Advanced Macro: Lecture 3, background for page 5

$\boxed{\text{LINEARIZATION OF EQNS (1) and (2) around } k^*, c^*}$

$$u'(c_t) = \beta\, u'(c_{t+1}) \left[f'(k_{t+1}) + 1 - \delta \right] \qquad (1)$$

$$c_t = f(k_t) - k_{t+1} + (1-\delta) k_t \qquad (2)$$

GOAL: find via 1st-order approximation

$$\begin{bmatrix} c_{t+1} - c^* \\ k_{t+1} - k^* \end{bmatrix} = A \cdot \begin{bmatrix} c_t - c^* \\ k_t - k^* \end{bmatrix}$$

(1) start with eq 2:

$$\cancel{c^*} + c_t - c^* = \cancel{f(k^*) - \delta k^*} + \left(f'(k^*) + 1 - \delta \right)(k_t - k^*) - (k_{t+1} - k^*)$$

$$\Rightarrow \boxed{k_{t+1} - k^* = -(c_t - c^*) + \left[f'(k^*) + 1 - \delta \right](k_t - k^*)} \qquad (2')$$

(2)
$$\cancel{u'(c^*)} + u''(c^*)(c_t - c^*) = \cancel{\beta\, u'(c^*)\left(f'(k^*) + 1 - \delta \right)}$$
$$+ \underbrace{\beta \left(f'(k^*) + 1 - \delta \right)}_{1} u''(c^*) (c_{t+1} - c^*)$$
$$+ \beta\, u'(c^*)\, f''(k^*) (k_{t+1} - k^*)$$

$$\Rightarrow c_{t+1} - c^* = c_t - c^* - \beta \frac{u'(c^*)}{u''(c^*)} f''(k^*) (k_{t+1} - k^*)$$

use (2') to substitute out for $k_{t+1} - k^*$:

$$\boxed{c_{t+1} - c^* = \left[1 + \beta \frac{u'(c^*)}{u''(c^*)} f''(k^*) \right] (c_t - c^*) - \frac{u'(c^*)}{u''(c^*)} f''(k^*) (k_t - k^*)} \qquad (1')$$