Advanced Macroeconomics

Leopold von Thadden
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Problem Set 5:
Saddlepath-stability

Problem 1: Saddlepath-stability of the dynamics of the centralized economy
Lecture 3 analyzes the dynamic sub-system in c_t and k_t s.t. $\forall t \geq 0$:

$$\beta \underbrace{\left[1 + f'(k_{t+1}) - \delta\right]}_{1+r_{t+1}} U'(c_{t+1}) = U'(c_t)$$

$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t,$$

with initial value k_0 and transversality condition $\lim_{t\to\infty} \beta^t \cdot U'(c_t) \cdot k_{t+1} = 0$.
In line with page 5, consider the first-order Taylor expansion of the system around the unique steady state values k^* and c^* such that

$$\begin{pmatrix} c_{t+1} - c^* \\ k_{t+1} - k^* \end{pmatrix} = A \cdot \begin{pmatrix} c_t - c^* \\ k_t - k^* \end{pmatrix},$$

with

$$A = \begin{bmatrix} 1 + \beta \cdot f''(k^*) \cdot \frac{U'(c^*)}{U''(c^*)} & -f''(k^*) \cdot \frac{U'(c^*)}{U''(c^*)} \\ -1 & f'(k^*) + 1 - \delta \end{bmatrix},$$

and show that the linearized dynamics are locally saddlepath-stable, satisfying the pattern $|\lambda_1| < 1$ and $|\lambda_2| > 1$, as claimed on page 14.