Advanced Macroeconomics

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Problem Set 4:
Systems of Difference Equations

Problem 1: $2 \times 2-$**Systems of first-order (linearized) difference equations**
Consider the linearized $2 \times 2-$system

$$h_{t+1} = \begin{pmatrix} h_{1,t+1} \\ h_{2,t+1} \end{pmatrix} = A \cdot \begin{pmatrix} h_{1,t} \\ h_{2,t} \end{pmatrix} = A \cdot h_t \tag{1}$$

with general solution (assuming $|\lambda_i| \neq 1$, $i = 1, 2$)

$$h_t = \begin{pmatrix} h_{1,t} \\ h_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \overline{q}_1 \cdot \mu_1 \end{pmatrix} \cdot \lambda_1^t + \begin{pmatrix} \mu_2 \\ \overline{q}_2 \cdot \mu_2 \end{pmatrix} \cdot \lambda_2^t \tag{2}$$

as derived in the Lecture Notes.

a) **Backwardlooking stability**
Let

$$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{9} \\ \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

  i) Illustrate the dynamics of (1) with a phase diagram.
  ii) Calculate the eigenvalues and eigenvectors of A.
  iii) Assume that the initial values of h_1 and h_2 in $t = 0$ are given (predetermined) by $h_{1,0} = h_{2,0} = 1$. Derive the definite solution of (2).

b) **Forwardlooking stability (one-dimensional)**
Let

$$A = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$$

  i) Illustrate the dynamics of (1) with a phase diagram.
  ii) Calculate the eigenvalues and eigenvectors of A.
  iii) Impose the terminal condition $\lim_{T \to \infty} h_{1,T} = 0$ and assume that the initial value of h_2 in $t = 0$ is given (predetermined) by $h_{2,0} = 1$. Derive the definite solution of (2).