Lecture 4
The Centralized Economy: Extensions

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I Motivation

This Lecture considers some **applications** and simple **extensions** of the basic dynamic general equilibrium model of a closed economy that was discussed in Lecture 2

Goal: we will do four things

- **1)** understand the concept of real business cycle dynamics
- **2)** make a quick detour and look at the numerical effects of a monetary policy shock in a medium-scale DSGE model
- **3)** allow for an endogenous choice of individual labour supply
- **4)** reconsider the role of investment if there are costs to installing new capital ($\rightarrow$ 'q-theory of investment')

$\rightarrow$ *Summary reference: Wickens, Chapter 2, Sections 2.5-2.7*
$\rightarrow$ *For details, see: Romer, Chapters 4, 8*

II Real-business-cycle dynamics

- Modern macroeconomics accounts for business cycles by considering systematic and typically persistent responses of dynamic macroeconomic systems to various shocks (which can be permanent or temporary, anticipated or unanticipated etc.)
- These shocks can have various origins (ie they may relate to the economy's technology, preferences of agents, attitudes of policymakers, price- and wage setting decisions, financial sector events, home or foreign channels etc.)
- To study an empirically plausible range of diverse shocks requires a model which allows for large-scale and stochastic extensions of the basic set-up discussed so far

II Real-business-cycle dynamics

- However, the existing set-up can be used to shed light on the role of **technology shocks**, in line with the first vintage of DSGE-models, the so-called **'real-business-cycle models'** (Long and Plosser, 1983) which focused exclusively on supply-side features
- Today, there is agreement that technology shocks should be studied in combination with many other types of shocks
- For the **euro area**, estimated versions of the widely used model of Smets and Wouters (2003) indicate that in the long run only about 12% of the variations of detrended output can be attributed to technology shocks. This is much less than initially conjectured by the real-business-cycle agenda
- Still, it is instructive to understand conceptually how technology shocks operate in the basic model set up so far

II Real-business-cycle dynamics

- To fix ideas, let us assume that a positive (negative) technology shock increases (decreases) output as well as the marginal product of capital, for any level of the predetermined capital shock
- Example: consider a Cobb-Douglas production function

$$y_t = f(k_t) = Z_I k_t^{\alpha},$$

  where Z_I measures a certain productivity level, and changes to Z_I shift both output and the marginal productivity of capital, ie $f'(k_t)$
- To be considered: **permanent** vs. **temporary** shocks to Z_I
- Assumption: the shock is known to everyone at the moment when it occurs

II Real-business-cycle dynamics

Permanent technology shock:

- Assume the economy is initially (ie in the period $t = 0$) in a steady-state equilibrium with k_I^* and c_I^* (ie conditional on the productivity level Z_I)
- What happens if Z changes once and for all, ie it increases from Z_I to the new level Z_{II}?
- In the **new steady state**, we will have

$$k_{II}^* > k_I^* \quad \text{and} \quad c_{II}^* > c_I^*$$

II Real-business-cycle dynamics

Permanent technology shock:

- To trace the **transitional dynamics**, consider the phase diagram developed in Lecture 2:
  $\longrightarrow$ when the shock occurs in $t = 0$, $k_0 = k_I^*$ is predetermined, ie k_0 cannot move
  $\longrightarrow$ but c_0 will move, ie it will jump on the stable saddlepath which ultimately converges against the new long-run values k_{II}^* and c_{II}^*
  $\longrightarrow$ on impact, c_0 jumps by less than the full long-run amount $(c_{II}^* - c_I^*)$ and there will be extra investment
  $\longrightarrow$ beginning in $t = 1$ there will be a higher capital stock in place and the economy will move along the saddlepath in geometrically declining steps until it settles down at the new steady state
  $\longrightarrow$ Thus, a permanent positive technology shock causes both **long-term** consumption and capital to increase, but in the first period - ie the **short run** - only consumption increases

II Real-business-cycle dynamics

Temporary technology shock:

- Assume the increase in Z_I to Z_{II} in period 0 lasts only temporarily, following, for example, a persistent process like:

$$Z_t = Z_I + \eta_t \quad \text{with: } \eta_t = \rho\eta_{t-1} + \varepsilon_t \quad \text{and: } \varepsilon_t \sim \text{iid } N(0, \sigma_\varepsilon^2), \quad \rho \in (0,1)$$

- The effects of such process ultimately fade away, ie technology will return to the initial level Z_I. Similarly, the long-term levels of consumption and capital will remain unchanged at c_I^* and k_I^*
- However, in the short run the productivity increase acts like a windfall gain.
- This gain will be partly directly consumed $(c_0 > c_I^*)$ and partly invested to facilitate some extra consumption in following periods, ie because of **consumption smoothing** c_t will slowly return to c_I^*, typically after the technology shock has died out

II Real-business-cycle dynamics

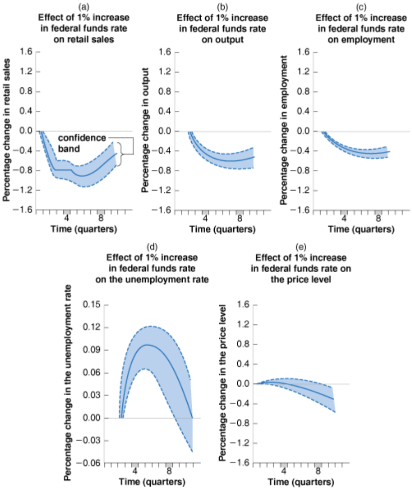
Temporary technology shock:

- This reasoning may be the starting point for a theory of business-cycle dynamics
- In particular, a sequence of temporary technology shocks (which can be positive or negative) triggers fluctuating patterns of consumption, investment, and output around some constant long-term values
- Empirically, such long-term values can be picked up from detrended time series

III Detour: Effects of a Monetary Policy Shock in a Medium-Scale DSGE model

- In practice, DSGE models can capture a large range of shocks
- Assuming saddlepath-stable dynamics, such models generate uniquely determined responses of the economy to any such shock
- A standard way to summarize such findings are impulse response functions
- *Illustration:* Effects of a monetary policy shock in a model estimated on US data (Source: Christiano, L., Eichenbaum, M., and Evans, C., The effects of monetary policy shocks: Evidence from the Flow of Funds, Review of Economics and Statistics, 78/1, February 1996.)
- *Assumption:* the Fed decides to raise the Fed funds rate by 1 percentage point.
- *Main finding:* in response to such contractionary monetary policy shock, output and unemployment respond on impact relatively strongly (before the effect ultimately fades away), while the effect on prices emerges only very slowly...

III Detour: Effects of a Monetary Policy Shock in a Medium-Scale DSGE model



IV Endogenous labour supply

To be done

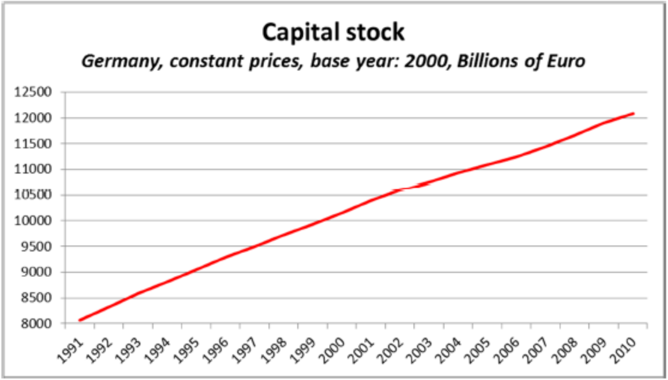
V Investment

Motivation

- The basic model focuses on the capital stock, while the treatment of investment dynamics is a bit simplistic
- In particular, while it takes time for k to reach its optimal long-run level, the basic model assumes that in each period investment can immediately adjust to its optimal level. This feature is unrealistic
- To correct for this feature, the so-called **q-theory of investment** stresses that firms face **adjustment costs** when they change the level of the capital stock
- Adjustment costs imply that it will be optimal to change the economy's capital stock more slowly than in the basic model discussed so far
- Moreover, the q-theory of investment can be used to see how investment decisions depend on the expected future productivity of capital
- In the spirit of Wickens (p. 33), one way to rationalize adjustment costs is to consider explicit **installation costs of new capital**

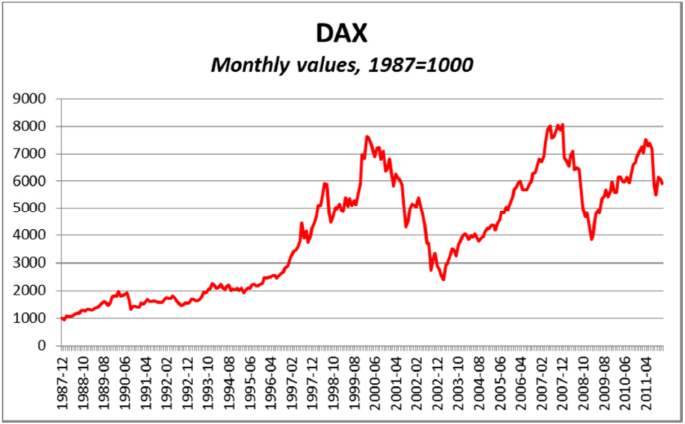
V Investment

Motivation - Chart 1: Capital stock in constant prices (state variable)



V Investment

Motivation - Chart 2: Market valuation of capital (forwardlooking variable)



V Investment
Motivation

- For illustration, suppose that for each unit of investment the installation is subject to an additional resource cost of

$$\frac{\phi}{2} \cdot \frac{i_t}{k_t}, \quad \text{with: } \phi > 0,$$

ie installation costs depend on the level of total investment relative to the capital stock in place

- This leads to the **modified resource constraint**:

$$f(k_t) = c_t + (1 + \frac{\phi}{2}\frac{i_t}{k_t}) \cdot i_t \tag{1}$$

- For convenience, let us make for the remainder of this Lecture the **simplifying assumption** that **capital does not depreciate** ($\delta = 0$), implying

$$i_t = k_{t+1} - k_t = \Delta k_{t+1}, \tag{2}$$

ie gross investment is identical to net investment

V Investment

Motivation

Implications of (1) and (2), ie

$$
\begin{aligned}
f(k_t) &= c_t + (1 + \frac{\phi}{2}\frac{i_t}{k_t}) \cdot i_t \\
i_t &= k_{t+1} - k_t = \Delta k_{t+1}
\end{aligned}
$$

- In the **long run**, with k^* being constant, investment i^* will be zero, ie within eqn (1) there will be **no permanent loss of resources** because of installation costs
- In the **short run**, with $k_t \neq k^*$, investment i_t will be different from 0, and adjustments are costly because of installation costs (assumed to be quadratic in i_t),
  ie the rate of transformation between period-t output and period-$t+1$ *installed capital* is different from 1

→ While preserving the simplicity of the functional form (1), this keeps the analysis close to the literature which stresses temporary adjustment costs to changes in the capital stock
→ see: Tobin (1969), Abel (1982), Hayashi (1982)

V Investment

Objective with investment subject to adjustment costs

- We ignore the labour-leisure decision discussed above
- The modified objective, addressed by the optimal solution, is to choose current and future **consumption** such that

$$V_0 = \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{3}$$

will be maximized $\forall t \geqslant 0$ subject to the **resource constraint** (1)**,** ie

$$f(k_t) = c_t + (1 + \frac{\phi}{2}\frac{i_t}{k_t}) \cdot i_t$$

and the **capital accumulation equation** (2), ie

$$i_t = k_{t+1} - k_t$$

- **Initial condition:** k is the single state variable with initial condition k_0; c and i are **forwardlooking (control) variables** w/o initial conditions

V Investment
Solution based on Lagrange multipliers

→ In order to maximize (3) s.t. (1) and (2) we optimize

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty}\{\beta^t U(c_t) + \lambda_t[f(k_t) - c_t - (1 + \frac{\phi}{2}\frac{i_t}{k_t}) \cdot i_t] \\ &\quad + \mu_t[i_t - k_{t+1} + k_t]\}\end{aligned}$$

over the choice variables $\{c_t,\ i_t,\ k_{t+1},\ \lambda_t$ and $\mu_t;\ \forall t \geqslant 0\}$

→ λ_t is a Lagrange multiplier t periods ahead, measuring the shadow value of an additional unit of period t **income** (in terms of utility of period 0)

→ μ_t is another Lagrange multiplier t periods ahead, measuring the shadow value of an additional unit of period t **investment**, ie of **installed capital** (in terms of utility of period 0)

V Investment

Solution based on Lagrange multipliers

Objective

$$\mathcal{L} = \sum_{t=0}^{\infty} \{\beta^t U(c_t) + \lambda_t [f(k_t) - c_t - (1 + \frac{\phi}{2}\frac{i_t}{k_t}) \cdot i_t] + \mu_t [i_t - k_{t+1} + k_t]\}$$

→ **FOCs** (interior) w.r.t. c_t, i_t, and k_t:

$$\frac{\partial \mathcal{L}_t}{\partial c_t} = \beta^t U'(c_t) - \lambda_t = 0 \qquad t \geqslant 0 \qquad (4)$$

$$\frac{\partial \mathcal{L}_t}{\partial i_t} = -\lambda_t (1 + \phi \frac{i_t}{k_t}) + \mu_t = 0 \qquad t \geqslant 0 \qquad (5)$$

$$\frac{\partial \mathcal{L}_t}{\partial k_t} = \lambda_t [f'(k_t) + \frac{\phi}{2} \cdot (\frac{i_t}{k_t})^2] - \mu_{t-1} + \mu_t = 0 \qquad t > 0 \qquad (6)$$

→ **FOCs** (interior) w.r.t. λ_t and μ_t reproduce (1) and (2)

→ **TV-condition**: $\lim_{t \to \infty} \mu_t \cdot k_{t+1} = 0$ (7)

V Investment

Tobin's q

- The key trade-off driving investment is given by eqn (5), ie

$$\lambda_t(1+\phi\frac{i_t}{k_t}) = \mu_t \qquad\qquad t \geqslant 0$$

- Eqn (5) says that, at the margin, the utility loss from sacrificing resources to install a new unit of capital needs to be equal to the utility gain from having one extra unit of installed capital
- Let

$$q_t \equiv \frac{\mu_t}{\lambda_t},$$

  where **Tobin's q** measures the **ratio between the market value of installed capital to its replacement cost** (see Tobin, 1969)
- Using this definition, eqn (5) can be rewritten to express investment as a function of the capital stock and of Tobin's q

$$i_t = \frac{1}{\phi}(q_t - 1)\cdot k_t \qquad\qquad t \geqslant 0 \qquad\qquad (8)$$

V Investment

Tobin's q

Interpretation of eqn (8), ie

$$i_t = \frac{1}{\phi}(q_t - 1) \cdot k_t$$

- For any predetermined level of the capital stock k_t, the capital stock will increase over time (ie $i_t > 0$) if Tobin's q exceeds unity
- A situation with $q_t > 1$ indicates that it is valuable to invest since the market value of installed capital exceeds the replacement cost of capital
- The speed at which changes in k take place depends on ϕ, ie significant installation costs ('high value of ϕ') imply that the capital stock should change slowly over time

V Investment

Consolidated intertemporal equilibrium conditions

- The five first-order conditions derived above - ie eqns (1), (2), and (4)-(6) - form a dynamic system in 5 variables: c, k, i, λ, and μ
- Using the definition of Tobin's q (ie $q_t \equiv \frac{\mu_t}{\lambda_t}$) this system can be consolidated to a system of 4 eqns in c, k, i, and q
- This system consists of three **familiar eqns**, namely the resource constraint (1), the capital accumulation eqn (2), and the investment eqn (8), ie

$$
\begin{aligned}
f(k_t) &= c_t + (1 + \frac{\phi}{2}\frac{i_t}{k_t}) \cdot i_t \\
i_t &= k_{t+1} - k_t \\
i_t &= \frac{1}{\phi}(q_t - 1) \cdot k_t
\end{aligned}
$$

as well as the **modified Euler equation**

$$
f'(k_{t+1}) = \frac{U'(c_t)}{\beta U'(c_{t+1})} q_t - q_{t+1} - \frac{1}{2\phi}(q_{t+1} - 1)^2, \qquad (9)
$$

with k_0 given and subject to the transversality condition (7)

V Investment

Consolidated intertemporal equilibrium conditions

Background: The modified Euler equation (9) results from combining the FOCs (4)-(6):

- Consider eqn (6)

$$\lambda_t[f'(k_t) + \frac{\phi}{2} \cdot (\frac{i_t}{k_t})^2] - \mu_{t-1} + \mu_t = 0 \quad t > 0$$

- Update the eqn by 1 and isolate $f'(k_{t+1})$:

$$f'(k_{t+1}) = -\frac{\mu_{t+1}}{\lambda_{t+1}} + \frac{\mu_t}{\lambda_{t+1}} - \frac{\phi}{2} \cdot (\frac{i_{t+1}}{k_{t+1}})^2 \quad t \geqslant 0$$

- Use eqn (5), ie $\mu_t = \lambda_t(1 + \phi\frac{i_t}{k_t})$, to substitute out for the second term

$$f'(k_{t+1}) = -\frac{\mu_{t+1}}{\lambda_{t+1}} + \frac{\lambda_t}{\lambda_{t+1}}(1 + \phi\frac{i_t}{k_t}) - \frac{\phi}{2} \cdot (\frac{i_{t+1}}{k_{t+1}})^2$$

- Use the definition of $q_t = \frac{\mu_t}{\lambda_t} = 1 + \phi\frac{i_t}{k_t}$ as well as $\frac{\lambda_t}{\lambda_{t+1}} = \frac{U'(c_t)}{\beta U'(c_{t+1})}$ to establish the modified Euler equation (9), ie

$$f'(k_{t+1}) = -q_{t+1} + \frac{U'(c_t)}{\beta U'(c_{t+1})} q_t - \frac{1}{2\phi}(q_{t+1} - 1)^2$$

V Investment

Steady-state solution

- In steady state: $\Delta k_t = \Delta c_t = \Delta i_t = \Delta q_t = 0$
- The system of consolidated equilibrium conditions admits a unique steady state
- Moreover, the system has a structure which allows for a recursive solution of all steady state values:
  $\rightarrow$ the capital accumulation eqn (2) implies $i^* = 0$
  $\rightarrow$ the investment eqn (8) implies $q^* = 1$
  $\rightarrow$ the modified Euler eqn (9) implicitly defines k^* via the expression

$$f'(k^*) = \frac{1}{\beta} - 1 = \theta$$

  $\rightarrow$ finally, the resource constraint (1) implies $c^* = f(k^*)$

Notice: When interpreting these values recall that we assumed $\delta = 0$

V Investment

General equilibrium dynamics

- To analyze the general equilibrium dynamics of the four eqns (1), (2), (8) and (9), ie

$$
\begin{aligned}
f(k_t) &= c_t + (1 + \frac{\phi}{2}\frac{i_t}{k_t}) \cdot i_t \\
i_t &= k_{t+1} - k_t \\
i_t &= \frac{1}{\phi}(q_t - 1) \cdot k_t \\
f'(k_{t+1}) &= \frac{U'(c_t)}{\beta U'(c_{t+1})} q_t - q_{t+1} - \frac{1}{2\phi}(q_{t+1} - 1)^2
\end{aligned}
$$

  is more involved...
- ...in general, it can be done if one uses eqns (2) and (8) to substitute out for i_t and q_t (and q_{t+1}) in (1) and (9), leading to a system in c and k, with k_0 given and subject to the TV-condition (7)
- ...rather than to study these general equilibrium dynamics, we will do something simpler and more instructive, namely we will study the **relationship between** q **and** k from a **partial equilibrium perspective**, holding c constant at its steady state value c^*

V Investment

Partial equilibrium dynamics in q and k

To analyze the dynamic relationship between q and k from a partial equilibrium perspective, we will consider two equations:

- First, we combine the accumulation eqn (2) and the investment equation (8), implying
$$\frac{1}{\phi}(q_t - 1) \cdot k_t = k_{t+1} - k_t = \Delta k_{t+1} \tag{10}$$
- Second, we consider the modified Euler eqn (9), with c assumed to be constant
$$f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1} - \frac{1}{2\phi}(q_{t+1} - 1)^2 \tag{11}$$

Comment: The assumption of c being constant isolates the investment problem faced by the (representative) firm

V Investment

Partial equilibrium dynamics in q and k

Two things to be done with the two eqns (10) and (11), ie

$$\begin{aligned}\frac{1}{\phi}(q_t - 1)\cdot k_t &= k_{t+1} - k_t = \Delta k_{t+1}\\ f'(k_{t+1}) &= \frac{1}{\beta}q_t - q_{t+1} - \frac{1}{2\phi}(q_{t+1}-1)^2\end{aligned}$$

1) We will consider a **phase diagram** to study the dynamic **interaction** between the **quantity of capital** and its **shadow price**, ie k and q ($\longrightarrow$ problem: eqn (11) needs first to be linearized)

2) The linearized version of eqn (11) can be used to establish an **alternative** and intuitive **interpretation of Tobin's q**

V Investment

Partial equilibrium dynamics in q and k

Linearization of eqn (11), ie

$$f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1} - \frac{1}{2\phi}(q_{t+1} - 1)^2$$

- Consider the RHS of eqn (11). Replace the terms containing q_{t+1}, ie

$$q_{t+1} + \frac{1}{2\phi}(q_{t+1} - 1)^2$$

  against a first-order Taylor approximation around the steady-state value $q = 1$, ie

$$q + (q_{t+1} - q) + \frac{1}{2\phi}(\underbrace{q-1}_{=0})^2 + \frac{1}{\phi}(\underbrace{q-1}_{=0})(q_{t+1} - q) = q_{t+1}$$

- Thus, the linearized version of eqn (11) reduces to

$$f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1} \tag{12}$$

V Investment

Partial equilibrium dynamics in q and k

Eqn (12), ie

$$f'(k_{t+1}) = \frac{1}{\beta} q_t - q_{t+1}$$

can be **solved forward** to express q_0 as follows

$$\begin{aligned}
q_0 &= \beta f'(k_1) + \beta q_1 \\
&= \beta f'(k_1) + \beta^2 f'(k_2) + \beta^2 q_2 \\
&= \beta f'(k_1) + \beta^2 f'(k_2) + \beta^3 f'(k_3) + \beta^3 q_3
\end{aligned}$$

or, equivalently,

$$q_0 = \sum_{t=1}^{\infty} \beta^t f'(k_t) + \lim_{t \to \infty} \beta^t q_t \qquad (13)$$

V Investment

Partial equilibrium dynamics in q and k

Interpretation of Tobin's q via eqn (13), ie

$$1+\phi\frac{i_0}{k_0}=q_0=\sum_{t=1}^{\infty}\beta^t f'(k_t)+\underbrace{\lim_{t\to\infty}\beta^t q_t}_{=0}$$

- The LHS measures the resource cost to install one additional unit of capital in the initial period 0
- The RHS measures the marginal contribution of this additional unit of installed capital to future output, ie the discounted sum of all future marginal products of capital
  Notice: The term β^t used for discounting can be linked to the real interest rate, ie $\beta^t=(\frac{1}{1+r})^t$, since $c=c^*$ in the consumption Euler eqn

- In equilibrium these two measures need to be identical, because otherwise there would be unexploited arbitrage opportunities
- Hence, $\lim_{t\to\infty}\beta^t q_t=0$ is necessary for optimality
  $\rightarrow$ this can also be deduced from the TV-condition (7)

V Investment

Partial equilibrium dynamics in q and k

Background:

Link between $\lim_{t\to\infty} \beta^t q_t = 0$ and TV-condition (7), ie $\lim_{t\to\infty} \mu_t \cdot k_{t+1} = 0$

- Use $\mu_t = \lambda_t q_t$ and $\lambda_t = \beta^t U'(c_t)$. Moreover, by assumption, $U'(c_t) = U'(c^*)$
- Hence, we can rewrite the TV-condition (7) as

$$\lim_{t\to\infty} \mu_t \cdot k_{t+1} = U'(c^*) \cdot \lim_{t\to\infty} \beta^t \cdot q_t \cdot k_{t+1} = 0$$

- Recall from eqn (10):

$$k_{t+1} = [\frac{1}{\phi}(q_t - 1) + 1] \cdot k_t$$

- Assume $\lim_{t\to\infty} \beta^t q_t > 0$. Then $k_{t+1} > k_t > 0$, implying that the TV-condition will not be satisfied
- Thus, $\lim_{t\to\infty} \beta^t q_t = 0$ must be satisfied for the TV-condition to be satisfied

V Investment

Partial equilibrium dynamics in q and k

Phase diagram: linearized dynamics in k_t and q_t

- Consider eqns (10) and (12). Rewrite eqn (12) as

$$f'(k_{t+1}) = \frac{1}{\beta}q_t - q_{t+1} \qquad \Longleftrightarrow \qquad q_{t+1} - q_t = \Delta q_{t+1} = \frac{1-\beta}{\beta}q_t - f'(k_{t+1})$$

to obtain the pair of eqns

$$\begin{aligned} \Delta k_{t+1} &= \frac{1}{\phi}(q_t - 1) \cdot k_t \\ \Delta q_{t+1} &= \frac{1-\beta}{\beta}q_t - f'(k_{t+1}) \end{aligned}$$

V Investment

Partial equilibrium dynamics in q and k

- Consider eqns (10) and (12), ie

$$\Delta k_{t+1} = \frac{1}{\phi}(q_t - 1)\cdot k_t$$

$$\Delta q_{t+1} = \frac{1-\beta}{\beta} q_t - f'(k_{t+1})$$

- Notice that if $q_t = q^* = 1$ and $k_t = k^*$ (such that $f'(k^*) = \frac{1}{\beta} - 1 = \theta$) then $\Delta k_{t+1} = \Delta q_{t+1} = 0$
- **Dynamic implication** of **eqn (10):** it features no dynamics in q, only in k such that

$$\Delta k_{t+1} \gtreqless 0 \text{ if } q_t \gtreqless 1$$

- **Dynamic implication** of **eqn (12):** it features no dynamics in k, only in q such that

$$\Delta q_{t+1} \gtreqless 0 \text{ if } q_t \gtreqless \frac{\beta}{1-\beta} f'(k_{t+1}),$$

- These informations can be combined to represent the dynamics in q_t and k_t via a **phase diagram**

V Investment

Partial equilibrium dynamics in q and k

Phase diagram: linearized dynamics in k_t and q_t

- Dynamics in k and q are characterized by a single state variable (k) with initial condition k_0 and a single control variable (q) w/o initial condition
- Tobin's q corresponds to the price of capital. Like a stock price q_0 adjusts flexibly and in a forwardlooking way, reflecting changes in the valuation of capital
- Arrows indicate regions of stability and instability around $k^* > 0$, $q^* = 1$
- For any initial departure of the state variable such that $k_0 \neq k^*$: **Saddlepath-stable configuration**, i.e. there exists a unique choice of the control variable q_0 such that the economy 'jumps' on the saddlepath and converges over time towards the steady state k^*, q^*

V Investment

Partial equilibrium dynamics in q and k

Phase diagram: linearized dynamics in k_t and q_t

- Assume $k_0 < k^*$
  Then $q_0 > q^* = 1$, ie Tobin's q indicates that it is valuable to invest since capital is scarce relative to the optimal k^*: the market value of an extra unit of installed capital (which captures the present value of all future returns earned by this unit) exceeds its installation costs
- Assume $k_0 > k^*$
  Then $q_0 < q^* = 1$, ie Tobin's q indicates that it is not valuable to invest, ie the capital stock should decline until k^* has been reached