The Decision Making of Risk and Uncertainty

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1 Introduction

In the last half century or so, the expected utility model has become the work horse model that economists fall back upon when modeling the choices made by decision makers faced with uncertainty in the environment. Examples of its application range from the 'dating market' to the financial market. My five lectures will be an introduction to the expected utility model—its foundations, as well as, some of its limitations.

The origin of the expected utility model goes back to the 1730s. Daniel Bernoulli proposed this model as a way of accommodating the famous *St. Petersburg Paradox*.[1] The modern treatment of the model starts with the pioneering work of vonNeumann and Morgenstern. They provided an axiomatic foundation for this model. Their treatment assumed a situation of *risk*, that is, a situation in which the uncertainty faced by a decision maker can be captured via objective probabilities. For example, consider a decision maker who is faced with a gamble that gives her \$100 if an unbiased coin toss shows up heads and \$0 if it shows up tails. Here the decision maker may summarize the uncertain prospect that she is faced with using objective probabilities, namely, she receives \$100 with probability 0.5 and \$0 with complementary probability. As opposed to this we can imagine a situation of 'pure uncertainty.' Consider changing the gamble that the decision maker faces to the following one: She will get \$100 if a South American team wins the next soccer world cup and nothing otherwise. Is such a decision maker able to attach a unique probability to an event like a South American team winning the next world? And, if so, are the choices that

[1] Originally published in 1738 in the *Commentaries of the Imperial Academy of Science of Saint Petersburg*; translated by Sommer, Lousie (1954): "Exposition of a New Theory on the Measurement of Risk," *Econometrica* 22 (1): 22 – 36.

she makes when faced with such gambles (for instance, how much to pay for this gamble) informed by her probability assessments? Furthermore, if we as analysts were to observe her choices, will we be able to provide unambiguous answers to these questions? These are the kind of questions that the pioneering work of Leonid Savage tries to answer.[2] David Kreps calls this model "the crowning glory of decision theory."[3] This very elegant model is however technically fairly demanding. Fortunately for us, Anscombe and Aumann present a much simpler model which addresses a similar set of issues.

In these lectures, we will rigorously go over the vonNeumann-Morgenstern (NM hereafter) and Anscombe-Aumann (AA hereafter) models. Here rigorously should be interpreted to mean deriving everything from first principle. We will supplement the theoretical considerations of the expected utility model with a discussion of some of its limitations. In addition, if we have time, we will go over the Savage model without getting bogged down by its technical details.

Before getting there though, we will address something even more foundational. We will begin the course with a discussion of how decision makers are modeled in economics. In this discussion we will introduce the theory of revealed preference. Revealed preference theory, which was formally introduced by Samuelson, although its intellectual roots can be traced back to Pareto around the turn of the twentieth century, insists on building theories of individual decision making from sound empirical foundations. It prescribes that the starting point of a theory of choice should be *what* a decision maker chooses, which, at least in principle, is observable, rather than the *motivation* for *why* she chooses what she chooses, which, it is alleged, is unobservable. However, this claim that the motivation behind a decision maker's choices is unobservable to the analyst has become a contentious one. For instance, the project of neuro-economics is premised on the claim that there is something important and useful that can be learnt from understanding the mental processes behind a decision maker's choices. That is, the fundamental disagreement between the revealed preference approach and a field like neuro-economics comes down to what kind of data ought to be admissible in building an empirically grounded theory of choice. Whereas the revealed preference approach insists that theories of choice should be founded exclusively on choice data, its critics, like neuro-economists, advocate considering non-choice data as well. For the purpose of this workshop, the discussion of revealed preference theory shall, therefore, be useful to understand the contemporary debates that have taken place amongst economists on the question of what an appropriate theory of choice in economics should involve.

[2]Savage (1954): *The Foundations of Statistics*

[3]In my opinion Savage's work is one of the five most important theoretical contributions in what constitutes modern economic theory; the others being Arrow's Impossibility Theorem, Nash's existence result for what we today call Nash equilibrium, Arrow-Debreu-McKinzie's existence result for a general competitive equilibrium and Harsyani's analysis of games of incomplete information.

2 Objective of the Lectures

The objective of these lectures may be summarized as two-fold.

- The first goal is methodological. The lectures will attempt to provide foundations for concepts that you may be already familiar with. Given that almost all of modern economic analysis starts with an investigation of the choices that decision makers like consumers, firms etc. make in their given environments, it is critical that you have a clear grasp of the foundations of how such choices are modeled in economics.

- The second goal is normative. Here we will emphasize that the answers, which we as students of economics arrive at for the questions that interests us, critically depends on the modeling choices that we make. Therefore, the question of what a "good" model of decision making in economics *should* involve is something that we need to engage with. By elucidating the foundations of the most widely used decision model, namely expected utility, the lectures hope to provide you with the tools to engage with this normative goal. Indeed some of the most innovative work that has taken place in economics in the last two decades or so, as well as the extension of the domain of economics into fields like psychology and neuro-science, may be said to have come about as ways of engaging with this normative goal.

3 Outline of the Lectures

- How is a decision maker modeled in economics?
  - Choice functions and Houthakker's axiom.
  - Choice, preference and utility functions (representations).
  - A word about axioms
- The NM model
  - The "NM axioms" – (reduction), weak order, continuity and independence.
  - Deriving the expected utility representation from the NM axioms.
- Anomalies I
  - The Allais Paradox: Certainty effect
  - Preference for one shot resolution of uncertainty: Violation of reduction
  - Concerns for "procedures" : Violation of stochastic dominance

- Risk and Time
  - Non-negative value of information
  - Consequentialism and Dynamic Consistency
- Anomalies II
  - Dynamic Inconsistency
  - Early versus late resolution of uncertainty : Kreps-Porteus preferences
- The AA model
  - The AA axioms – weak order, continuity, independence, non-triviality and monotonicity.
  - Eliciting probabilities and deriving the expected utility representation from the AA axioms.
- The Savage model
  - The Savage axioms
  - Bayes rule
- Anomalies III
  - The Sure Thing Principle and the Ellseberg paradox
  - Ambiguity, coarse contingencies and beyond

4 References

We will draw primarily from the following sources.

- Gollier, Christian (2001): *The Economics of Risk and Time*, The MIT Press
- Kreps, David (1988): *Notes on the Theory of Choice*, Westview Press/Boulder and London
- Machina, Mark (1989): *Dynamic Consistency and Non-Expected Utility Models of Choice Under Uncertainty*, Journal of Economics Literature, Vol. XXVII (December 1989), pp. 1622 - 1668.

I will add more references to this list.