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Complementary solution to problem set 3

Using the period budget constraint we have

$$\begin{aligned}
s_0^* &= e_0 + R_0 s_{-1} - c_0^*. && (1)\\
s_1^* &= e_1 + R_1 s_0^* - c_1^* && (2)\\
&= e_1 + e_0 R_1 + R_0 R_1 s_{-1} - c_0^* R_1 - c_1^* && (3)\\
&= \frac{1}{q_1}(e_1 q_1 + e_0 + R_0 s_{-1}) - \frac{1}{q_1}(c_0^* + c_1^* q_1) && (4)\\
&= \frac{1}{q_1}(e_1 q_1 + e_0 + R_0 s_{-1} - M(\bar{c}_0 + \bar{c}_1)). && (5)\\
s_2^* &= e_2 + R_2 s_1^* - c_2^* && (6)\\
&= \frac{1}{q_2}(e_2 q_2 + e_1 q_1 + e_0 + R_0 s_{-1}) - \frac{1}{q_2}(c_0^* + c_1^* q_1 + c_2^* q_2) && (7)\\
&= \frac{1}{q_2}(e_2 q_2 + e_1 q_1 + e_0 + R_0 s_{-1} - M(\bar{c}_0 + \bar{c}_1 + \bar{c}_2)). && (8)
\end{aligned}$$

By induction one can obtain

$$s_t^* = \frac{1}{q_t}\Big(\sum_{i=0}^{t} e_i q_i + R_0 s_{-1} - M \sum_{i=0}^{t} \bar{c}_i\Big) \tag{9}$$

Notice that $\lim_{t\to\infty} q_t s_t^* = 0$, which constitutes the so-called TVC.