Lecture 4
The Centralized Economy: Extensions

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I Motivation

This Lecture considers some **applications** and simple **extensions** of the basic dynamic general equilibrium model of a closed economy that was discussed in Lecture 2.

**Goal:** we will do four things

1) understand the concept of real business cycle dynamics

2) make a quick detour and look at the numerical effects of a monetary policy shock in a medium-scale DSGE model

3) allow for an endogenous choice of individual labour supply

4) reconsider the role of investment if there are costs to installing new capital (→ ‘q-theory of investment’)

→ **Summary reference:** Wickens, Chapter 2, Sections 2.5-2.7
→ **For details, see:** Romer, Chapters 4, 8
II Real-business-cycle dynamics

- Modern macroeconomics accounts for business cycles by considering systematic and typically persistent responses of dynamic macroeconomic systems to various shocks (which can be permanent or temporary, anticipated or unanticipated etc.)

- These shocks can have various origins (ie they may relate to the economy’s technology, preferences of agents, attitudes of policymakers, price- and wage setting decisions, financial sector events, home or foreign channels etc.)

- To study an empirically plausible range of diverse shocks requires a model which allows for large-scale and stochastic extensions of the basic set-up discussed so far
II Real-business-cycle dynamics

- However, the existing set-up can be used to shed light on the role of technology shocks, in line with the first vintage of DSGE-models, the so-called ‘real-business-cycle models’ (Long and Plosser, 1983) which focused exclusively on supply-side features.

- Today, there is agreement that technology shocks should be studied in combination with many other types of shocks.

- For the euro area, estimated versions of the widely used model of Smets and Wouters (2003) indicate that in the long run only about 12% of the variations of detrended output can be attributed to technology shocks. This is much less than initially conjectured by the real-business-cycle agenda.

- Still, it is instructive to understand conceptually how technology shocks operate in the basic model set up so far.
II Real-business-cycle dynamics

- To fix ideas, let us assume that a positive (negative) technology shock increases (decreases) output as well as the marginal product of capital, for any level of the predetermined capital shock.

- Example: consider a Cobb-Douglas production function

\[ y_t = f(k_t) = Z_I k_t^\alpha, \]

where \( Z_I \) measures a certain productivity level, and changes to \( Z_I \) shift both output and the marginal productivity of capital, ie \( f'(k_t) \).

- To be considered: **permanent** vs. **temporary** shocks to \( Z_I \).

- Assumption: the shock is known to everyone at the moment when it occurs.
II Real-business-cycle dynamics

Permanent technology shock:

- Assume the economy is initially (ie in the period $t = 0$) in a steady-state equilibrium with $k_i^*$ and $c_i^*$ (ie conditional on the productivity level $Z_i$)
- What happens if $Z$ changes once and for all, ie it increases from $Z_i$ to the new level $Z_{II}$?
- In the new steady state, we will have

$$ k_{II}^* > k_i^* \quad \text{and} \quad c_{II}^* > c_i^* $$
II Real-business-cycle dynamics

Permanent technology shock:

- To trace the **transitional dynamics**, consider the phase diagram developed in Lecture 2:
  - when the shock occurs in $t = 0$, $k_0 = k_I^*$ is predetermined, ie $k_0$ cannot move
  - but $c_0$ will move, ie it will jump on the stable saddlepath which ultimately converges against the new long-run values $k_{II}^*$ and $c_{II}^*$
  - on impact, $c_0$ jumps by less than the full long-run amount $(c_{II}^* - c_I^*)$ and there will be extra investment
  - beginning in $t = 1$ there will be a higher capital stock in place and the economy will move along the saddlepath in geometrically declining steps until it settles down at the new steady state
  - Thus, a permanent positive technology shock causes both **long-term** consumption and capital to increase, but in the first period - ie the **short run** - only consumption increases
II Real-business-cycle dynamics

Temporary technology shock:

- Assume the increase in $Z_I$ to $Z_{II}$ in period 0 lasts only temporarily, following, for example, a persistent process like:

$$Z_t = Z_I + \eta_t \quad \text{with: } \eta_t = \rho \eta_{t-1} + \epsilon_t \quad \text{and: } \epsilon_t \sim \text{iid } N(0, \sigma^2_\epsilon), \quad \rho \in (0, 1)$$

- The effects of such process ultimately fade away, i.e., technology will return to the initial level $Z_I$. Similarly, the long-term levels of consumption and capital will remain unchanged at $c_i^*$ and $k_i^*$.

- However, in the short run, the productivity increase acts like a windfall gain.

- This gain will be partly directly consumed ($c_0 > c_i^*$) and partly invested to facilitate some extra consumption in following periods, i.e., because of consumption smoothing $c_t$ will slowly return to $c_i^*$, typically after the technology shock has died out.
II Real-business-cycle dynamics

Temporary technology shock:

- This reasoning may be the starting point for a theory of business-cycle dynamics
- In particular, a sequence of temporary technology shocks (which can be positive or negative) triggers fluctuating patterns of consumption, investment, and output around some constant long-term values
- Empirically, such long-term values can be picked up from detrended time series
III Detour: Effects of a Monetary Policy Shock in a Medium-Scale DSGE model

- In practice, DSGE models can capture a large range of shocks.
- Assuming saddlepath-stable dynamics, such models generate uniquely determined responses of the economy to any such shock.
- A standard way to summarize such findings are impulse response functions.


Assumption: the Fed decides to raise the Fed funds rate by 1 percentage point.

Main finding: in response to such contractionary monetary policy shock, output and unemployment respond on impact relatively strongly (before the effect ultimately fades away), while the effect on prices emerges only very slowly...
III Detour: Effects of a Monetary Policy Shock in a Medium-Scale DSGE model
IV Endogenous labour supply

*To be done*
V Investment
Motivation

- The basic model focuses on the capital stock, while the treatment of investment dynamics is a bit simplistic.
- In particular, while it takes time for \( k \) to reach its optimal long-run level, the basic model assumes that in each period investment can immediately adjust to its optimal level. This feature is unrealistic.
- To correct for this feature, the so-called \textit{q-theory of investment} stresses that firms face \textit{adjustment costs} when they change the level of the capital stock.
- Adjustment costs imply that it will be optimal to change the economy’s capital stock more slowly than in the basic model discussed so far.
- Moreover, the q-theory of investment can be used to see how investment decisions depend on the expected future productivity of capital.
- In the spirit of Wickens (p. 33), one way to rationalize adjustment costs is to consider explicit \textit{installation costs of new capital}.
V Investment

Motivation - Chart 1: Capital stock in constant prices (state variable)
V Investment
Motivation - Chart 2: Market valuation of capital (forward-looking variable)
V Investment

Motivation

• For illustration, suppose that for each unit of investment the installation is subject to an additional resource cost of

$$\frac{\phi}{2} \cdot \frac{i_t}{k_t}, \quad \text{with: } \phi > 0,$$

ie installation costs depend on the level of total investment relative to the capital stock in place.

• This leads to the **modified resource constraint**:

$$f(k_t) = c_t + (1 + \frac{\phi}{2} \cdot \frac{i_t}{k_t}) \cdot i_t \quad (1)$$

• For convenience, let us make for the remainder of this Lecture the **simplifying assumption** that **capital does not depreciate** ($\delta = 0$), implying

$$i_t = k_{t+1} - k_t = \Delta k_{t+1}, \quad (2)$$

ie gross investment is identical to net investment.
**Implications** of (1) and (2), ie

\[
    f(k_t) = c_t + (1 + \phi \frac{i_t}{k_t}) \cdot i_t
\]

\[
i_t = k_{t+1} - k_t = \Delta k_{t+1}
\]

- **In the long run**, with \( k^* \) being constant, investment \( i^* \) will be zero, ie within eqn (1) there will be **no permanent loss of resources** because of installation costs

- **In the short run**, with \( k_t \neq k^* \), investment \( i_t \) will be different from 0, and adjustments are costly because of installation costs (assumed to be quadratic in \( i_t \)), ie the rate of transformation between period-\( t \) output and period-\( t + 1 \) **installed capital** is different from 1

→ **While preserving the simplicity of the functional form (1)**, this keeps the analysis close to the literature which stresses temporary adjustment costs to changes in the capital stock

→ see: Tobin (1969), Abel (1982), Hayashi (1982)
V Investment
Objective with investment subject to adjustment costs

• We ignore the labour-leisure decision discussed above
• The modified objective, addressed by the optimal solution, is to choose current and future consumption such that

\[ V_0 = \sum_{t=0}^{\infty} \beta^t U(c_t) \]  \hspace{1cm} (3)

will be maximized \( \forall t \geq 0 \) subject to the resource constraint (1), ie

\[ f(k_t) = c_t + (1 + \frac{\phi \cdot i_t}{2 \cdot k_t}) \cdot i_t \]

and the capital accumulation equation (2), ie

\[ i_t = k_{t+1} - k_t \]

• Initial condition: \( k \) is the single state variable with initial condition \( k_0 \); \( c \) and \( i \) are forwardlooking (control) variables w/o initial conditions
In order to maximize (3) s.t. (1) and (2) we optimize

\[
\mathcal{L} = \sum_{t=0}^{\infty} \left\{ \beta^t U(c_t) + \lambda_t [f(k_t) - c_t - (1 + \frac{\phi}{2} i_t) \cdot i_t] + \mu_t [i_t - k_{t+1} + k_t] \right\}
\]

over the choice variables \( \{c_t, i_t, k_{t+1}, \lambda_t \text{ and } \mu_t; \forall t \geq 0\} \)

\( \lambda_t \) is a Lagrange multiplier \( t \) periods ahead, measuring the shadow value of an additional unit of period \( t \) income (in terms of utility of period 0)

\( \mu_t \) is another Lagrange multiplier \( t \) periods ahead, measuring the shadow value of an additional unit of period \( t \) investment, ie of installed capital (in terms of utility of period 0)
V Investment

Solution based on Lagrange multipliers

Objective

\[ \mathcal{L} = \sum_{t=0}^{\infty} \left\{ \beta^t U(c_t) + \lambda_t \left[ f(k_t) - c_t - \left(1 + \frac{\phi}{2} \frac{i_t}{k_t}\right) i_t \right] \right. \\
\left. + \mu_t [i_t - k_{t+1} + k_t] \right\} \]

→ **FOCs (interior)** w.r.t. \( c_t, i_t, \) and \( k_t \):

\[ \frac{\partial \mathcal{L}_t}{\partial c_t} = \beta^t U'(c_t) - \lambda_t = 0 \quad t \geq 0 \quad (4) \]

\[ \frac{\partial \mathcal{L}_t}{\partial i_t} = -\lambda_t \left(1 + \phi \frac{i_t}{k_t}\right) + \mu_t = 0 \quad t \geq 0 \quad (5) \]

\[ \frac{\partial \mathcal{L}_t}{\partial k_t} = \lambda_t \left[f'(k_t) + \phi \cdot \left(\frac{i_t}{k_t}\right)^2 - \mu_{t-1} + \mu_t \right] = 0 \quad t > 0 \quad (6) \]

→ **FOCs (interior)** w.r.t. \( \lambda_t \) and \( \mu_t \) reproduce (1) and (2)

→ **TV-condition:** \[ \lim_{t \to \infty} \mu_t \cdot k_{t+1} = 0 \quad (7) \]
The key trade-off driving investment is given by eqn (5), i.e.

$$\lambda_t (1 + \phi \frac{i_t}{k_t}) = \mu_t \quad t \geq 0$$

Eqn (5) says that, at the margin, the utility loss from sacrificing resources to install a new unit of capital needs to be equal to the utility gain from having one extra unit of installed capital.

Let

$$q_t = \frac{\mu_t}{\lambda_t},$$

where Tobin's q measures the ratio between the market value of installed capital to its replacement cost (see Tobin, 1969).

Using this definition, eqn (5) can be rewritten to express investment as a function of the capital stock and of Tobin's q

$$i_t = \frac{1}{\phi} (q_t - 1) \cdot k_t \quad t \geq 0 \quad (8)$$
Interpretation of eqn (8), ie

\[ i_t = \frac{1}{\phi} (q_t - 1) \cdot k_t \]

- For any predetermined level of the capital stock \( k_t \), the capital stock will increase over time (ie \( i_t > 0 \)) if Tobin’s \( q \) exceeds unity.

- A situation with \( q_t > 1 \) indicates that it is valuable to invest since the market value of installed capital exceeds the replacement cost of capital.

- The speed at which changes in \( k \) take place depends on \( \phi \), ie significant installation costs (‘high value of \( \phi \)’) imply that the capital stock should change slowly over time.
V Investment

Consolidated intertemporal equilibrium conditions

- The five first-order conditions derived above - ie eqns (1), (2), and (4)-(6) - form a dynamic system in 5 variables: $c, k, i, \lambda$, and $\mu$
- Using the definition of Tobin’s $q$ (ie $q_t \equiv \frac{\mu_t}{\lambda_t}$) this system can be consolidated to a system of 4 eqns in $c, k, i$, and $q$
- This system consists of three **familiar eqns**, namely the resource constraint (1), the capital accumulation eqn (2), and the investment eqn (8), ie

$$f(k_t) = c_t + (1 + \frac{\phi}{2} \frac{i_t}{k_t}) \cdot i_t$$

$$i_t = k_{t+1} - k_t$$

$$i_t = \frac{1}{\phi} (q_t - 1) \cdot k_t$$

as well as the **modified Euler equation**

$$f'(k_{t+1}) = \frac{U'(c_t)}{\beta U'(c_{t+1})} q_t - q_{t+1} - \frac{1}{2\phi} (q_{t+1} - 1)^2,$$  \hspace{1cm} (9)

with $k_0$ given and subject to the transversality condition (7).
Background: The modified Euler equation (9) results from combining the FOCs (4)-(6):

- Consider eqn (6)

\[ \lambda_t [f'(k_t) + \frac{\phi}{2} \cdot (\frac{i_t}{k_t})^2] - \mu_{t-1} + \mu_t = 0 \quad t > 0 \]

- Update the eqn by 1 and isolate \( f'(k_{t+1}) \):

\[ f'(k_{t+1}) = -\frac{\mu_{t+1}}{\lambda_{t+1}} + \frac{\mu_t}{\lambda_{t+1}} - \frac{\phi}{2} \cdot (\frac{i_{t+1}}{k_{t+1}})^2 \quad t \geq 0 \]

- Use eqn (5), ie \( \mu_t = \lambda_t (1 + \phi \frac{i_t}{k_t}) \), to substitute out for the second term

\[ f'(k_{t+1}) = -\frac{\mu_{t+1}}{\lambda_{t+1}} + \frac{\lambda_t}{\lambda_{t+1}} (1 + \phi \frac{i_t}{k_t}) - \frac{\phi}{2} \cdot (\frac{i_{t+1}}{k_{t+1}})^2 \]

- Use the definition of \( q_t = \frac{\mu_t}{\lambda_t} = 1 + \phi \frac{i_t}{k_t} \) as well as \( \frac{\lambda_t}{\lambda_{t+1}} = \frac{\beta U'(c_t)}{U'(c_{t+1})} \) to establish the modified Euler equation (9), ie

\[ f'(k_{t+1}) = -q_{t+1} + \frac{U'(c_t)}{\beta U'(c_{t+1})} q_t - \frac{1}{2\phi} (q_{t+1} - 1)^2 \]
In steady state: $\Delta k_t = \Delta c_t = \Delta i_t = \Delta q_t = 0$

The system of consolidated equilibrium conditions admits a unique steady state.

Moreover, the system has a structure which allows for a recursive solution of all steady state values:

→ the capital accumulation eqn (2) implies $i^* = 0$
→ the investment eqn (8) implies $q^* = 1$
→ the modified Euler eqn (9) implicitly defines $k^*$ via the expression

$$f'(k^*) = \frac{1}{\beta} - 1 = \theta$$

→ finally, the resource constraint (1) implies $c^* = f(k^*)$

Notice: When interpreting these values recall that we assumed $\delta = 0$
V Investment
General equilibrium dynamics

- To analyze the general equilibrium dynamics of the four eqns (1), (2), (8) and (9), ie

\[
\begin{align*}
f(k_t) &= c_t + (1 + \frac{\phi_i t}{2 k_t}) \cdot i_t \\
i_t &= k_{t+1} - k_t \\
i_t &= \frac{1}{\phi} (q_t - 1) \cdot k_t \\
f'(k_{t+1}) &= \frac{U'(c_t)}{\beta U'(c_{t+1})} q_t - q_{t+1} - \frac{1}{2\phi} (q_{t+1} - 1)^2
\end{align*}
\]

is more involved...

- ...in general, it can be done if one uses eqns (2) and (8) to substitute out for \(i_t\) and \(q_t\) (and \(q_{t+1}\)) in (1) and (9), leading to a system in \(c\) and \(k\), with \(k_0\) given and subject to the TV-condition (7)

- ...rather than to study these general equilibrium dynamics, we will do something simpler and more instructive, namely we will study the relationship between \(q\) and \(k\) from a partial equilibrium perspective, holding \(c\) constant at its steady state value \(c^*\).
Partial equilibrium dynamics in $q$ and $k$

To analyze the dynamic relationship between $q$ and $k$ from a partial equilibrium perspective, we will consider two equations:

- First, we combine the accumulation eqn (2) and the investment equation (8), implying
  \[
  \frac{1}{\phi}(q_t - 1) \cdot k_t = k_{t+1} - k_t = \Delta k_{t+1}
  \]  
  (10)

- Second, we consider the modified Euler eqn (9), with $c$ assumed to be constant
  \[
  f'(k_{t+1}) = \frac{1}{\beta}q_t - q_{t+1} - \frac{1}{2\phi}(q_{t+1} - 1)^2
  \]  
  (11)

**Comment:** The assumption of $c$ being constant isolates the investment problem faced by the (representative) firm
Two things to be done with the two eqns (10) and (11), ie

\[
\frac{1}{\phi}(q_t - 1) \cdot k_t = k_{t+1} - k_t = \Delta k_{t+1}
\]

\[
f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1} - \frac{1}{2\phi}(q_{t+1} - 1)^2
\]

1) We will consider a **phase diagram** to study the dynamic **interaction** between the **quantity of capital** and its **shadow price**, ie \( k \) and \( q \)

(\( \rightarrow \) **problem:** eqn (11) needs first to be linearized)

2) The linearized version of eqn (11) can be used to establish an **alternative** and intuitive **interpretation of Tobin’s q**
V Investment
Partial equilibrium dynamics in q and k

Linearization of eqn (11), ie

\[ f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1} - \frac{1}{2\phi} (q_{t+1} - 1)^2 \]

- Consider the RHS of eqn (11). Replace the terms containing \( q_{t+1} \), ie

\[ q_{t+1} + \frac{1}{2\phi} (q_{t+1} - 1)^2 \]

against a first-order Taylor approximation around the steady-state value \( q = 1 \), ie

\[ q + (q_{t+1} - q) + \frac{1}{2\phi} (q - 1)^2 + \frac{1}{\phi} (q - 1) (q_{t+1} - q) = q_{t+1} \]

= 0

Thus, the linearized version of eqn (11) reduces to

\[ f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1} \]  \( (12) \)
V Investment
Partial equilibrium dynamics in \( q \) and \( k \)

Eqn (12), ie

\[
f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1}
\]

can be solved forward to express \( q_0 \) as follows

\[
q_0 = \beta f'(k_1) + \beta q_1
\]
\[
= \beta f'(k_1) + \beta^2 f'(k_2) + \beta^2 q_2
\]
\[
= \beta f'(k_1) + \beta^2 f'(k_2) + \beta^3 f'(k_3) + \beta^3 q_3
\]

or, equivalently,

\[
q_0 = \sum_{t=1}^{\infty} \beta^t f'(k_t) + \lim_{t \to \infty} \beta^t q_t
\]  \hspace{2cm} (13)
V Investment
Partial equilibrium dynamics in $q$ and $k$

**Interpretation of Tobin’s $q$ via eqn (13), ie**

$$1 + \phi \frac{i_0}{k_0} = q_0 = \sum_{t=1}^{\infty} \beta^t f'(k_t) + \lim_{t \to \infty} \beta^t q_t$$

- The LHS measures the resource cost to install one additional unit of capital in the initial period 0.
- The RHS measures the marginal contribution of this additional unit of installed capital to future output, ie the discounted sum of all future marginal products of capital.

*Notice:* The term $\beta^t$ used for discounting can be linked to the real interest rate, ie $\beta^t = \left( \frac{1}{1+r} \right)^t$, since $c = c^*$ in the consumption Euler eqn.

- In equilibrium these two measures need to be identical, because otherwise there would be unexploited arbitrage opportunities.
- Hence, $\lim_{t \to \infty} \beta^t q_t = 0$ is necessary for optimality. This can also be deduced from the TV-condition (7).
V Investment
Partial equilibrium dynamics in q and k

**Background:**
Link between \( \lim_{t \to \infty} \beta^t q_t = 0 \) and TV-condition (7), ie \( \lim_{t \to \infty} \mu_t \cdot k_{t+1} = 0 \)

- Use \( \mu_t = \lambda_t q_t \) and \( \lambda_t = \beta^t U'(c_t) \). Moreover, by assumption, \( U'(c_t) = U'(c^*) \)
- Hence, we can rewrite the TV-condition (7) as

\[
\lim_{t \to \infty} \mu_t \cdot k_{t+1} = U'(c^*) \cdot \lim_{t \to \infty} \beta^t \cdot q_t \cdot k_{t+1} = 0
\]

- Recall from eqn (10):

\[
k_{t+1} = \left[ \frac{1}{\phi} (q_t - 1) + 1 \right] \cdot k_t
\]

- Assume \( \lim_{t \to \infty} \beta^t q_t > 0 \). Then \( k_{t+1} > k_t > 0 \), implying that the TV-condition will not be satisfied
- Thus, \( \lim_{t \to \infty} \beta^t q_t = 0 \) must be satisfied for the TV-condition to be satisfied
Phase diagram: linearized dynamics in $k_t$ and $q_t$

Consider eqns (10) and (12). Rewrite eqn (12) as

$$f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1} \quad \iff \quad q_{t+1} - q_t = \Delta q_{t+1} = \frac{1 - \beta}{\beta} q_t - f'(k_{t+1})$$

to obtain the pair of eqns

$$\Delta k_{t+1} = \frac{1}{\phi} (q_t - 1) \cdot k_t$$

$$\Delta q_{t+1} = \frac{1 - \beta}{\beta} q_t - f'(k_{t+1})$$
V Investment
Partial equilibrium dynamics in $q$ and $k$

- Consider eqns (10) and (12), ie

\[
\Delta k_{t+1} = \frac{1}{\phi} (q_t - 1) \cdot k_t
\]

\[
\Delta q_{t+1} = \frac{1 - \beta}{\beta} q_t - f'(k_{t+1})
\]

- Notice that if $q_t = q^* = 1$ and $k_t = k^*$ (such that $f'(k^*) = \frac{1}{\beta} - 1 = \theta$) then $\Delta k_{t+1} = \Delta q_{t+1} = 0$

- **Dynamic implication of eqn (10):** it features no dynamics in $q$, only in $k$ such that

\[
\Delta k_{t+1} \geq 0 \text{ if } q_t \geq 1
\]

- **Dynamic implication of eqn (12):** it features no dynamics in $k$, only in $q$ such that

\[
\Delta q_{t+1} \geq 0 \text{ if } q_t \geq \frac{\beta}{1 - \beta} f'(k_{t+1}),
\]

- These informations can be combined to represent the dynamics in $q_t$ and $k_t$ via a **phase diagram**
Phase diagram: linearized dynamics in $k_t$ and $q_t$

- Dynamics in $k$ and $q$ are characterized by a single state variable ($k$) with initial condition $k_0$ and a single control variable ($q$) w/o initial condition.

- Tobin’s $q$ corresponds to the price of capital. Like a stock price $q_0$ adjusts flexibly and in a forwardlooking way, reflecting changes in the valuation of capital.

- Arrows indicate regions of stability and instability around $k^* > 0$, $q^* = 1$.

- For any initial departure of the state variable such that $k_0 \neq k^*$: **Saddlepath-stable configuration**, i.e. there exists a unique choice of the control variable $q_0$ such that the economy ‘jumps’ on the saddlepath and converges over time towards the steady state $k^*$, $q^*$.
V Investment
Partial equilibrium dynamics in $q$ and $k$

Phase diagram: linearized dynamics in $k_t$ and $q_t$

- Assume $k_0 < k^*$
  Then $q_0 > q^* = 1$, ie Tobin’s $q$ indicates that it is valuable to invest since capital is scarce relative to the optimal $k^*$: the market value of an extra unit of installed capital (which captures the present value of all future returns earned by this unit) exceeds its installation costs

- Assume $k_0 > k^*$
  Then $q_0 < q^* = 1$, ie Tobin’s $q$ indicates that it is not valuable to invest, ie the capital stock should decline until $k^*$ has been reached