

Advanced Macroeconomics

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Klaus Wälde (lecture) and Jean Roch Donsimoni (tutorial)

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Tutorial 1: Economic Growth

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1) Solow growth model

- a) Derive the dynamics of motion for the auxiliary variable $\tilde{k} = \frac{K}{AL}$. Discuss its meaning.
- b) Derive the long-run value of $\tilde{k}$ and discuss its meaning.
- c) Discuss the long-run growth rates of output and output per capita.

2) Optimal consumption behaviour

Compute the Keynes-Ramsey rule for the following optimal saving problem (See Ch. 5.1 in AIO [Wälde, 2010]):

$$\max_{c(\tau)} \int_t^{\infty} e^{-\rho(\tau-t)} u(c(\tau)) d\tau$$

Subject to:

$$\dot{a}(\tau) = r(\tau)a(\tau) + w(\tau) - c(\tau)$$

3) Differentiated goods

- a) What does the budget constraint mean?
- b) How do individuals behave optimally? (Using the following function, where θ denotes tastes for different varieties, and constraint, find the optimal consumption level of the individual):

$$\max_{c_j} U = c_1^\theta + c_2^\theta + \dots + c_n^\theta = \sum_{j=1}^n c_j^\theta, \quad \text{where } 0 < \theta < 1$$

Subject to:

$$\sum_{j=1}^n p_j c_j = w_j \equiv E$$

- c) How do firms behave optimally when choosing the price? What does monopolistic competition mean in this context? (*Using the following Technology and Profit functions, set up and solve the maximisation problem of the firm*):

$$\begin{aligned} x_j &= al_j - \phi, & \text{technology function} \\ \pi_j &= p(x_j)x_j - w^L l_j, & \text{profit function} \end{aligned}$$

4) Innovation and growth

- a) Compute optimal allocation of expenditure over time (i.e. $\dot{E}/E$) using the following function and the result in (3b) above for optimal consumption levels (Hint: *use the indirect utility function and Keynes-Ramsey rule*):

$$\int_t^\infty e^{-\rho(\tau-t)} \ln c(\tau) d\tau$$

- b) What is a reduced form of the equilibrium? Solve for $\dot{n}$ and $\dot{E}/E$ (*Use the following results, and standard profit and consumption maximisation functions*):

Where $\dot{n}$ is the accumulation of knowledge, L_R is the total share of the labour force engaged in R&D, and φ is a parameter:

$$\dot{n} = \varphi L_R, \quad v \geq w/\varphi, \quad v = w/\varphi$$

Where v can represent the price of a firm. When free markets have access to R&D, we have equality.

Also, technology in the economy is equal to labour directed towards building that particular good, for that firm:

$$x(i) = l(i)$$

Thus the maximum amount of labour L , is given by the sum of these two activities (R&D and production, where R&D is common for all firms):

$$L = \int_0^n l(i) di + L_R$$

Assets accumulation is given by the interest value of assets plus wage minus expenditure:

$$\dot{a} = ra + w - e, \quad r = \frac{\dot{v} + \pi}{v}$$

Firms seek to maximise their profits, given by:

$$\pi(i) = p(i)x(i) - wl(i)$$

Households seek to maximise their utility, given by:

$$\int_t^\infty e^{-\rho(\tau-t)} \ln u(c(\tau)) d\tau$$

- c) Derive the phase diagram
d) Discuss the long-run implications of this growth model