delta}{1-\delta}\frac{w(1-r)}{\alpha r^{\beta}}\frac{1}{\rho/g_L - \nu - 1/\eta}.$$

Substituting V into the condition for an interior labor market equilibrium (7.6) and solving for r gives:

$$r = \frac{\beta\frac{\delta}{1-\delta}}{\rho/g_L - \nu - 1/\eta + \beta\frac{\delta}{1-\delta}}.$$

Note that (7.12) ensures that $r < 1$. Again, of course, $\beta = 0$ implies $r = 0$, but for $\beta > 0$, $r > 0$. Inspection of this expression delivers:

Proposition 11 *Under semi-endogenous growth, research effort is independent of research productivity α and increasing in the population growth rate g_L relative to the discount factor ρ.*

Even though higher research productivity α has no effect on research effort, it implies a higher level of technology per worker τ, since the same research effort delivers more research output. A consequence is a higher T and a higher real wage for any given size of the labor force. While endogenous growth implies that a large labor force generates a higher growth rate, semi-endogenous growth implies that a larger labor force generates a higher standard of living, since τ is invariant to L, while T, which is relevant for living standards, is equal to τL.

Again, the particular case of Bertrand competition yields a particularly stark result:

$$r = \frac{\beta}{\rho\theta/g_L - (1-\beta)}$$

which, if $\beta = 1$, simplifies to $r = g_L/\rho\theta$. Again, we leave the two cases of monopolistic competition as exercises.

In Chapter 4 we posited an exogenous process for the arrival of ideas. We have now provided two explanations for such an arrival rate, both of which generate a process of ongoing growth like that experienced by most countries over the last century, as discussed in Chapter 2. But before we can address other features of the cross-country data on productivity and research effort, we need to think about how countries interact. On the one hand, treating the world as a single economy is unsatisfactory since we observe vast differences in living standards and in research activity. On the other hand, treating each country as a separate entity is equally unsatisfactory. Under endogenous growth large countries would grow forever faster, while under semi-endogenous growth they would be systematically richer. Neither implication stands out in the data. Moreover, neither model delivers a compelling explanation for research specialization. Endogenous growth would imply a strong correlation between research specialization and size, inconsistent with the presence of Finland, Sweden, and Luxembourg among the top five in terms of research intensity in Table 2 of Chapter 1. Semi-endogenous growth can only explain cross-country differences in research intensity by arbitrary heterogeneity in parameters. Neither could explain why an inventor would ever seek patent protection abroad.

A

7.2 International Diffusion

To address these shortcomings we need to recognize that the world consists of multiple countries, each with its own ability to generate ideas, with the potential for ideas to drift from country to country. We amend the previous analysis to allow for N countries and posit conditions under which the world will achieve balanced growth, with each country growing at the same rate, but with the potential for cross-country differences in living standards.

We use the wage in country 1 as numeraire, but continue to include it in the relevant equations. Under balanced growth, wages everywhere are then constant over time, but can differ across countries.

Chapter 6 focussed on how differences in what countries know generates comparative advantage and gains from trade. Since diffusion acts to eliminate such differences, simultaneously studying trade in goods embodying ideas and the diffusion of the ideas themselves is daunting. To isolate the role of diffusion we assume no direct trade in individual goods. Suppressing all trade, however, forces balance in net foreign profit flows. Instead, we adopt the interpretation that different goods are intermediates that go into the production of a final good that is costlessly traded. Hence the price level

$P(t)$, which is the unit cost of the final good, is common across countries, but falls over time as knowledge accumulates. Trade in this final good offsets any imbalances in international profit flows.

We now revisit in this international setting the elements that go into the determination of research effort and growth, as we did for the isolated economy above.

B

7.2.1 The Creation and Diffusion of Ideas

We continue to assume an idea production function of the form given in expression (7.1). Every idea is potentially usable everywhere, but we now introduce a friction in the form of a time delay between the creation of an idea and its entry into the stock of knowledge somewhere. Specifically, we assume that the time it takes for an idea from country i to enter country n's usable knowledge T_n is a random variable τ_{ni} which is exponentially distributed with parameter $\epsilon_{ni} \geq 0$; that is:

$$\Pr[\tau_{ni} \leq t] = 1 - \exp(-\epsilon_{ni} t).$$

An implication is that the mean diffusion lag is $1/\epsilon_{ni}$, so that a higher ϵ_{ni} means faster diffusion.

The notion that an idea is available locally, on average, sooner than abroad can be captured by specifying $\epsilon_{ii} > \epsilon_{ni}$, $n \neq i$. Just as the d_{ni}'s in the previous chapter represented barriers to the movement of goods, here ϵ_{ni}'s capture (inversely) barriers to the diffusion of ideas. The first part of this chapter dealt with the special case $\epsilon_{ii} \to \infty$

and $\epsilon_{ni} = 0,\ n \neq i$, instantaneous diffusion at home with none abroad.

Incorporating these assumptions, then, we modify equation (7.1) to introduce multiple sources of ideas and diffusion lags. The change in the stock of usable knowledge in country n thus becomes:

$$\dot{T}_n(t) = \sum_{i=1}^{N} \epsilon_{ni} \left[\int_{-\infty}^{t} \exp[-\epsilon_{ni}(t-s)] \alpha_i(s) r_i(s)^{\beta} L_i(s) ds \right] \quad n = 1, ..., N \qquad (7.13)$$

That is, each country's stock of technology grows as ideas arrive that were generated by the history of past research around the world.

B

7.2.2 The Value of an Idea

We can calculate the value of an idea from country i in country n at time of invention t, again, not conditioning on its quality. But now we need to take into account the expected wait for it to be usable there. The probability that it is used there by date $s \geq t$ is $1 - \exp[-\epsilon_{ni}(s-t)]$. Hence:

$$V_{ni}(t) = \int_{t}^{\infty} \exp[-\rho(s-t)] \left\{1 - \exp[-\epsilon_{ni}(s-t)]\right\} \frac{\Pi_n(s)}{T_n(s)} \frac{P(t)}{P(s)} ds \quad i, n = 1, ..., N \quad (7.14)$$

where $\Pi_n(s)$ is total profit generated in market n at date s.

Summing across all destinations, the total value of an idea from country i at time t is:

$$V_i(t) = \sum_{n=1}^{N} V_{ni}(t) \quad i = 1, ..., N \qquad (7.15)$$

B
7.2.3 Balanced Growth

Parallel to our treatment of the isolated economy, we can relate the growth rate of usable knowledge in country n at date t, $g_{T_n}(t)$ to research effort in country i at date s, $r_i(s)$ using expression (7.13):

$$g_{T_n}(t) = \sum_{i=1}^{N} \epsilon_{ni} \left[\int_{-\infty}^{t} \exp[-\epsilon_{ni}(t-s)]\alpha_i(s) r_i(s)^{\beta} \frac{L_i(s)}{T_i(s)} \frac{T_i(s)}{T_i(t)} ds \right] \frac{T_i(t)}{T_n(t)} \quad n = 1, ..., N \tag{7.16}$$

We now assume that the labor force in each country grows at a common, constant rate $g_L \geq 0$.

The multicountry analog to our definition of a balanced growth path is a dynamic equilibrium entailing a constant share of the labor force $r_i(t) = r_i$ in each country i along with a growth rate in usable knowledge g_T constant over time and common across countries. A consequence of the last condition is constancy over time in the ratio T_i/T_n for any i and n. Under these conditions the expression above simplifies to:

$$g_T = \sum_{i=1}^{N} \epsilon_{ni} r_i{}^{\beta} \left[\int_{-\infty}^{t} \exp[-(\epsilon_{ni} + g_T)(t-s)]\Upsilon_i(s) ds \right] \frac{T_i}{T_n} \quad n = 1, ..., N \tag{7.17}$$

where now:

$$\Upsilon_i(s) = \alpha_i(s) \frac{L_i(s)}{T_i(s)}$$

Now we need Υ_i not to change over time. Hence we can integrate to get:

$$g_T = \sum_{i=1}^{N} \frac{\epsilon_{ni}}{\epsilon_{ni} + g_T} r_i^{\beta} \Upsilon_i \frac{T_i}{T_n} \quad n = 1, ..., N \tag{7.18}$$

As before, we consider two cases which deliver constant Υ_i's, endogenous growth with $g_L = 0$ and with $\alpha_i(s) = \alpha_i T_i$, and semi-endogenous growth, with $g_L > 0$ and $\alpha_i(s) = \alpha_i$. In either case, α_i is a parameter reflecting country i's research prowess.

Since it's a bit simpler, we limit our discussion to Bertrand competition, leaving monopolistic competition as an exercise for the reader.

C Endogenous Growth

In this case we can write $\Upsilon_i = \alpha_i L_i$ and rewrite (7.18) to obtain:

$$\dot{T}_n(t) = \sum_{i=1}^{N} \frac{\epsilon_{ni}}{\epsilon_{ni+g_T}} \alpha_i r_i^{\beta} L_i T_i(t) \quad n = 1, ..., N \tag{7.19}$$

where L_i is the (constant) labor force in country i.

While we will eventually turn to the determination of the r_i, it's worth first examining the properties of (7.19) for given r_i. First note that (7.19) represents a generalization of the international technology dynamics in Krugman (1979) to N countries, all of whom can be innovating, with arbitrary bilateral diffusion lags. As in Krugman's model, under certain assumptions the system will evolve toward one with a common growth rate of technology, with a particular pecking order. Finding the growth rate and the pecking order requires substantially more work, however, to which we now turn.

It is useful to stack the equations in (7.19) and write them in matrix form as:

$$\dot{T}(t) = \Delta(g_T) T(t) \tag{7.20}$$

where $\Delta(g)$ is a matrix with representative element:

$$\Delta_{ni}(g) = \frac{\epsilon_{ni}}{\epsilon_{ni+}g}\alpha_i r_i^{\beta} L_i,$$

where $T(t)$ is the vector:

$$T(t) = \begin{bmatrix} T_1(t) \\ T_2(t) \\ T_3(t) \\ . \\ . \\ T_N(t) \end{bmatrix}$$

and where $\dot{T}(t)$ is the vector:

$$\dot{T}(t) = \begin{bmatrix} \dot{T}_1(t) \\ \dot{T}_2(t) \\ \dot{T}_3(t) \\ . \\ . \\ \dot{T}_N(t) \end{bmatrix}$$

For any given $g_T = g$, (7.20) is a system of linear differential equations. An interesting feature of the matrix $\Delta(g)$, which is invariant to any finite value of g, is its indecomposability. It is indecomposable if there is no way to order countries so that the matrix takes the form:

$$\Delta = \begin{bmatrix} \Delta_{AA} & \Delta_{AB} \\ 0 & \Delta_{BB} \end{bmatrix}.$$

where Δ_{AA} and Δ_{BB} are square matrices. If the matrix is indecomposable then every country is connected directly or indirectly to research in every other country. Otherwise, the world can be broken into separate research "blocs" A and B such that ideas from bloc A never make it into bloc B.

While there is no analytic solution for g_T, we can establish, for given r_i's, its existence and uniqueness:

Proposition 12 *If the matrix $\Delta(g)$ is indecomposable then there exists a unique positive balanced growth rate of technology $g_T > 0$ given research intensities r_i. Associated with that growth rate is a vector T (defined up to a scalar multiple), with every element positive, which reflects each country's relative level of knowledge along that balanced growth path.*

G *Proof.* We seek a balanced growth rate g_T and an associated vector T such that:

$$g_T T = \Delta(g_T)T.$$

The Frobenius theorem guarantees that if $\Delta(g)$ is indecomposable then it has a single, positive dominant root $\widetilde{g}(g)$ (the Frobenius root) that has a strictly positive associated Eigenvector. The Frobenius root is increasing in each element of $\Delta(g)$. Since each element of.$\Delta(g)$ is decreasing in g it follows that $\widetilde{g}(g)$ is also decreasing in g. Since, as an Eigenvalue, $\widetilde{g}(g)$ is continuous in the elements of $\Delta(g)$ and since each element of $\Delta(g)$ is continuous in g then $\widetilde{g}(g)$ is also continuous in g. Since $\widetilde{g}(0) \in (0, \infty)$, and $\widetilde{g}(g)$ continuously decreases in g, there exists a unique fixed point $g_T = \widetilde{g}(g_T)$. Associated with this solution, which is the Frobenius root of $\Delta(g_T)$, is a strictly positive Eigenvector T unique up to a scalar multiple. See, e.g., Gandolfo (1996) and Sydsæter, Strøm, and Berck (1999). ■

Hence with all countries connected by diffusion, knowledge in each of them ends up growing at the same rate although, depending on parameters, some countries have more advanced levels of technology than others.

What can go wrong if Δ is decomposable? Say that on it's own research bloc B has a lower growth rate than research bloc A (meaning that the Frobenius root of B is smaller than A's). Since ideas never flow from countries in A to countries in B, this second group converges to a lower constant growth rate. Indecomposability is sufficient but not necessary for convergence to a single balanced growth rate, however. If bloc B would grow faster on its own and if ideas can trickle down from B to A then A's growth rate will eventually catch up.

Having established conditions for the existence and uniqueness of a balanced growth rate g_T, given r_i we can provide the following result on how it responds to changes in parameters of interest.

Proposition 13 *The balanced growth rate* g_T *is increasing in research intensity* r_i, *research productivity* α_i, *scale* L_i, *and the speed of diffusion* ϵ_{ni} *for any country* i *or country pair* n *and* i.

G *Proof.* The proof is by contradiction. We show that g_T cannot fall or remain constant. Write the Frobenius root of the matrix $\Delta(g, \theta)$ as $\widetilde{g}(g, \theta)$, where θ is a representative parameter in the statement of the proposition. As pointed out in the the proof of the previous proposition, given $\theta, \widetilde{g}$ is decreasing in g. Since, given g, no element of $\Delta(g, \theta)$ is decreasing in θ and at least one element is increasing in θ, $\widetilde{g}$ is increasing in θ. Consider a change $\theta' > \theta$. Say that the associated $g'_T \leq g_T$. Then:

$$g'_T = \widetilde{g}(g'_T, \theta') \geq \widetilde{g}(g_T, \theta') > \widetilde{g}(g_T, \theta) = g_T,$$

a contradiction. ■

This result is premised on the existence of a balanced growth path for the set of countries. A closely related result considers the extreme in which cross-country diffusion vanishes ($\epsilon_{ni} = 0$ for $i \neq n$) in which case each country generates its own growth rate, g_{T_n}.

The balanced growth rate g_T achieved in an technologically integrated world exceeds the balanced growth rate that any country would achieve in isolation, given its research effort. To see why, note that for any n (7.19) can, under balanced growth, be written:

$$
\begin{aligned}
g_T &= \sum_{i=1}^{N} \frac{\epsilon_{ni}}{\epsilon_{ni+} g_T} \alpha_i r_i^{\beta} L_i \frac{T_i(t)}{T_n(t)} \\
&= \frac{\epsilon_{ii}}{\epsilon_{ii+} g_T} \alpha_i r_i^{\beta} L_i + \sum_{i=1, i \neq n}^{N} \frac{\epsilon_{ni}}{\epsilon_{ni+} g_T} \alpha_i r_i^{\beta} L_i \frac{T_i(t)}{T_n(t)} \\
&> \frac{\epsilon_{ii}}{\epsilon_{ii+} g_T} \alpha_i r_i^{\beta} L_i,
\end{aligned}
\tag{7.21}
$$

while, in isolation, country n's balanced growth rate would solve:

$$
g_{T_n} = \frac{\epsilon_{ii}}{\epsilon_{ii+} g_{T_n}} \alpha_i r_i^{\beta} L_i.
$$

The same argument by contradiction used in the proof of the last proposition establishes that $g_T > g_{T_n}$.

Not only is the growth of technology of interest, as it governs the growth in the standard of living, so are the relative levels of technology, as they determine cross-country differences in living standards at any moment. From our analysis in Chapter 5, any two countries n and n', relative technologies translate into relative wages as:

$$
\frac{w_{n'}}{w_n} = \left(\frac{T_{n'}}{T_{n'}} \right)^{1/\theta}.
$$

Since trade equalizes the price of the final good internationally, this ratio reflects relative living standards.

We now provide a result relating relative technology levels to research effort and diffusion. A rearrangement of (7.21) yields:

$$T_n(t) = \frac{1}{g_T} \sum_{i=1}^{N} \widetilde{\epsilon}_{ni} \alpha_i r_i^{\beta} L_i T_i(t). \tag{7.22}$$

where:

$$\widetilde{\epsilon}_{ni} = \frac{\epsilon_{ni}}{\epsilon_{ni} + g_T}, \tag{7.23}$$

which increases monotonically in ϵ_{ni}.

Using this expression we can establish the following:

Proposition 14 *Consider two countries n and n'. (1) Say that the countries are the same with respect to the speed with which they receive ideas from third countries (i.e., $\epsilon_{ni} = \epsilon_{n'i}$ $\forall i \neq n, n'$) and are symmetric with respect to the speed with which they receive ideas from each other and from themselves, with faster diffusion at home (i.e., $\epsilon_{nn} = \epsilon_{n'n'} = \epsilon_D$, $\epsilon_{nn'} = \epsilon_{n'n} = \epsilon_F$, and $\epsilon_D > \epsilon_F$) then the country that generates more ideas relative to its stock of technology will have a larger stock of technology, .i.e., $\alpha_{n'} r_{n'}^{\beta} L_{n'} > \alpha_n r_n^{\beta} L_n$ implies $T_{n'} > T_n$. (2) Say that $\alpha_{n'} r_{n'}^{\beta} L_{n'} = \alpha_n r_n^{\beta} L_n$ while $\epsilon_{nn} = \epsilon_{n'n'} = \epsilon_D$ and $\epsilon_{nn'} = \epsilon_{n'n} = \epsilon_F$, then if $\epsilon_{n'i} \geq \epsilon_{ni}$ $\forall i \neq n, n'$, with strict inequality for at least one i, then $T_{n'} > T_n$.*

G *Proof.* *(1) We can use (7.22) to write:*

$$T_{n'}(t) - T_n(t) = \frac{1}{g_T} \left(\alpha_{n'} r_{n'}^{\beta} L_{n'} T_{n'}(t) - \alpha_n r_n^{\beta} L_n T_n(t) \right) \left(\widetilde{\epsilon}_D - \widetilde{\epsilon}_F \right).$$

Dividing by $T_{n'}(t)$:

$$\begin{aligned}\frac{T_{n'}}{T_n} &= 1+\frac{1}{g_T}\left(\alpha_{n'}r_{n'}^{\beta}L_{n'}\frac{T_{n'}}{T_n}-\alpha_n r_n^{\beta}L_n\right)(\widetilde{\epsilon}_D-\widetilde{\epsilon}_F)\\ &= \frac{g_T-\alpha_n r_n^{\beta}L_n(\widetilde{\epsilon}_D-\widetilde{\epsilon}_F)}{g_T-\alpha_{n'}r_{n'}^{\beta}L_{n'}(\widetilde{\epsilon}_D-\widetilde{\epsilon}_F)}.\end{aligned}$$

Both numerator and denominator of this last expression are positive since, from (7.21):

$$g_T>\alpha_m r_m^{\beta}L_m\widetilde{\epsilon}_D>\alpha_m r_m^{\beta}L_m(\widetilde{\epsilon}_D-\widetilde{\epsilon}_F)\quad m=n,n'.$$

Hence, since we assume $\epsilon_D>\epsilon_F, \alpha_{n'}r_{n'}^{\beta}L_{n'}>\alpha_n r_n^{\beta}L_n$ implies that $T_{n'}>T_n$. (2) We can use (7.22) to write:

$$T_{n'}(t)-T_n(t)=\frac{1}{g_T}\sum_{i=1,i\neq n,n'}^{N}(\widetilde{\epsilon}_{n'i}-\widetilde{\epsilon}_{ni})\,\alpha_i r_i^{\beta}L_i>0.$$

■

Hence, all else equal, countries that are faster to absorb ideas from abroad and (with home bias in the speed of diffusion) countries that are more research productive have larger states of technology.

We now turn to the determination of the r_i's. We first derive the value of an idea from country i. Using (7.15) we get:

$$V_i=\frac{1}{\theta}\sum_{n=1}^{N}K_{ni}\frac{(1-r_n)L_n}{T_n(t)}\quad i=1,...,N$$

where:

$$K_{ni}=\frac{1}{\rho+(1-1/\theta)g_T}-\frac{1}{\rho+(1-1/\theta)g_T+\epsilon_{ni}}. \tag{7.24}$$

Combining these expressions with the conditions for an interior labor-market allocation (7.6) for country i we get:

$$r_i^{1-\beta} = \frac{\alpha_i \beta}{\theta} \sum_{n=1}^{N} K_{ni}(1 - r_n) L_n \left(\frac{T_n(t)}{T_i(t)} \right)^{(1-\theta)/\theta} \qquad i = 1, ..., N. \tag{7.25}$$

Together, the solution to (7.20) and (7.25) gives us r_i, g_T, and T_i (up to a scalar multiple) as functions of labor forces L_i, research productivities α_i, and diffusion parameters ϵ_{ni} (which vary with geography) as well as the parameters θ, β, and (in the case of monopolistic competition) σ. While the presence of the Eigenvector system precludes an analytic solution for r_i, a numerical solution is readily obtainable.

Two special cases bring us back to the single economy solution above. In one, $\epsilon_{ni} = 0$ for $n \neq i$ and $\epsilon_{ii} \to \infty$, so that countries are autarkic. The world economy decomposes into N closed economies each one generating its own single-economy growth rate depending on its α_i and L_i. In the other case $\epsilon_{ni} \to \infty$ and $\alpha_i = \alpha \; \forall n, i$; the world behaves like a single economy with $L = \sum_{n=1}^{N} L_n$.

Moreover, we get much insight into the solution by taking the ratio of (7.25) between two countries i' and i, which can be written as for $\beta < 1$:

$$\frac{r_{i'}}{r_i} = \left(\frac{\alpha_{i'}}{\alpha_i} \right)^{1/(1-\beta)} \left(\frac{T_{i'}}{T_i} \right)^{(\theta-1)/[\theta(1-\beta)]} \left(\frac{\sum_{n=1}^{N} K_{ni'}(1 - r_n) L_n T_n(t)^{(1-\theta)/\theta}}{\sum_{n=1}^{N} K_{ni}(1 - r_n) L_n T_n(t)^{(1-\theta)/\theta}} \right)^{1/(1-\beta)}. \tag{7.26}$$

Note that the determinants of relative research intensity (on the left-hand side) can be broken up into three factors. The first factor involves relative research productivity

$\alpha_{i'}/\alpha_i$: Given the other two factors, the more research productive country does more research, with elasticity $1/(1-\beta) > 1$. The second involves relative technology stocks $T_{i'}/T_i$. Given the first and third factor, the country with the larger technology stock does more research with an elasticity $(\theta-1)/\left[\theta(1-\beta)\right]$. This factor combines two effects going in opposite directions: (1) a negative cost of research effect: researchers in the country with the higher T_i have a greater opportunity cost of doing research since the wage is higher there by a factor of $(T_{i'}/T_i)^{1/\theta}$; (2) a positive technology spillover effect: the country with the larger stock of knowledge is more research productive by a factor of $T_{i'}/T_i$. Since $\theta > 1$, this positive effect dominates. The third factor involves the countries' ability to access world markets for its technologies. Since K_{ni} is increasing in ϵ_{ni}, given the first and second factor, the country whose ideas disseminate more rapidly does more research. Note that a destination n provides a more lucrative market for ideas the larger its labor force in production $(1-r_n)L_n$, but the smaller its stock of technology T_n. This last effect combines a positive wage effect with a negative effect arising from the fact that a higher T_n mean that an idea is less likely to constitute a breakthrough there. Our requirement that $\theta > 1$ ensures that this second effect dominates.

In general these effects can't be thought of as independent since the T's and r 's in the second two factors are determined jointly with the r's on the left-hand side. In a limiting case the decomposition is pure: As $L_i, L_{i'} \to 0$ the two countries under comparison are too small as researchers to influence the T's and too small as markets to influence the r's of other countries.

Moving away from this pure case, with home bias in diffusion ($K_{ii} > K_{ni}$, $n \neq i$) the feedback from r to T reinforces research concentration through the second factor, as countries that do more research have higher T's. But the feedback from r to market size, with home bias, attenuates research concentration as fewer production workers mean a smaller domestic market for ideas.[4]

In summary, the model relates research specialization to research productivity itself, with a more research productive country doing more research, and to a country's position in the global flow of ideas. Countries that are more receptive to ideas from the rest of the world have a greater knowledge base on which to base research, while those whose ideas the world is most receptive to have a larger market for them. Home bias in diffusion would then imply that, given its research productivity, a large country has an advantage both in having more home-grown ideas to work with and in having a larger market. Until recently, large countries did tend to devote a bigger share of resource to research. The recent emergence of Finland as a research center may reflect its greater integration into the global market for technology.

[4] The solution for $\beta = 1$ is more opaque as, given the T 's, the market size effect serves as the only equilibrating mechanism. As a country gets small in terms of both its effect on T 's around the world and the world market for ideas, it will almost surely either specialize completely in research or do none at all.

C Semi-Endogenous Growth

Assume now that all countries have labor forces that grow at the same rate $g_L > 0$, continuing to impose the restriction (7.12). Again, we first condition on the r_i's to characterize the balanced growth path of the stocks of technology. We then derive the value of an idea in different countries along a balanced growth path in order to solve for research intensity.

We now specify Υ_i in equation (7.18) as $\alpha_i L_i(t)/T_i(t)$ and require, for balanced growth, a constant ratio τ_i of ideas to workers in each country, where:

$$\tau_i = \frac{T_i(t)}{L_i(t)}.$$

Hence:

$$\Upsilon_i = \frac{\alpha_i}{\tau_i}$$

and:

$$g_T = g_L.$$

To solve for τ_i we rewrite equation (7.18) as:

$$\frac{\dot{T}_n(t)}{L_n(t)} = \sum_{i=1}^{N} \widetilde{\epsilon}_{ni} \alpha_i r_i^{\beta} \frac{L_i}{L_n} \quad n = 1, ..., N.$$

Here $\widetilde{\epsilon}_{ni}$ is as in (7.23) with $g_T = g_L$ and we have replaced $L_i(t)/L_n(t)$ with L_i/L_n since the ratio is constant. Under balanced growth:

$$\frac{\dot{T}_n(t)}{L_n(t)} = \frac{\dot{T}_n(t)}{T_n(t)} \frac{T_n(t)}{L_n(t)} = g_L \tau_n \quad n = 1, ..., N.$$

Equating the two:

$$\tau_n = \frac{1}{g_L} \sum_{i=1}^{N} \widetilde{\epsilon}_{ni} \alpha_i r_i^{\beta} \frac{L_i}{L_n} \quad n = 1, ..., N. \tag{7.27}$$

This expression gives each country's stock of knowledge per worker as a function of research done around the world, relative labor forces, and parameters of research productivity and diffusion. Note that a country has more technology per worker the bigger the stock of ideas flowing into it relative to its size.

To make this expression more parallel to that for the case of endogenous growth, (7.22), we multiply by $L_n(t)$ to get:

$$T_n(t) = \frac{1}{g_T} \sum_{i=1}^{N} \widetilde{\epsilon}_{ni} \alpha_i r_i^{\beta} L_i(t) \quad n = 1, ..., N. \tag{7.28}$$

Note that it is identical except for the absence of $T_i(t)$ in the summation on the right-hand side. The reason is that we no longer assume that research productivity increases in proportion to the stock of ideas. Otherwise, the two formulations, while providing different explanations for the world growth rate, deliver a similar explanation for differences in living standards: Rich countries are the ones that absorb ideas faster. With home bias in diffusion, countries that create more ideas, whether it's because they are more research productive, more research intensive, or simply larger, are richer.

We now turn to how market forces determine research intensity, the r_i's. Continuing with Bertrand competition, from (7.14) and (7.15), the value of an idea in each country i is:

$$V_i = \frac{1}{\theta}\sum_{n=1}^{N} K_{ni}\frac{w_n(1-r_n)}{\tau_n} \quad i = 1, ..., N$$

where K_{ni} remains as in (7.24), setting $g_T = g_L$.

Incorporating this expression into condition (7.6) for labor-market equilibrium gives, for $\beta < 1$, at an interior solution for country i:

$$r_i^{1-\beta} = \frac{\alpha_i\beta}{\theta}\sum_{n=1}^{N} K_{ni}\frac{(1-r_n)}{\tau_n}\left(\frac{\tau_n L_n}{\tau_i L_i}\right)^{1/\theta} \quad i = 1, ..., N, \tag{7.29}$$

where we have used the fact that:

$$\frac{w_n}{w_i} = \left(\frac{T_n}{T_i}\right)^{1/\theta} = \left(\frac{\tau_n L_n}{\tau_i L_i}\right)^{1/\theta}, \quad i, n = 1, ..., N.$$

Together (7.27) and (**??**) determine relative knowledge stocks per worker and research activity around the world as functions of research productivity, labor forces, diffusion parameters, and the parameters β, g_L, and θ.

Again, substituting T for τL creates *deja vu,* an expression very close to the one we obtained for endogenous growth (7.25):

$$r_i^{1-\beta} = \frac{\alpha_i\beta}{\theta}\sum_{n=1}^{N} K_{ni}\frac{(1-r_n)L_n(t)}{T_n(t)}\left(\frac{T_n(t)}{T_i(t)}\right)^{1/\theta} \quad i = 1, ..., N.$$

The only difference is the exponent on $T_i(t)$, which is $-1/\theta$ rather than $1 - 1/\theta$. Since the specification of the research production function does not entail any technology spillovers,.a higher stock of technology $T_i(t)$ now has only a negative effect on country i's research effort through its wage. Otherwise, even though semi-endogenous and endogenous growth identify very different forces determining the growth rate, the determinants of research specialization are the same. Research specialization is driven by research productivity and access to world markets for technology.

A

7.3 Conclusion

The endogenous and semi-endogenous growth specifications each provide an elegant closure to the model of world technology accumulation. With technology diffusion both can deliver balanced growth with each country growing at the same rate. They tie the world growth rate of ideas to very different factors, however. Under endogenous growth it depends on a complex interaction of parameters of preferences and technology around the world, as well as the scale of the world economy. Under semi-endogenous growth it depends only on the growth rate of the labor force. We know of no definitive case for either, and a more convincing mechanism may yet emerge.

While we make no strong case for one over the other, it is comforting that, combined with a mechanism for the international flow of ideas, the two deliver very similar explanations for differences in living standards across countries, and for specialization in research. Countries quick to adopt new ideas, whether or not their own, are richer. The share of resources a country devotes to research reflects its own research productivity as well as the strength of its technological links to other countries. If ideas are most quickly adopted at home, greater research output is reflected in higher living standards.

Gandolfo, Giancarlo (1996) *Economic Dynamics.* Berlin: Springer.

Grossman, Gene M. and Elhanan Helpman (1991), *Innovation and Growth in the Global Economy.* Cambridge, MA: MIT Press.

Sydsæter, Knut, Arne Strøm, and Peter Berck (1999), *Economists' Mathematical Manual.* Berlin: Springer.

TABLE 1: Country Coverage (first of three panels)

#	Country	Code	Data Availability 1970-72	1986	1995-97
1	AFGHANISTAN	AFG		+	+
2	ALBANIA	ALB		+	+
3	ALGERIA	ALG	+	+	+
4	ANGOLA	ANG		+	+
5	ARGENTINA	ARG	+	+	+
6	AUSTRALIA	AUL	+	+	+
7	AUSTRIA	AUT	+	+	+
8	BAHRAIN	BAH			+
9	BANGLADESH	BAN	+	+	+
10	BARBADOS	BAR	+		+
11	BELIZE	BEZ			+
12	BELGIUM-LUXEMBOURG	BEL	+	+	
13	BENIN	BEN	+	+	+
14	BHUTAN	BHU			+
15	BOLIVIA	BOL	+	+	+
16	BRAZIL	BRA	+	+	+
17	BULGARIA	BUL		+	+
18	BURKINA FASO	BUK		+	+
19	BURUNDI	BUR		+	+
20	CAMBODIA	CAB			+
21	CAMEROON	CAM	+	+	+
22	CANADA	CAN	+	+	+
23	CENTRAL AFRICAN REPUBLIC	CEN		+	+
24	CHAD	CHA		+	+
25	CHILE	CHI	+	+	+
26	CHINA	CHN	+	+	+
27	COLOMBIA	COL	+	+	+
28	COMOROS	COM			+
29	COSTA RICA	COS	+	+	+
30	COTE D'IVOIRE	COT	+	+	+
31	CUBA	CUB		+	
32	CYPRUS	CYP	+		+
33	CZECHOSLOVAKIA(FORMER)	CZE	+	+	
34	DENMARK	DEN	+	+	+
35	DJIBOUTI	DJI			+
36	DOMINICAN REPUBLIC	DOM	+	+	+
37	ECUADOR	ECU	+	+	+
38	EGYPT	EGY	+	+	+
39	EL SALVADOR	ELS	+	+	+
40	ETHIOPIA	ETH	+	+	
41	FIJI	FIJ	+		+
42	FINLAND	FIN	+	+	+
43	FRANCE	FRA	+	+	+
44	GABON	GAB	+		+
45	GERMANY(EAST)	GEE		+	
46	GERMANY(WEST)	GER	+	+	+
47	GHANA	GHA	+	+	+
48	GREECE	GRE	+	+	+

TABLE 1: Country Coverage (second of three panels)

#	Country	Code	Data Availability 1970-72	1986	1995-97
49	GUATEMALA	GUA	+	+	+
50	GUINEA-BISSAU	GBI	+		+
51	HONDURAS	HON	+	+	+
52	HONG KONG	HOK		+	
53	HUNGARY	HUN	+	+	+
54	ICELAND	ICE	+		+
55	INDIA	IND	+	+	+
56	INDONESIA	INO	+	+	+
57	IRAN	IRN	+	+	+
58	IRAQ	IRQ	+	+	
59	IRELAND	IRE	+	+	+
60	ISRAEL	ISR	+	+	+
61	ITALY	ITA	+	+	+
62	JAMAICA	JAM	+	+	+
63	JAPAN	JAP	+	+	+
64	JORDAN	JOR	+	+	+
65	KENYA	KEN	+	+	+
66	KOREA(SOUTH)	KOR	+	+	+
67	KUWAIT	KUW	+	+	+
68	LAOS	LAO			+
69	LEBANON	LEB	+		+
70	LIBERIA	LIB		+	
71	LIBYA	LIY	+	+	
72	MADAGASCAR	MAD	+	+	+
73	MALAWI	MAW	+	+	+
74	MALAYSIA	MAY	+	+	+
75	MALI	MAL		+	+
76	MALTA	MAT	+		+
77	MAURITANIA	MAU		+	+
78	MAURITIUS	MAS	+	+	+
79	MEXICO	MEX	+	+	
80	MONGOLIA	MON			+
81	MOROCCO	MOR	+	+	+
82	MOZAMBIQUE	MOZ	+	+	+
83	NEPAL	NEP	+	+	+
84	NETHERLANDS	NET	+	+	+
85	NEW ZEALAND	NZE	+	+	+
86	NICARAGUA	NIC	+	+	+
87	NIGER	NIG		+	+
88	NIGERIA	NIA	+	+	+
89	NORWAY	NOR	+	+	+
90	OMAN	OMA		+	+
91	PAKISTAN	PAK	+	+	+
92	PANAMA	PAN	+	+	+
93	PAPUA NEW GUINEA	PAP	+	+	+
94	PARAGUAY	PAR	+	+	+
95	PERU	PER	+	+	+
96	PHILIPPINES	PHI	+	+	+

TABLE 1: Country Coverage (third of three panels)

#	Country	Code	Data Availability 1970-72	1986	1995-97
97	POLAND	POL	+		+
98	PORTUGAL	POR	+	+	+
99	ROMANIA	ROM		+	+
100	RWANDA	RWA		+	+
101	SAUDI ARABIA	SAU	+	+	+
102	SENEGAL	SEN	+	+	+
103	SEYCHELLES	SEY			+
104	SIERRA LEONE	SIE	+	+	+
105	SINGAPORE	SIN	+	+	
106	SOMALIA	SOM	+	+	
107	SOUTH AFRICA	SOU	+	+	+
108	SPAIN	SPA	+	+	+
109	SRI LANKA	SRI	+	+	+
110	SUDAN	SUD	+	+	+
111	SWEDEN	SWE	+	+	+
112	SWITZERLAND	SWI		+	+
113	SYRIAN ARAB REPUBLIC	SYR	+	+	+
114	TAIWAN	TAI		+	+
115	TANZANIA	TAN		+	
116	THAILAND	THA	+	+	+
117	TOGO	TOG	+	+	+
118	TRINIDAD AND TOBAGO	TRI	+	+	+
119	TUNISIA	TUN	+	+	+
120	TURKEY	TUR	+	+	+
121	UGANDA	UGA	+	+	+
122	UNITED KINGDOM	UNK	+	+	+
123	UNITED STATES	USA	+	+	+
124	URUGUAY	URU	+	+	+
125	USSR(FORMER)	USR		+	
126	VENEZUELA	VEN	+	+	+
127	VIETNAM	VIE		+	
128	YEMEN	YEM			+
129	YUGOSLAVIA(FORMER)	YUG		+	
130	ZAIRE	ZAI		+	
131	ZAMBIA	ZAM		+	+
132	ZIMBABWE	ZIM	+	+	+

TABLE 2

Business Sector Research Scientists
(per 1000 Industrial Workers)

COUNTRY	Scientists	Income	Population
Finland	12.2	69	5176
United States	10.2	100	275423
Japan	9.8	73	126919
Sweden	7.7	69	8871
Luxembourg	6.8	138	441
Russia	6.6	28	145555
Belgium	6.2	70	10254
Norway	6.0	90	4491
Canada	5.9	81	30750
Germany	5.5	67	82168
Singapore	5.3	80	4018
France	5.1	66	60431
Denmark	4.5	80	5338
Ireland	4.4	76	3787
Korea	4.2	42	47275
United Kingdom	4.2	68	59756
Taiwan	4.2	55	21777
Austria	3.9	70	8110
Netherlands	3.6	72	15920
Australia	2.4	76	19157
Slovenia	2.0	48	1988
Spain	1.8	53	39927
New Zealand	1.7	56	3831
Italy	1.6	64	57728
Slovak Republic	1.6	35	5401
Czech Republic	1.4	42	10272
Hungary	1.4	31	10024
Romania	1.4	14	22435
Poland	0.8	27	38646
Portugal	0.7	48	10005
China	0.7	11	1258821
Greece	0.5	44	10558
Turkey	0.2	21	66835
Mexico	0.1	27	97221

Data are for 2000 or the previous available year
Income is relative to the United States (100)
Population is in 1000's
Sources: OECD (2004) and Heston, Summers, and Aten (2002).

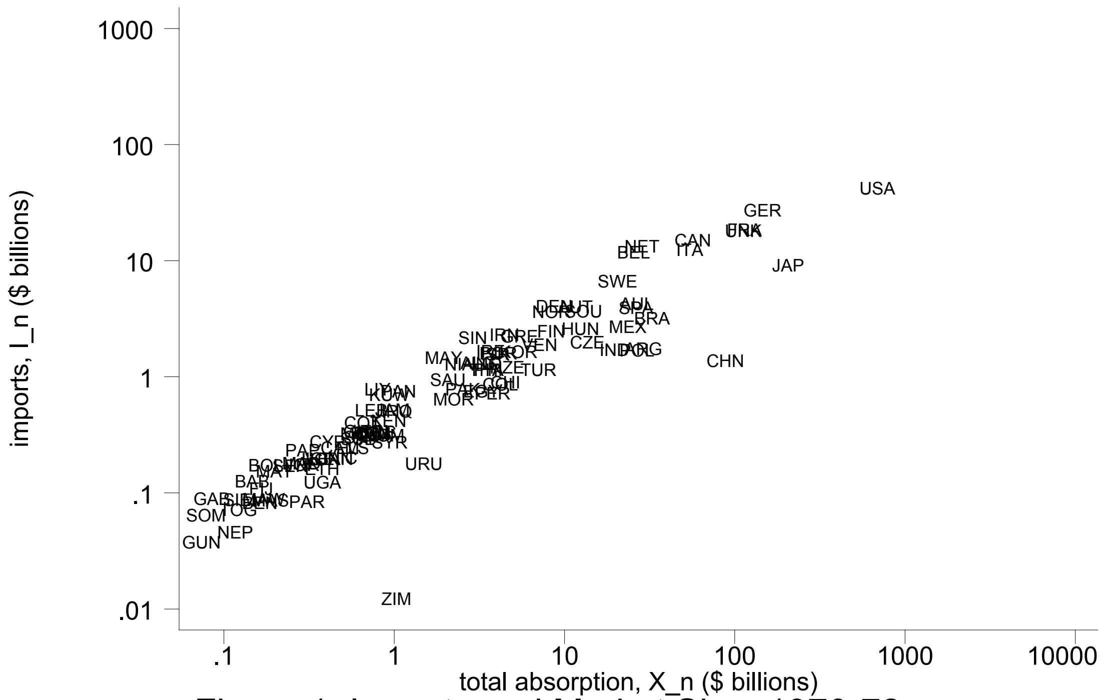

Figure 1: Imports and Market Size, 1970-72

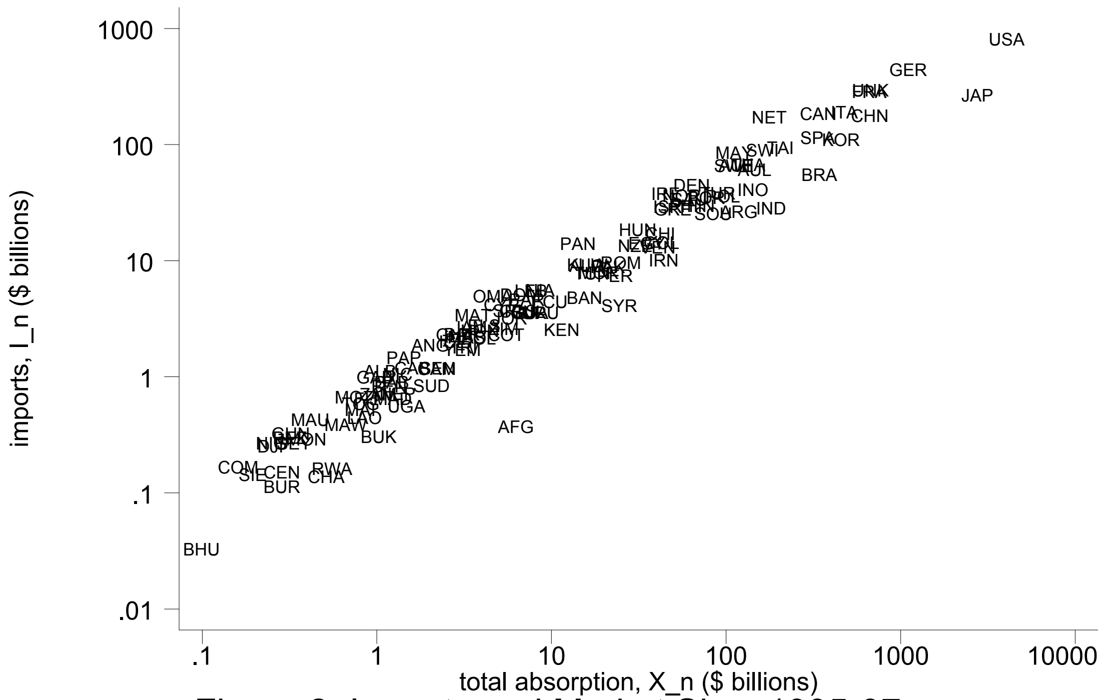

Figure 2: Imports and Market Size, 1995-97

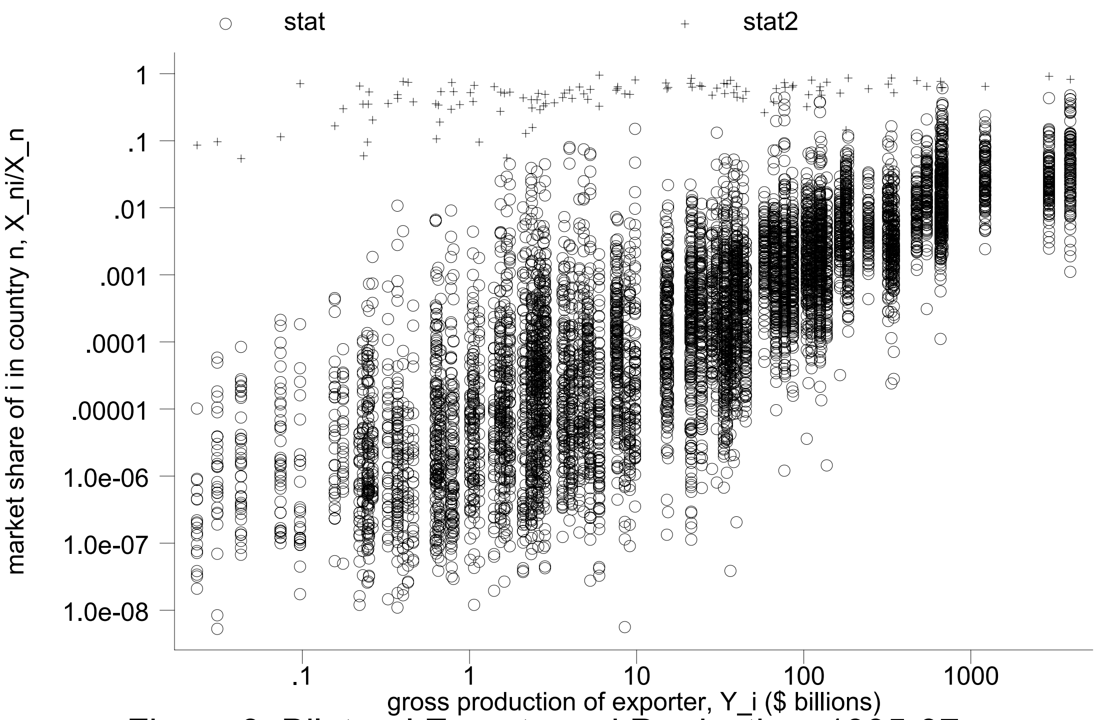

Figure 3: Bilateral Exports and Production, 1995-97

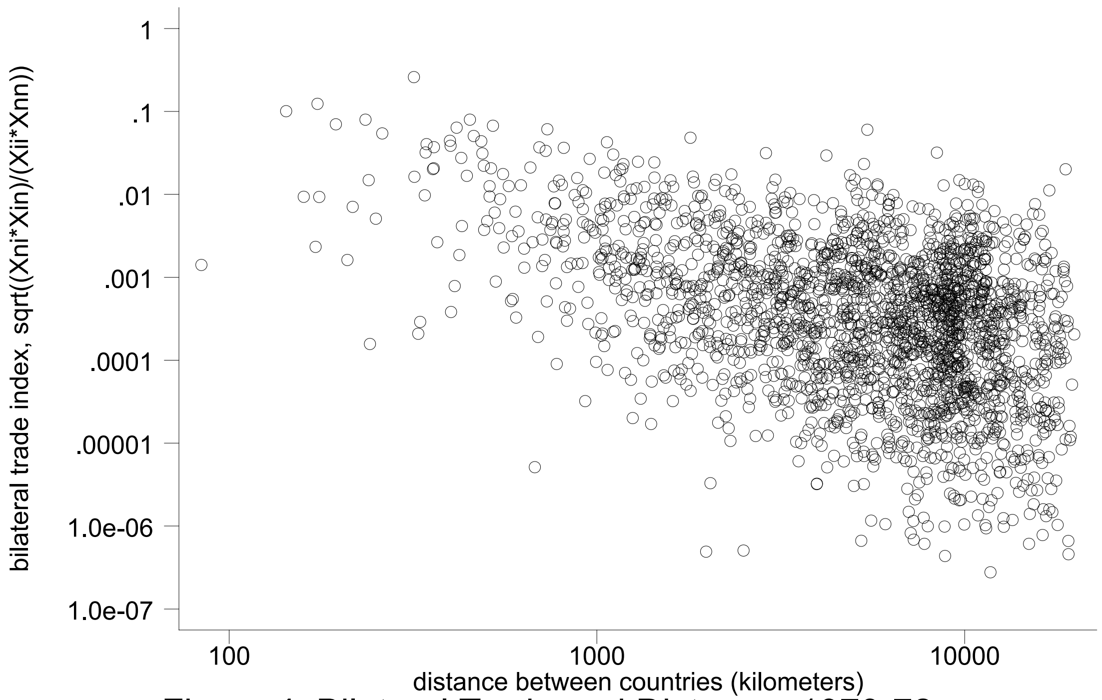

Figure 4: Bilateral Trade and Distance, 1970-72

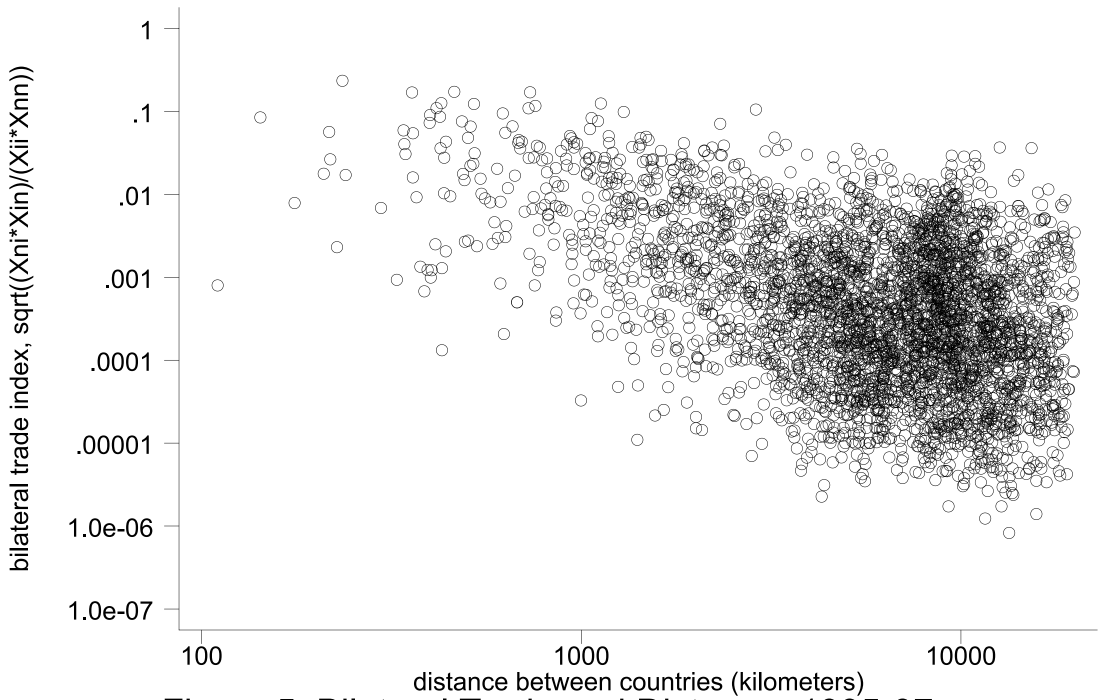

Figure 5: Bilateral Trade and Distance, 1995-97

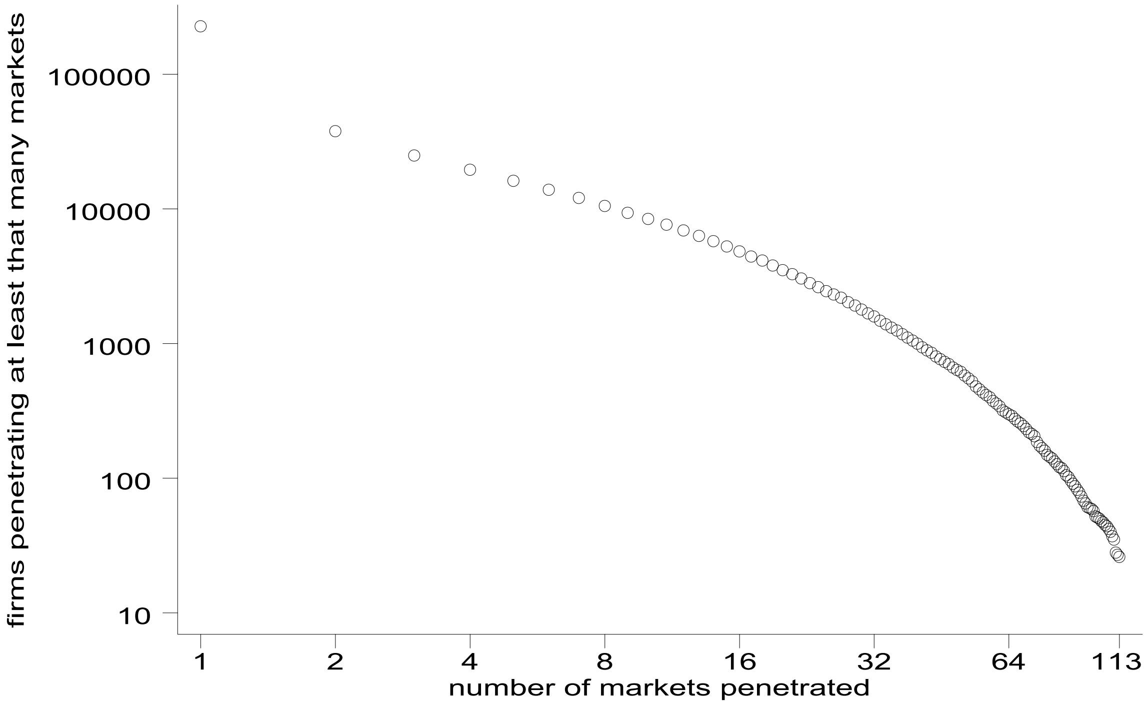

Figure 6: Frequency of Selling in Multiple Markets

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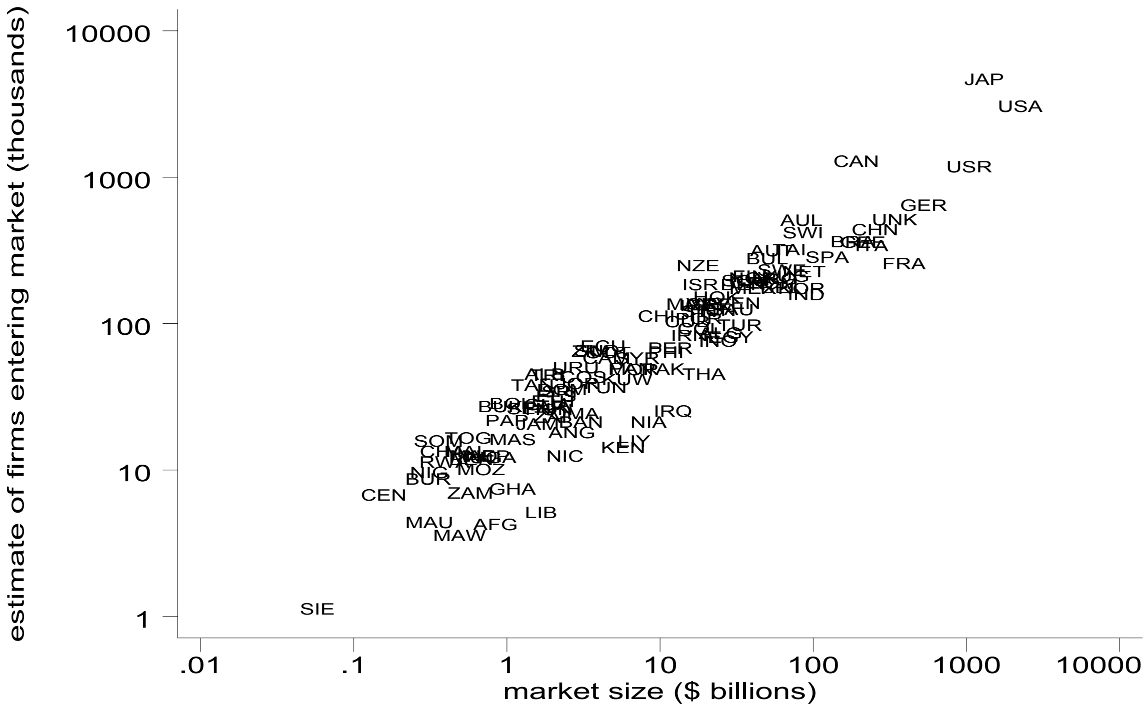


Figure 7: Entry and Market Size

Figure 8: Industry Financed Business Enterprise R&D

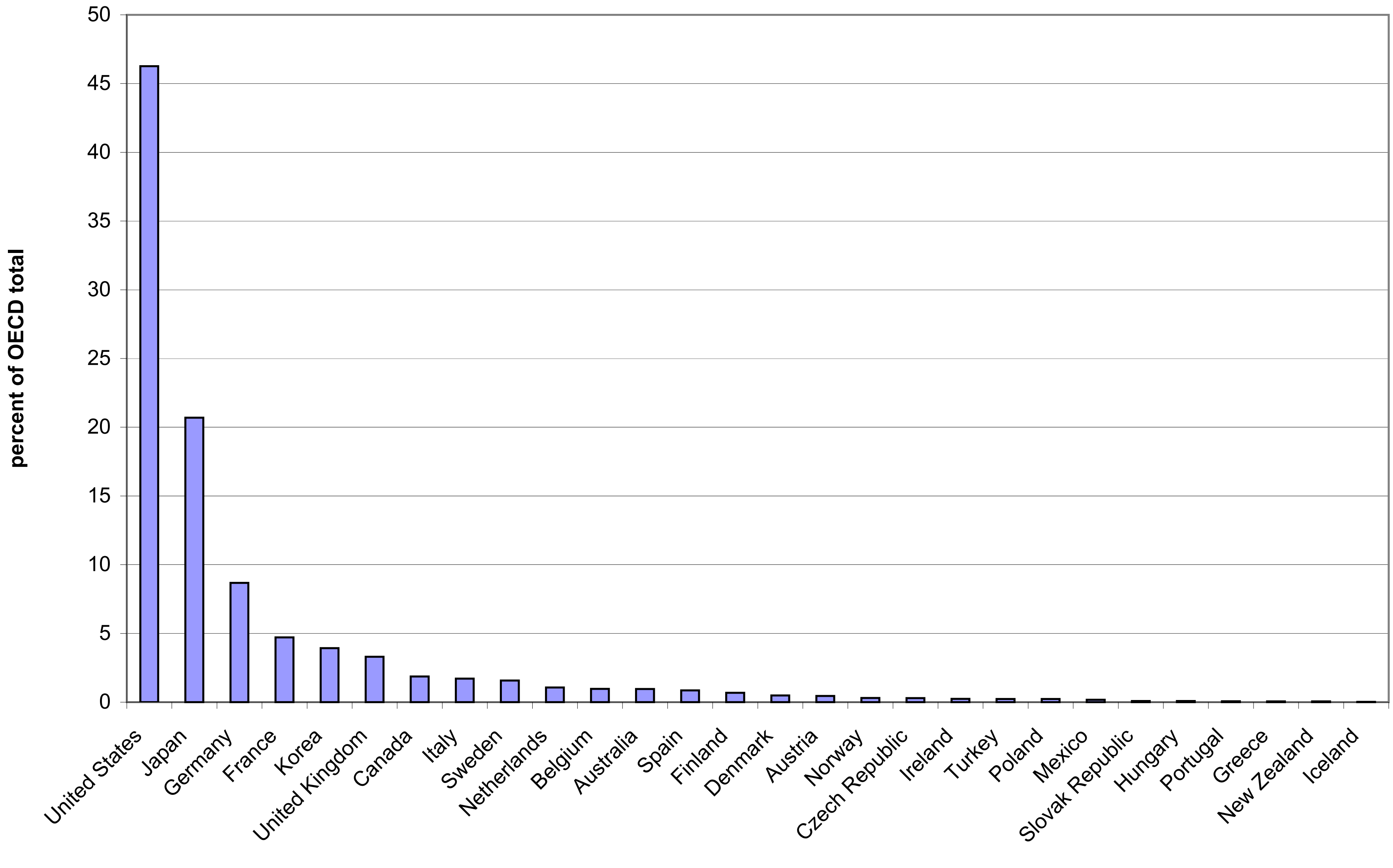


Figure 9: R&D and Patents

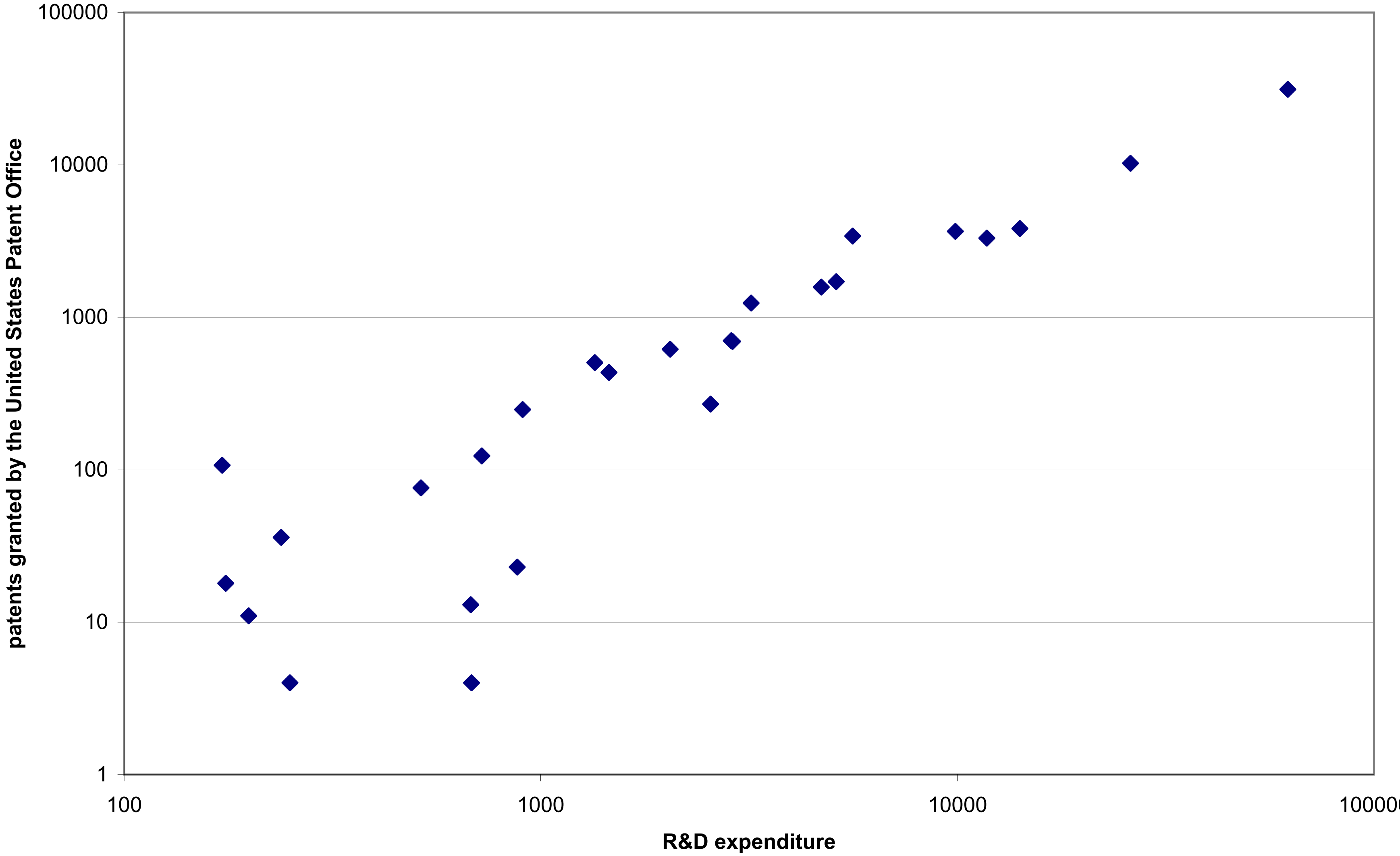
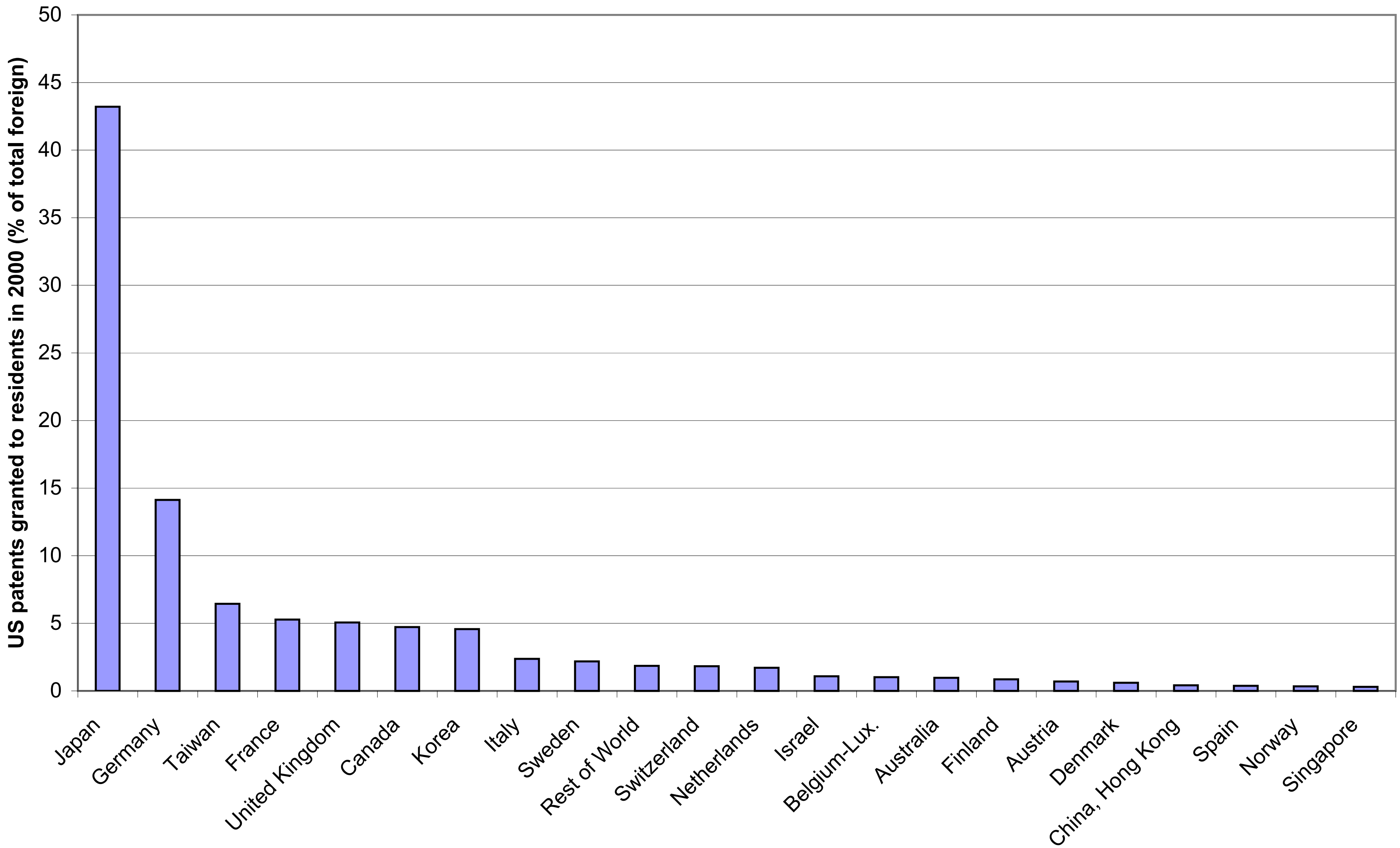


Figure 10: Foreign Patenting in the United States



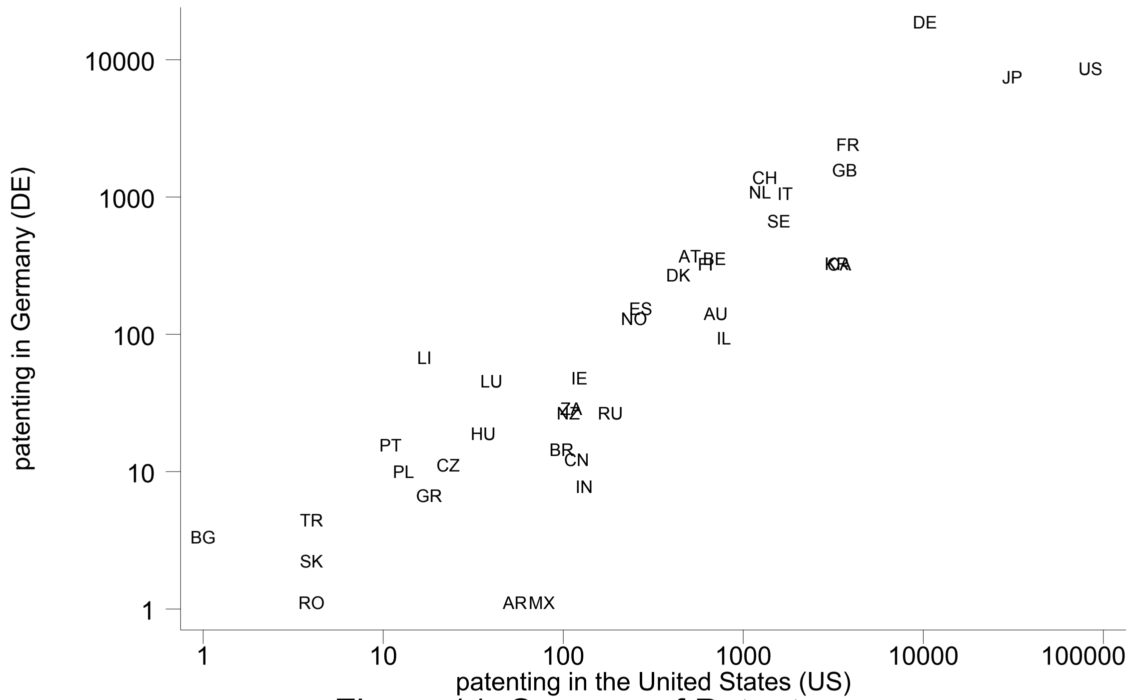


Figure 11: Sources of Patents

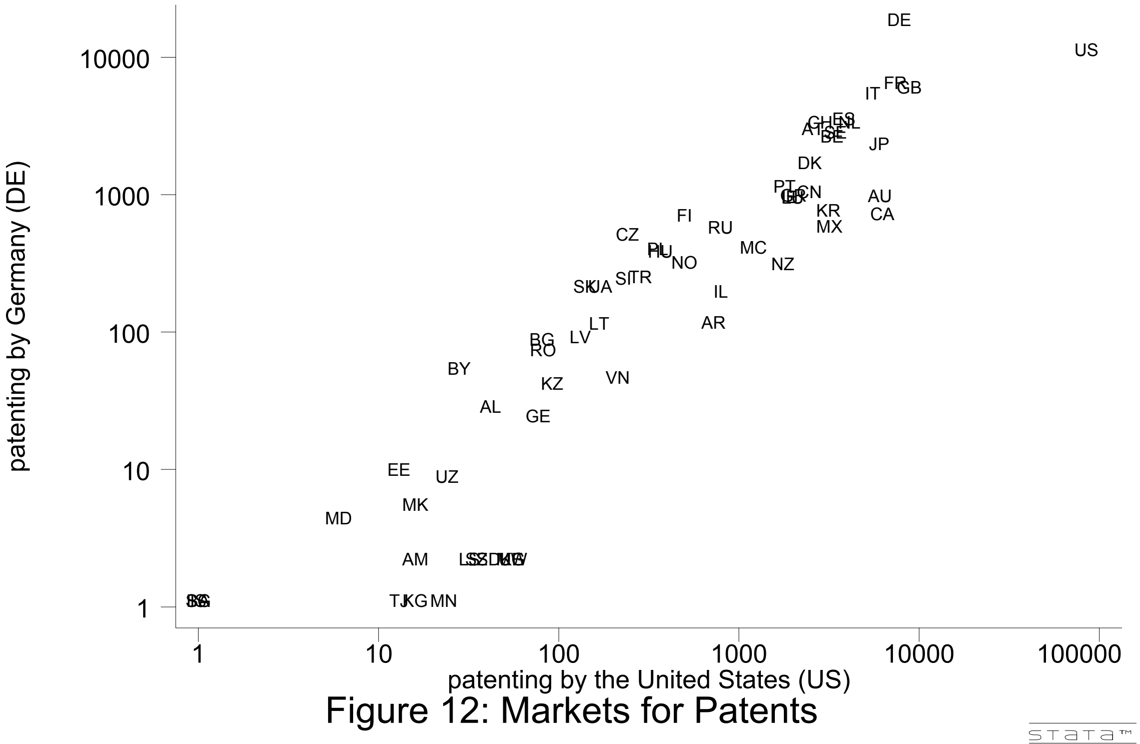


Figure 12: Markets for Patents

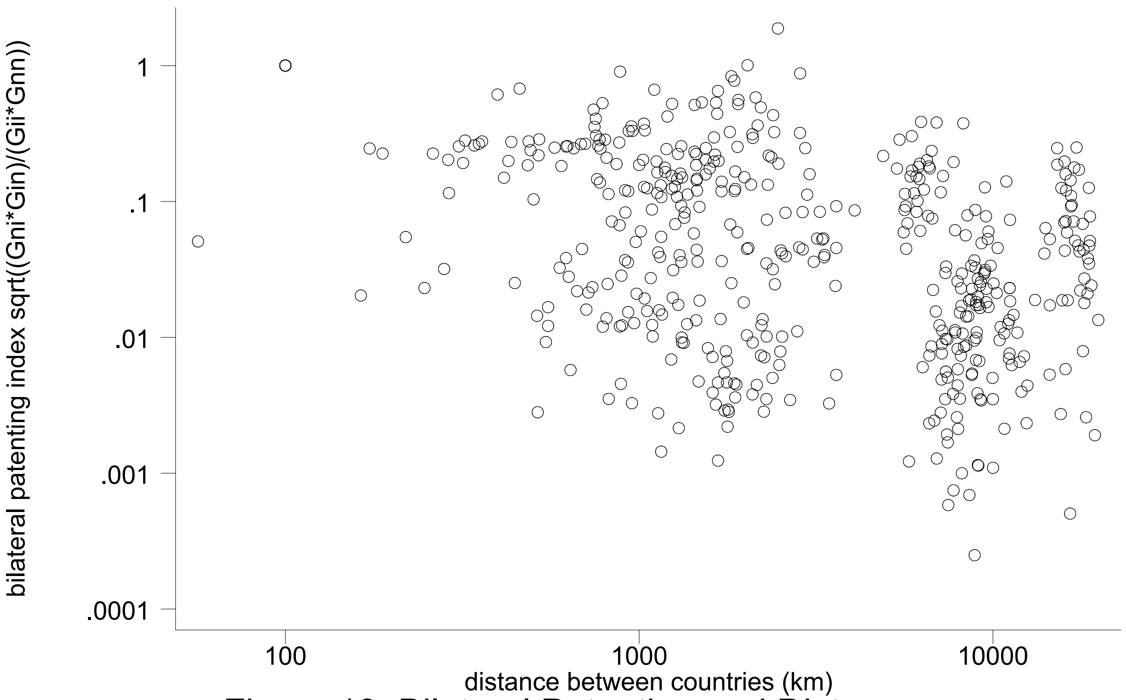


Figure 13: Bilateral Patenting and Distance

Figure 14: Evolution of Productivity in High Productivity Countries

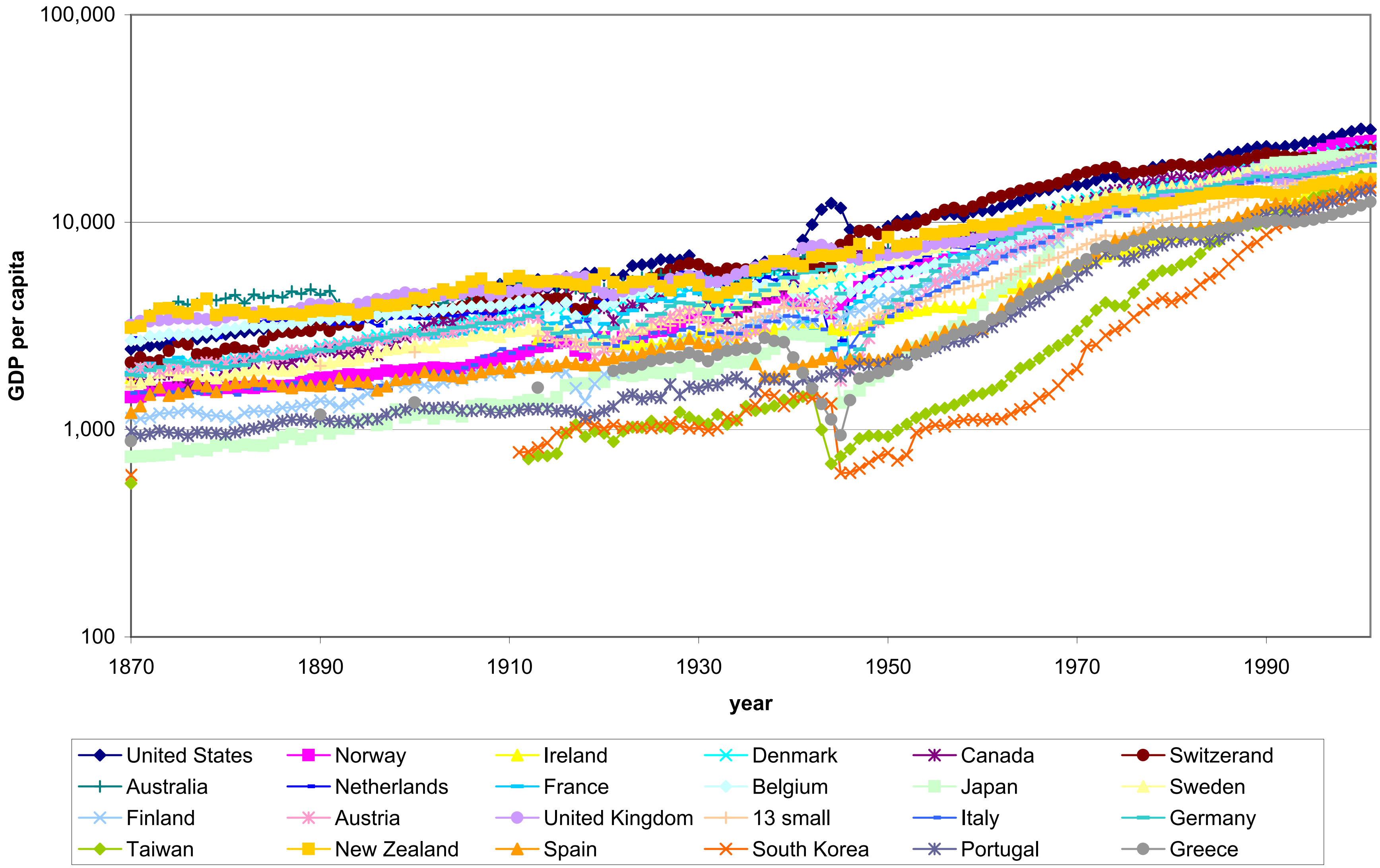


Figure 15: Evolution of Productivity in Low Productivity Countries