Partial wave analysis of η photoproduction data with analyticity constraint

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Outline

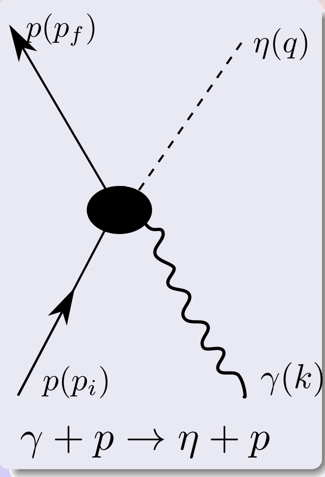
- η production generaly
  - Invariant amplitudes in η photoproduction
  - Analytic structure of invariant amplitudes
- Single energy partial wave analysis - SE PWA
- Imposing the fixed-t analyticity in PWA
- How does it work in the PWA of the η photoproduction data?
- Preliminary results

η photoproduction

p_i -four momentum of incoming nucleon
p_f - four momentum of outgoing nucleon
k - four momentum of incident photon
p - four momentum of η meson

Madelstam variable

$s = w^2 = (p_i + k)^2$
$t = (q - k)^2$
$u = (p_i - q)^2$
$\nu = \frac{s-u}{4m}$
$s + t + u = 2m^2 + m_\eta^2$;
m - mass of nucleon,
m_η - mass of eta meson



Kinematics in η photoproduction

In the $N\eta$ CMS:

$p_i^\mu = (E_i, -\vec{k})$, $p_f^\mu = (E_f, -\vec{q})$

$k^\mu = (|\vec{k}|, \vec{k})$, $q^\mu = (\omega, \vec{q})$

$|\vec{k}| = \frac{s-m^2}{2\sqrt{s}}$ incident photon momentum

$\omega = \frac{s+m_\eta^2-m^2}{2\sqrt{s}}$ - η meson energy

$|\vec{q}| = \left[\left(\frac{s-m_\eta^2+m^2}{2\sqrt{s}}\right)^2 - m^2\right]^{\frac{1}{2}}$

$E_i = \frac{s-m^2}{2\sqrt{s}}$ - energy of incident nucleon

$E_f = \frac{s+m^2+m_\eta^2}{2\sqrt{s}}$- energy of outgoing nucleon

$t = m_\eta^2 - 2|\vec{k}|(\omega - 2|\vec{q}|\cos\theta)$; θ- scattering angle in CMS

Invariant amplitudes in η photoproduction

Starting from reaction $\gamma + N \to \eta + N$ (s - channel), using crossing relation, one obtains another two channels:

$$\gamma + \eta \to N + \bar{N} \qquad \text{t-channel}$$

$$\gamma + \bar{N} \to \eta + \bar{N} \qquad \text{u-channel}$$

All three channels defined above are described by four invarint amplitudes (IA), B_1, B_2, B_6 and B_8 as defined in (I.G. Aznauryan, Phys. Rev. C 67, 015209 (2003); Phys. Rev. C 68, 065204 (2003)).

Analytic structure of invariant amplitudes

Singularities of B_i are defined from unitarity in s, t and u-channels:

- s - channel cut $(m + m_\eta)^2 \le s < \infty$

  $+$

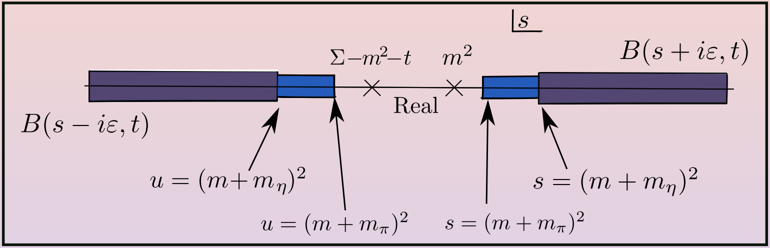
  unphysical cut $(m_\pi + m)^2 \le s \le (m_\eta + m)^2$
- u - channel cut $(m + m_\eta)^2 \le u < \infty$

  $+$

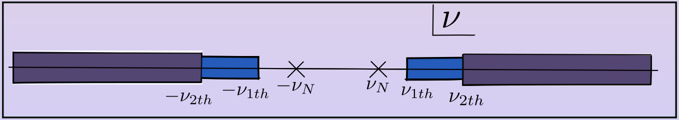
  unphysical cut $(m_\pi + m)^2 \le u \le (m_\eta + m)^2$
- t - channel cut $4m^2 \le t < \infty$

  $+$

  unphysical cut $4m_\pi^2 \le t \le 4m^2$
- nucleon pole at $s = m^2$, $u = m^2$



It is more practical to use crossing variable $\nu = \frac{s-u}{4m}$.
$s - u$ crossing implies sign change $\nu \to -\nu$



$$\nu_{1th} = m_\pi + \frac{t}{4m}, \quad \nu_{2th} = m_\eta + \frac{t}{4m}, \quad \nu_N = \frac{(t - m_\eta^2)}{4m}$$

Multipole expansion of invariant amplitudes

In partial wave analysis of η - photoproduction data it is convenient to work with CGLN amplitudes (ref: Chew, Goldberger, Low, Nambu, Phys. rev. 106 (1957),1345) having simple representation in terms of multipoles:

$$
\begin{aligned}
F_1 &= \sum_{l=0}^{\infty}[(lM_{l+} + E_{l+})P'_{l+1}(x) + ((l+1)M_{l+} + E_{l-})P'_{l-1}(x)], \\
F_2 &= \sum_{l=1}^{\infty}[(l+1)M_{l+} + lM_{l-}]P'_l(x), \\
F_3 &= \sum_{l=1}^{\infty}[(E_{l+} - M_{l+})P''_{l+1} + (E_{l-} + M_{l-})P''_{l-1}(x)], \\
F_4 &= \sum_{l=2}^{\infty}[M_{l+} - E_{l+} - M_{l-} - E_{l-}]P''_l(x),
\end{aligned}
$$

Multipole expansion of invariant amplitudes

Another set of amplitudes commonly used are helicity amplitudes. In terms of CGLN amplitudes they are given as follows:

$$
\begin{aligned}
H_1 &= -\frac{1}{\sqrt{2}} \sin\theta \cos\frac{\theta}{2}(F_3 + F_4), \\
H_2 &= \sqrt{2}\cos\frac{\theta}{2}\left[(F_2 - F_1) + \frac{1-\cos\theta}{2}(F_3 - F_4)\right], \\
H_3 &= \frac{1}{\sqrt{2}} \sin\theta \sin\frac{\theta}{2}(F_3 - F_4), \\
H_4 &= \sqrt{2}\sin\frac{\theta}{2}\left[(F_1 + F_2) + \frac{1+\cos\theta}{2}(F_3 + F_4)\right]
\end{aligned}
$$

Invariant amplitudes are given in terms of CGLN amplitudes by formula:

$$\begin{pmatrix} B_1 \\ B_2 \\ B_6 \\ B_8 \end{pmatrix} = M \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$$

where matrix M is:

$$M = \frac{1}{2W(s-m^2)} \begin{pmatrix} \frac{(s-m^2)}{a_1} & -\frac{(s-m^2)}{a_2} & 0 & 0 \\ 0 & 0 & -\frac{(t-m_\eta^2)(m-W)}{2a_3} & -\frac{(t-m_\eta^2)(m+W)}{2a_4} \\ -\frac{2(m+W)}{a_1} & \frac{2(m-W)}{a_2} & -\frac{(t-m_\eta^2)}{a_3} & -\frac{(t-m_\eta^2)}{a_4} \\ -\frac{(m+W)}{a_1} & \frac{(m-W)}{a_2} & -\frac{(s-u)}{2a_3} & -\frac{(s-u)}{2a_4} \end{pmatrix}$$

In formulas above:

$$a_1 = \frac{\sqrt{(E_1+m)(E_2+m)}}{8\pi W}$$

$$a_2 = \frac{\sqrt{(E_1-m)(E_2-m)}}{8\pi W}$$

$$a_3 = \frac{\sqrt{(E_1-m)(E_2-m)}(E_2+m)}{8\pi W} = a_2 \cdot (E_2+m)$$

$$a_4 = \frac{\sqrt{(E_1+m)(E_2+m)}(E_2-m)}{8\pi W} = a_1 \cdot (E_2-m)$$

Some authors use another set of invariant amplitudes, A_i:

$A_1 = B_1 - mB_6, \qquad A_2 = \frac{2B_2}{t-m_\eta^2}, \qquad A_3 = -B_8, \qquad A_4 = -\frac{1}{2}B_6.$

(J. S. Ball, Phys. Rev. 124, (1961), 2014)

Single energy partial wave analysis

At a given energy W minimize a quadratic form:

$$\chi^2_{data} = \sum_D \sum_{k=1}^{N_D} \left(\frac{D_k^{exp}(\theta_k) - D_k^{fit}(\theta_k)}{\Delta_{D_k}} \right)^2$$

$D_k^{exp}(\theta_k)$– values of observable D measured at angles θ_k with errors Δ_{D_k}.
$D_k^{fit}(\theta_k)$ - predictions calculated from partial waves (multipoles) which are parameters in the fit.
Serious problem in SE PWA - ambiguities, no unique solution.
How to resolve the problem? **Conditio sine qua non**: Smoothness of partial waves as function of energy.
Is it enough?

Single energy partial wave analysis

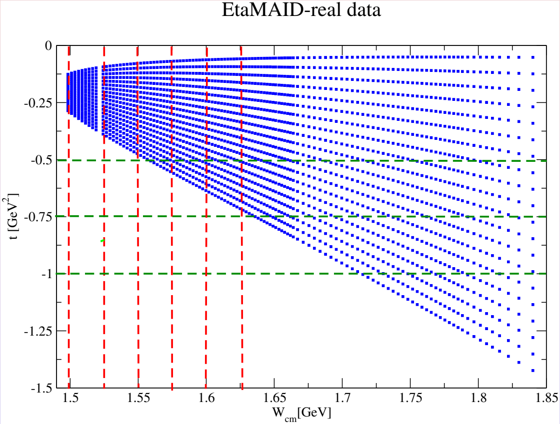
One must impose more stringent constraints taking into account analyticity of scattering amplitudes.
(J. S. Bowcock, H. Burkhardt, Rep. Prog Phys 38 (1975) 1099)
Important step forward:

- E. Pietarinen: Amplitude analysis using fixed-t analyticity of invariant amplitudes
  - E. Pietarinen, Nuovo Cim. 12 (1972) 522
  - E. Pietarinen, Nucl. Phys. B49 (1972) 315 Discussion of uniqueness problem
  - E. Pietarinen, Nucl. Phys. 8107 (1976) 21 Discussion of uniqueness problem
  - J. Hamilton, J. L. Peterson, New developments in dispersion theory, Vol.1, Nordita, 1975.

Imposing the fixed-t analyticity in PWA of scattering data

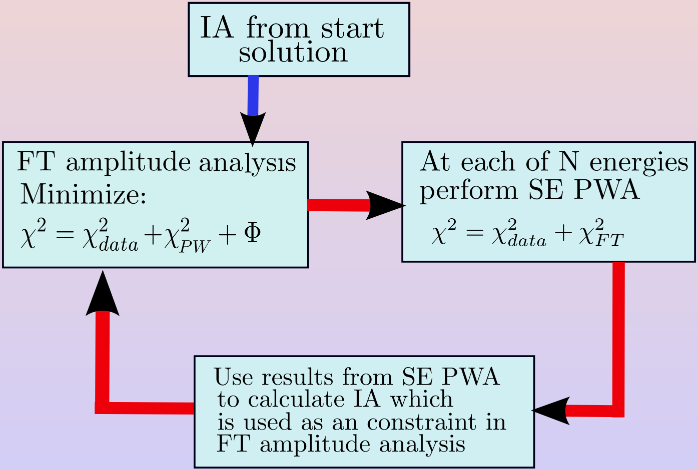
- The method consists of two separated analysis:
  - Fixed-t amplitude analysis - a method which can determine the scattering amplitudes from exp. data at fixed-t
  - Single energy partial wave analysis - SE PWA

- Fixed-t AA and SE PWA are coupled. Results from one analysis are used as constraint in another in an iterative procedure.
- Method was used in famous KH80 analysis of πN scattering data.
- In Mainz-Tuzla-Zagreb PWA of η- photoproduction data we apply the same principles.

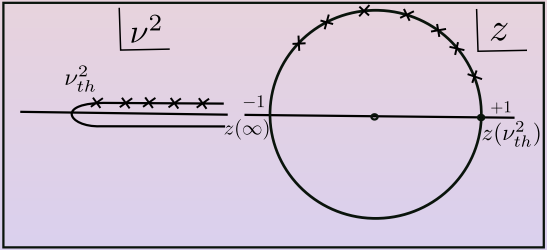


Red dashed lines-SE PWA, Green dashed lines - fixed-t amplitude analysis

Imposing the fixed-t analyticity in PWA of scattering data



The simplest case-πN elastic scattering at fixed-t.
Apart from nucleon poles, crossing symmetric invariant amplitudes are analytic function in a complex ν^2 plane $\nu_{th}^2 \leq \nu^2 < \infty$, $(\nu_{th} = m_\pi + \frac{t}{4m})$.



Conformal mapping:

$$z = \frac{\alpha - \sqrt{\nu_{th}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th}^2 - \nu^2}}$$

mapps a cut ν^2 plane inside and on the circle in a z plane.

Pietarinen's expansion method

Pietarinen expansion method: Invariant amplitudes $C^{\pm}$, $B^{\pm}$ represented by:

$$C^{\pm}(\nu^2, t) = C_N^{\pm}(\nu^2, t) + \hat{C}^{\pm}(\nu^2, t) \sum_{n=0}^{\infty} c_n^{\pm} z^n$$

$$B^{\pm}(\nu^2, t) = B_N^{\pm}(\nu^2, t) + \hat{B}^{\pm}(\nu^2, t) \sum_{n=0}^{\infty} b_n^{\pm} z^{\pm}$$

$C_N^{\pm}$, $B_N^{\pm}$ - nucleon pole contributions, $\hat{C}^{\pm}(\nu^2, t)$, $\hat{B}(\nu^2, t)$ describe high energy behaviour of IA.

Pietarinen's expansion method

Pietarinen: The best approximants of IA are to be determined by minimizing a quadratic form:

$$\chi^2 = \chi^2_{data} + \Phi.$$

Φ is a convergence test function:

$$\Phi = \lambda_1 \Phi_1 + \lambda_2 \Phi_2 + \lambda_3 \Phi_3 + \lambda_3 \Phi_4.$$

$$\Phi_1 = \sum_{n=0}^{N} (n+1)^3 (c_n^+)^2, \ldots, \Phi_4 = \sum_{n=0}^{N} (n+1)^3 (b_n^-)^2.$$

For $N \approx 30$:

$$\lambda_1 = \frac{N}{\sum_{n=0}^{N} (n+1)^3 (c_n^+)^2}, \ldots, \quad \lambda_4 = \frac{N}{\sum_{n=0}^{N} (n+1)^3 (b_n^-)^2}.$$

PWA of η photoproduction data with fixed-t analyticity

Our PWA of η photoproduction data consists of two analysis:

- Fixed-t amplitude analysis
- SE PWA

Fixed- t amplitude analysis requires experimental data at a given value of variable t. Experimental data have to be shifted to predefined t-values using a small steps in t- **t-binning.**
SE PWA requires experimental data at a given energy.
Experimental data have to be shifted to predefined energies- **energy binning**.

Fixed-t amplitude anlysis

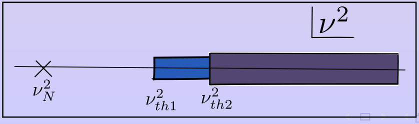
For a given t crossing symmetric invariant amplitudes are represented by two Pietarinen series:

$$B_1 = B_{1N} + \sum_{i=0}^{N_1} b_{1i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{1i}^{(2)} z_2^i, \quad B_2 = B_{2N} + \sum_{i=0}^{N_1} b_{2i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{2i}^{(2)} z_2^i$$

$$B_6 = B_{6N} + \sum_{i=0}^{N_1} b_{6i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{6i}^{(2)} z_2^i, \quad B_8 = \frac{B_{8N}}{\nu} + \sum_{i=0}^{N_1} b_{8i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{8i}^{(2)} z_2^i$$

B_{iN} are known nucleon pole contributions. Conformal variables z_1 and z_2 are defined as:

$$z_1 = \frac{\alpha - \sqrt{\nu_{th1}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th1}^2 - \nu^2}}, \qquad z_2 = \frac{\beta - \sqrt{\nu_{th2}^2 - \nu^2}}{\beta + \sqrt{\nu_{th2}^2 - \nu^2}}.$$



Fixed-t amplitude analysis

Coefficients $\{b_1^{(k)}\}$ and $\{b_2^{(k)}\}$ are obtained by minimizing a quadratic form

$$\chi^2 = \chi^2_{data} + \chi^2_{PW} + \Phi$$

.

$$\begin{aligned}
\chi^2_{data} &= \sum_{i=1}^{N^E}\left(\frac{\frac{d\sigma}{d\Omega}(W_i)^{exp} - \frac{d\sigma}{d\Omega}(W_i)^{fit}}{\Delta\frac{d\sigma}{d\Omega}(W_i)^{exp}}\right)^2 \\
&+ \sum_{i=1}^{N^E}\left(\frac{T(W_i)^{exp} - T(W_i)^{fit}}{\Delta T(W_i)^{exp}}\right)^2 \\
&+ \sum_{i=1}^{N^E}\left(\frac{F(W_i)^{exp} - F(W_i)^{fit}}{\Delta F(W_i)^{exp}}\right)^2 \\
&+ \sum_{i=1}^{N^E}\left(\frac{\Sigma(W_i)^{exp} - \Sigma(W_i)^{fit}}{\Delta\Sigma(W_i)^{exp}}\right)^2
\end{aligned}$$

Fixed-t amplitude analysis

χ^2_{PW} contains as a "data" the helicity amplitudes calculated from partial wave solution:

$$\chi^2_{PW} = q \sum \sum_{i=1}^{N^E} \left(\frac{Re\, H_k(W_i)^{PW} - Re\, H_k(W_i)^{fit}}{(\varepsilon_R)_{ki}} \right)^2 + q \sum_{k=1}^{4} \sum_{i=1}^{N^E} \left(\frac{Im\, H_k(W_i)^{PW} - Im\, H\,_k(W_i)^{fit}}{(\varepsilon_I)_{ki}} \right)^2$$

q - adjustable weight factor

Errors ε_{Rk} and ε_{Ik} are adjusted in such a way to get $\chi^2_{data} \approx \chi^2_{PW}$.

In a first iteration amplitudes H_k^{PW} are calculated from initial, already existing PW solution.
In subsequent iterations H_k^{PW} are calculated from multipoles obtained in SE PWA of the same set of experimental data.

Fixed-t amplitude analysis

Φ is Pietarinen's convergence test function

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4$$

$$\Phi_k = \lambda_{1k} \sum_{i=0}^{N_1} (b_{1i}^{(k)})^2 (n+1)^3 + \lambda_{2k} \sum_{i=0}^{N_2} (b_{2i}^{(k)})^2 (i+1)^3$$

$$\lambda_{1k} = \frac{N_1}{\sum_{i=0}^{N_1} (b_{1i}^{(k)})^2 (i+1)^3}, \quad \lambda_{2k} = \frac{N_2}{\sum_{i=0}^{N_2} (b_{2i}^{(k)})^2 (i+1)^3}$$

One starts with some initial values of coefficients $\{b_1^{(k)}\}$, $\{b_2^{(k)}\}$ and determines λ_{1k} and λ_{2k} in an iterative procedure.

Fixed-t amplitude analysis

Conection between fixed-t AA and SE PWA

After performing fixed-t amplitude analysis at predetermined t-values, helicity amplitudes may be calculated at any energy W at N_c values of scattering angle

$$cos\theta_i = \frac{t_i - m_\eta^2 + 2k\omega}{2kq} \qquad |cos\theta_i| \leq 1, \quad t_i \in [t_{min}, t_{max}]$$

These values of helicity amplitudes are used as constraint in SE PWA.

In a single energy partial wave analysis we minimize a quadratic form:

$$\chi^2 = \chi^2_{data} + \chi^2_{FT}$$

χ^2_{data} contains all experimental data at a given energy W:

$$\begin{aligned}
\chi^2_{data} &= \sum_{i=1}^{N_1^D} \left(\frac{\frac{d\sigma}{d\Omega}(\theta_i)^{exp} - \frac{d\sigma}{d\Omega}(\theta_i)^{fit}}{\Delta \frac{d\sigma}{d\Omega}(W_i)^{exp}} \right)^2 \\
&+ \sum_{i=1}^{N_2^D} \left(\frac{T(\theta_i)^{exp} - T(\theta_i)^{fit}}{\Delta T(W_i)^{exp}} \right)^2 \\
&+ \sum_{i=1}^{N_3^D} \left(\frac{F(\theta_i)^{exp} - F(\theta_i)^{fit}}{\Delta F(W_i)^{exp}} \right)^2 \\
&+ \sum_{i=1}^{N_4^D} \left(\frac{\Sigma(\theta_i)^{exp} - \Sigma(\theta_i)^{fit}}{\Delta \Sigma(W_i)^{exp}} \right)^2
\end{aligned}$$

Constrained SE PWA

χ^2_{FT} contains as the "data" the helicity amplitudes from the fixed-t amplitudes analysis.

$$\chi^2_{FT} = q \sum_{k=1}^{4} \sum_{i=1}^{N^C} \left(\frac{Re\, H_k(\theta_i)^{PW} - Re\, H_k(\theta_i)^{fit}}{(\varepsilon_R)_{ki}} \right)^2 + q \sum_{k=1}^{4} \sum_{i=1}^{N^C} \left(\frac{Im\, H_k(\theta_i)^{PW} - Im\, H_k(\theta_i)^{fit}}{(\varepsilon_I)_{ki}} \right)^2$$

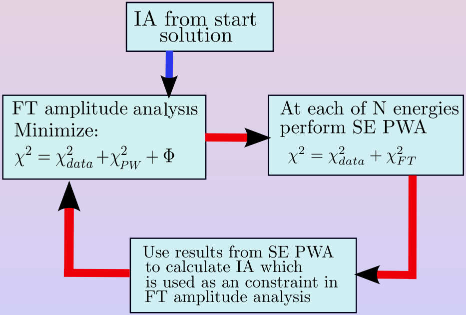
q - adjuastuble weight factor
N^C - number of angles at which constraining amplitudes are determined. Errors ε_{Rk} and ε_{Ik} are adjusted in such a way to get $\chi^2_{data} \approx \chi^2_{FT}$.

Connection between SE PWA and fixed-t AA

Multipoles obtained from SE PWA at N^E energies are used to calculate helicity amplitudes which are used as constraint in the fixed-t amplitude analysis.

Constrained PWA of η photoproduction data

The whole procedure has to be iterated until reaching reasonable agreement in two subsequent iterations



η photoproduction data base

Data base consists of following experimental data

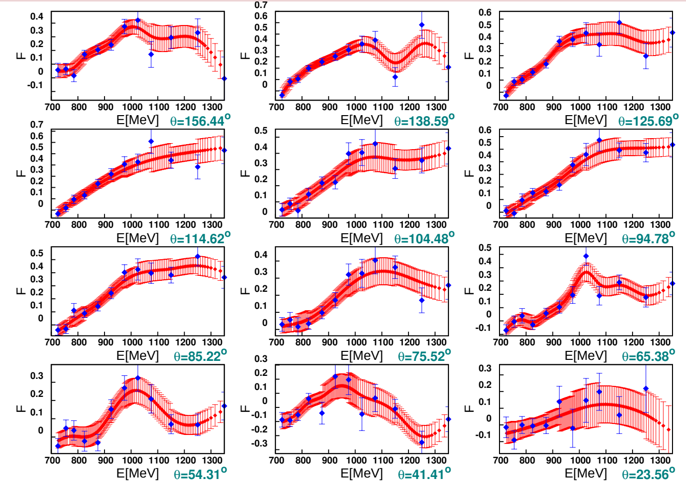
- Differential cross section $\frac{d\sigma}{d\Omega}$
  CBall/MAMI: E.McNicoll et al., PRC 82(2010) 035208
  $E_{lab} = 710, \dots 1395 MeV$
  2400 data points at 120 energies
- Beam asymmetry Σ
  GRAAL: O. Bartalini et al., EPJ A 33 (2007) 169
  $E_{lab} = 724, \dots 1472 MeV$
  150 data points at 15 energies
- Target asymmetry T
  CBall/MAMI: V. Kashevarov (preliminary)
  $E_{lab} = 725, \dots, 1350 MeV$
  144 data points at 12 energies
- Double-polarisation asymmetry F
  CBall/MAMI: V. Kashevarov (preliminary)
  $E_{lab} = 725, \dots, 1350 MeV$
  144 data points at 12 energies

Energy binning

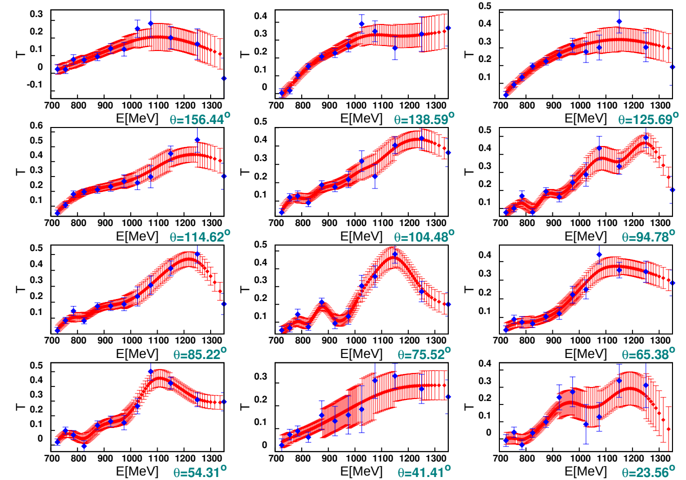
F, T,Σ

Experimental values of double-polarisation asymmetry F , target asymmetry T, and beam asymmetry Σ for given angles are interpolated to the energies where $\frac{d\sigma}{d\Omega}$ are available. We use a spline fit method. Errors of interpolated data are taken to be equal to errors of nearest measured data points.

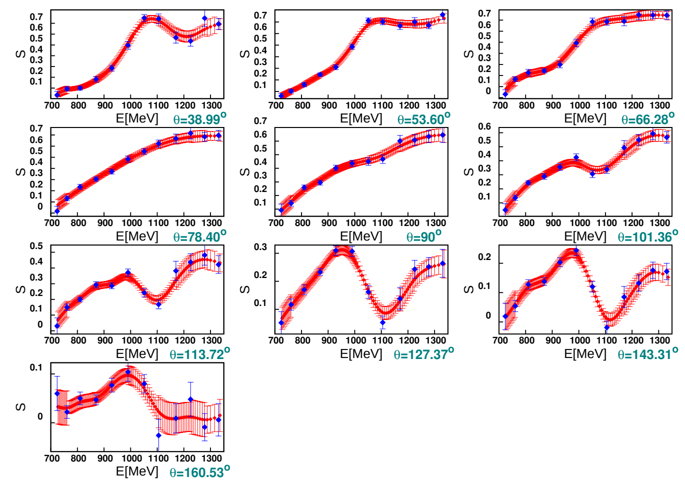
Interpolated values of double polarisation F



Interpolated values of target asymmetry T



Interpolated values of beam assymetry Σ

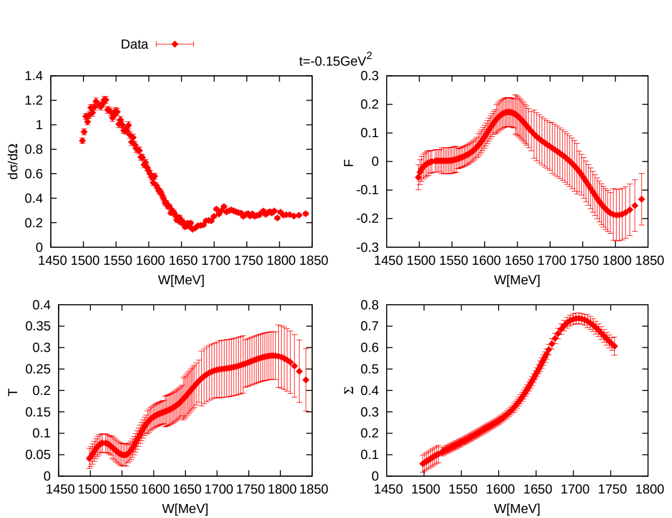


t-binning

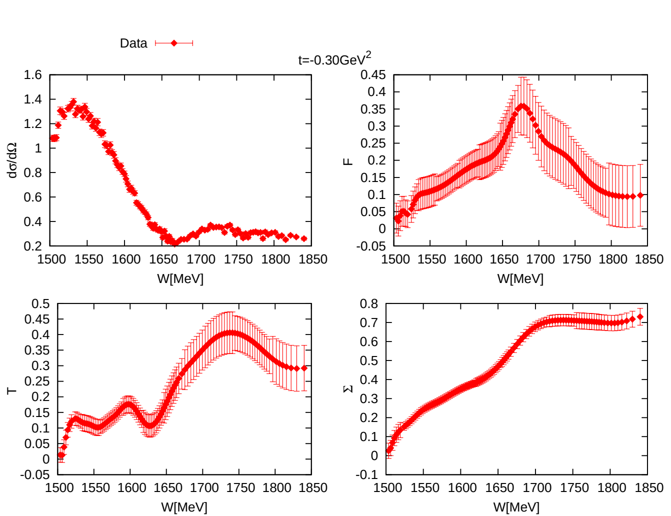
Input data $\frac{d\sigma}{d\Omega}$, T, F and Σ for t-binning are obtained from energy binning procedure (113 energies).

- Observables $\frac{d\sigma}{d\Omega}$, T, F and Σ are available at different t-values (different $\cos\theta$).
- Fixed-t amplitude analysis is performed at t-values in the range $-0.05 GeV^2 < t < -1.00 GeV^2$.
- Using spline fit, experimental data ($\frac{d\sigma}{d\Omega}$, T, F and Σ) are shifted to the predetermined t-values from above interval.

Interpolated values of measurable quantities at $t = -0.15\,GeV^2$



Interpolated values of measurable quantities at $t = -0.30 GeV^2$



Fixed-t invariant amplitudes $t = -0.10\,GeV^2$

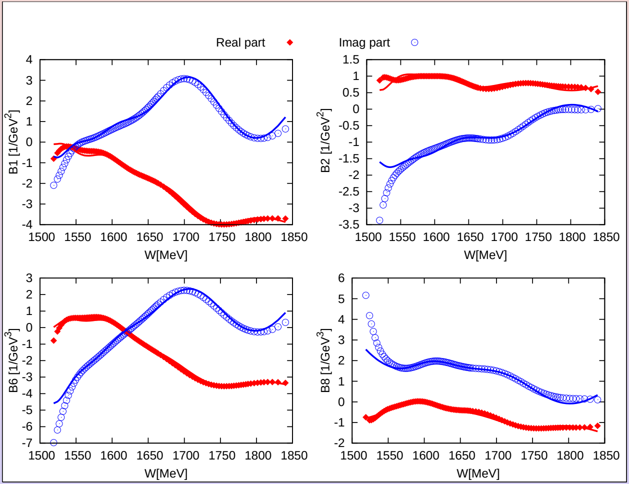
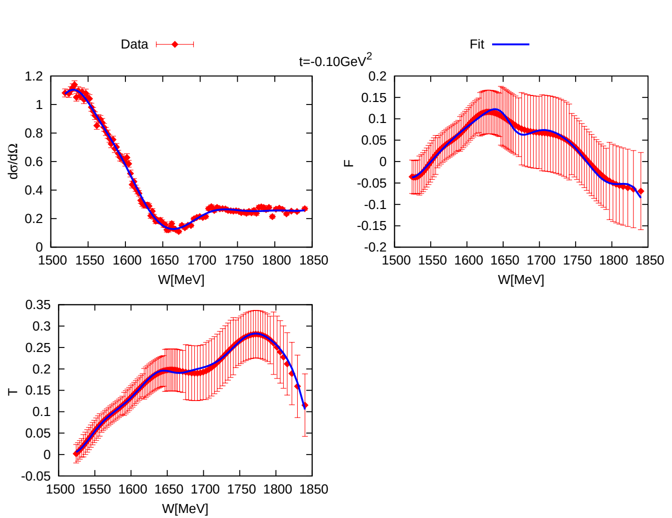


Figure: Corresponding fixed-t invariant (helicity) amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes B_i.

Fit of experimental data at $t = -0.10\, GeV^2$



Fixed-t invariant amplitudes $t = -0.30\,GeV^2$

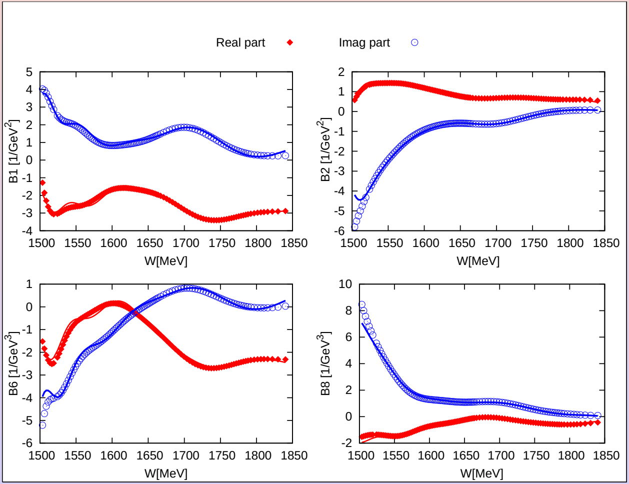
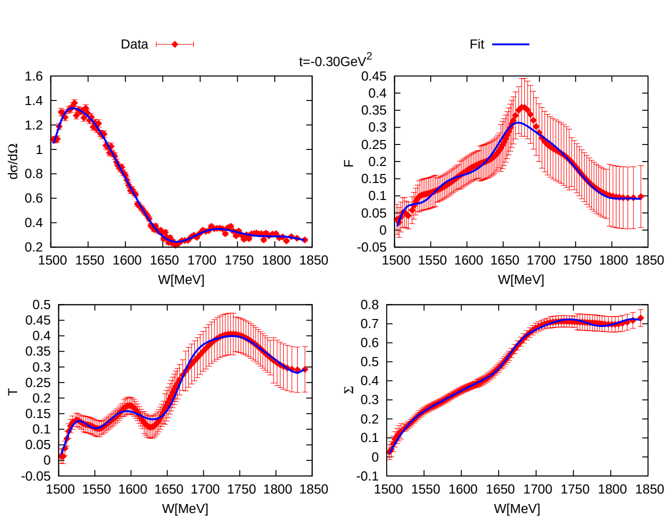


Figure: Corresponding fixed-t invariant (helicity) amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes B_i.

Fit of experimental data $t = -0.30 GeV^2$



Fixed-t invariant amplitudes $t = -0.50\,GeV^2$

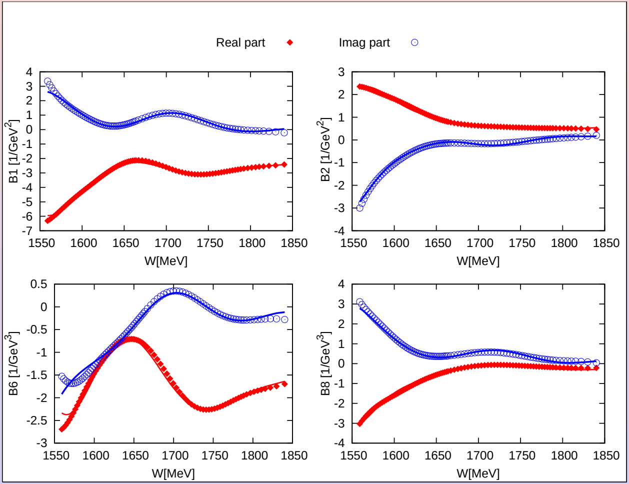
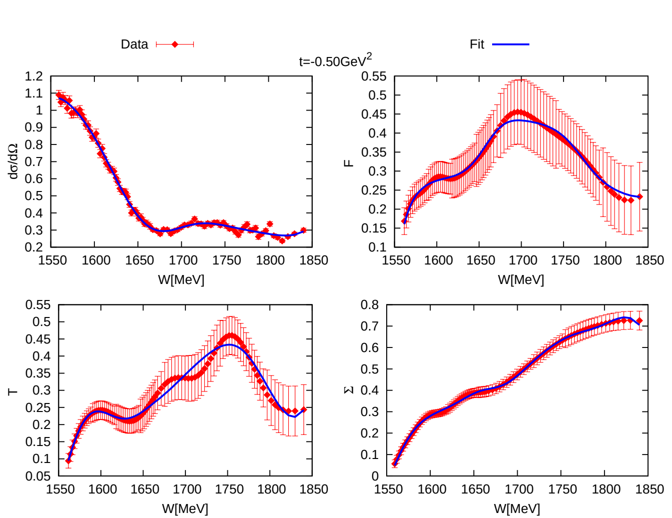


Figure: Corresponding fixed-t invariant (helicity) amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes B_i.

Fit of experimental data $t = -0.50 GeV^2$



Fixed-t invariant amplitudes $t = -0.70\,GeV^2$

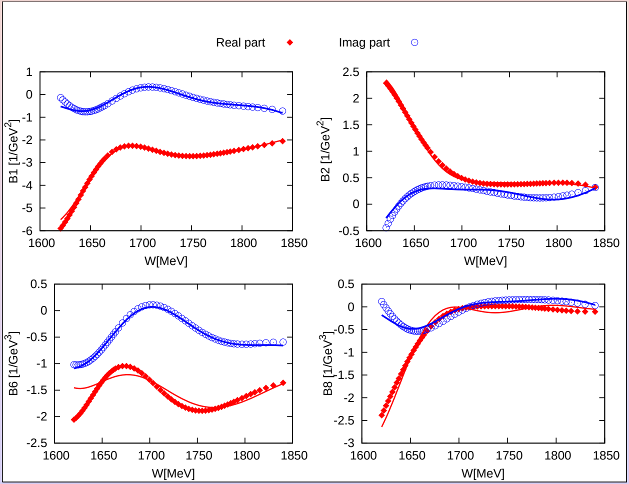
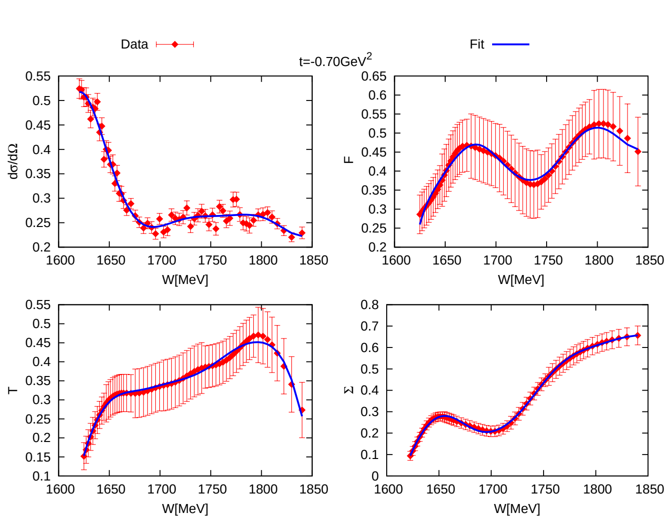


Figure: Corresponding fixed-t invariant (helicity) amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes B_i.

Fit of experimental data $t = -0.70 GeV^2$



Fixed-t invariant amplitudes $t = -1.00\,GeV^2$

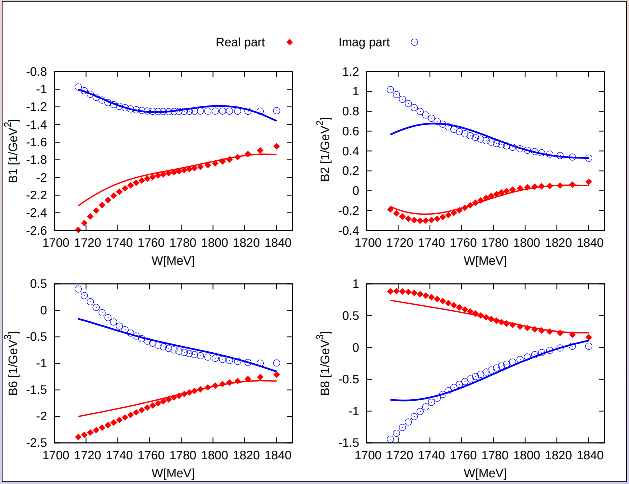
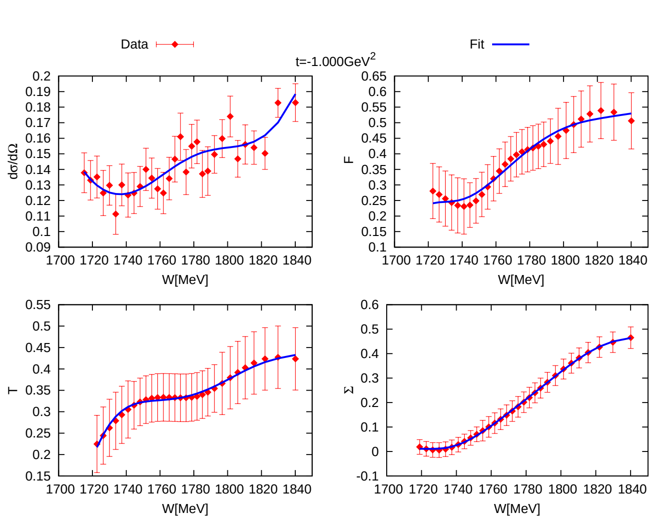


Figure: Corresponding fixed-t invariant (helicity) amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes B_i.

Fit of experimental data $t = -1.00 GeV^2$



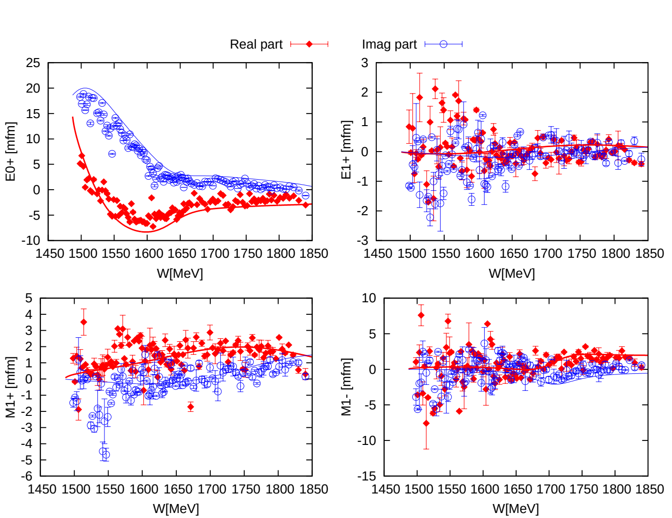


Figure: Red and blue solid lines-initial solution etaMAID2015b

SE PWA - Multipoles- Unconstrained fit

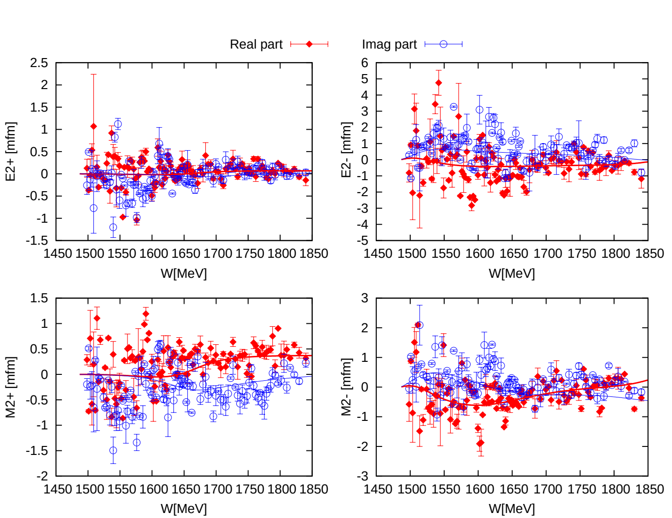


Figure: Red and blue solid lines-initial solution etaMAID2015b

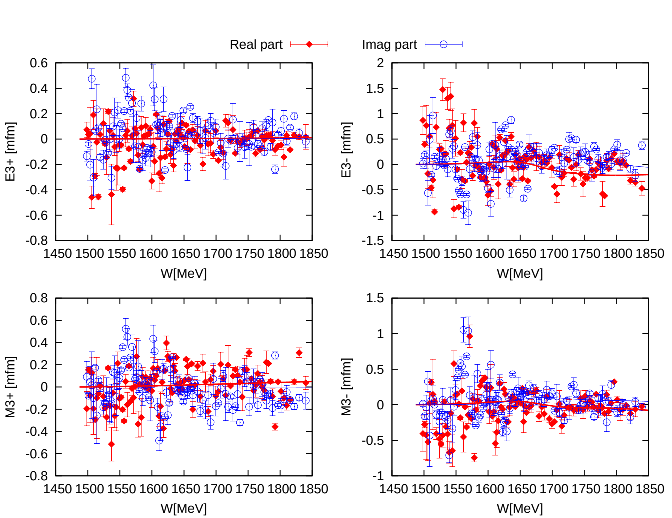


Figure: Red and blue solid lines-initial solution etaMAID2015b

SE PWA - Multipoles - Constrained fit

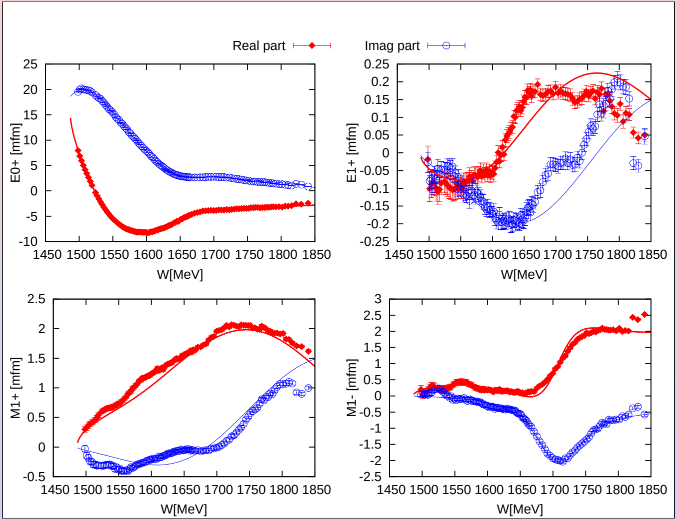


Figure: Red and blue solid lines-initial solution etaMAID2015b

SE PWA - Multipoles - Constrained fit

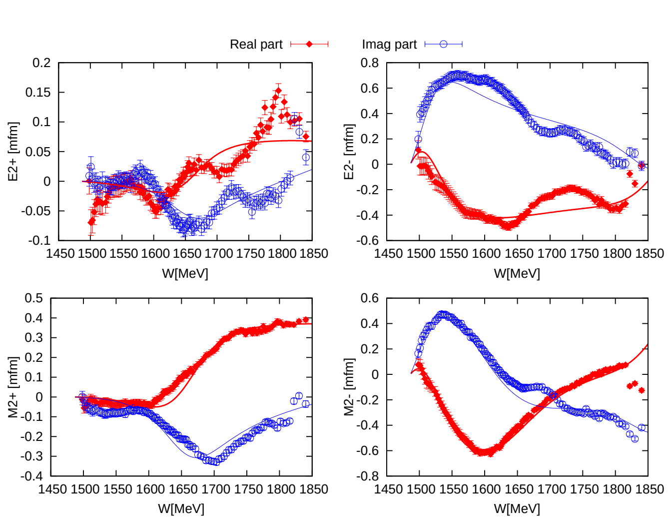


Figure: Red and blue solid lines-initial solution etaMAID2015b

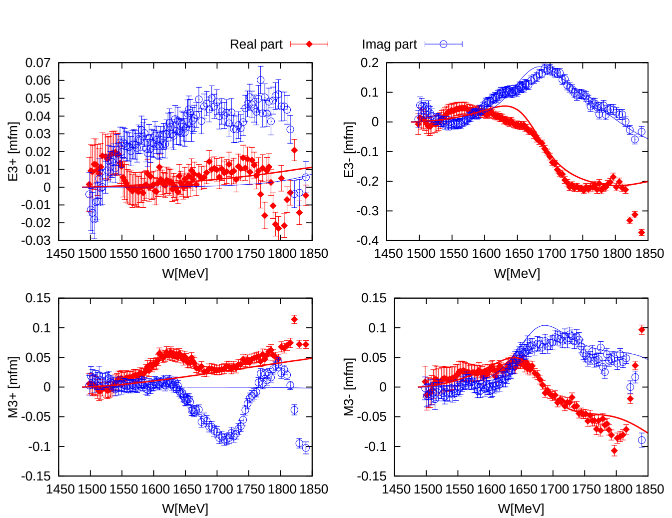


Figure: Red and blue solid lines-initial solution etaMAID2015b

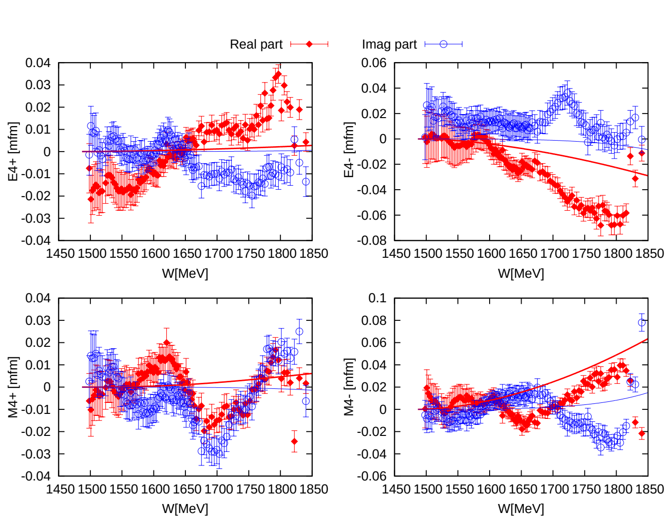


Figure: Red and blue solid lines-initial solution etaMAID2015b

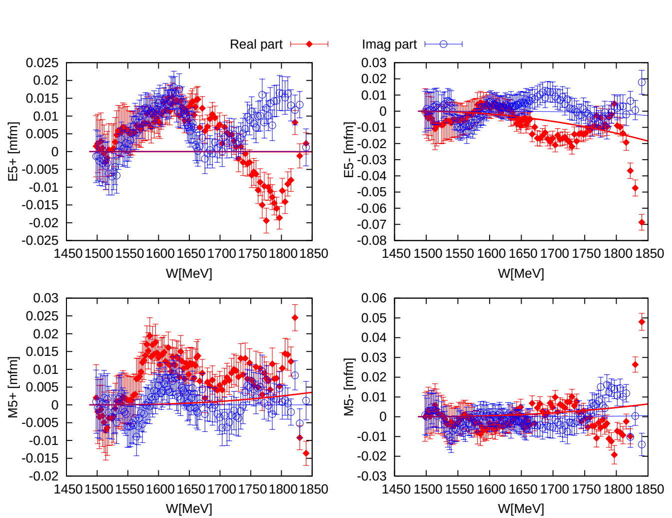
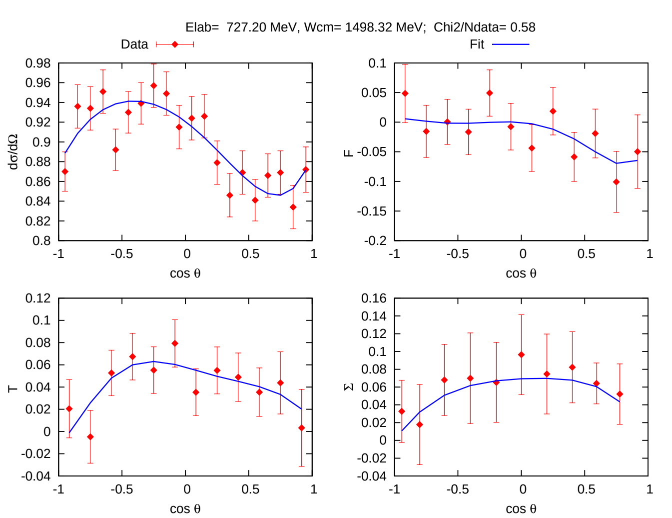
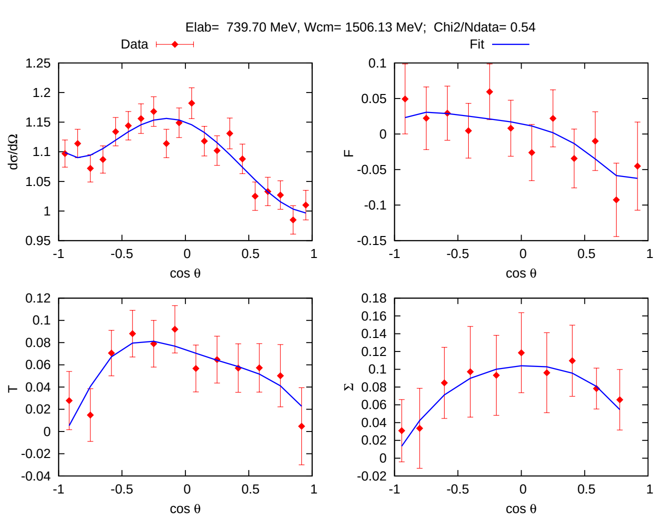
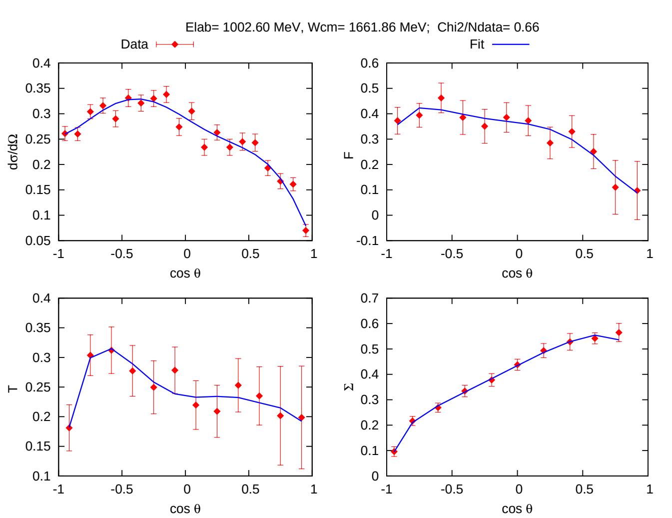


Figure: Red and blue solid lines-initial solution etaMAID2015b

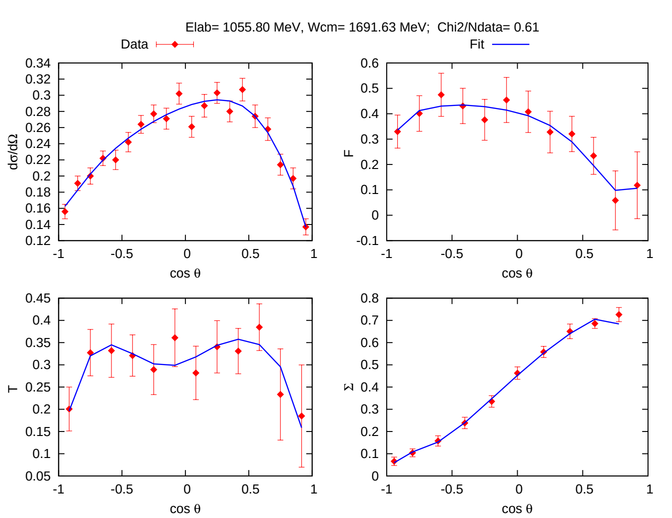
SE PWA - Exp.data



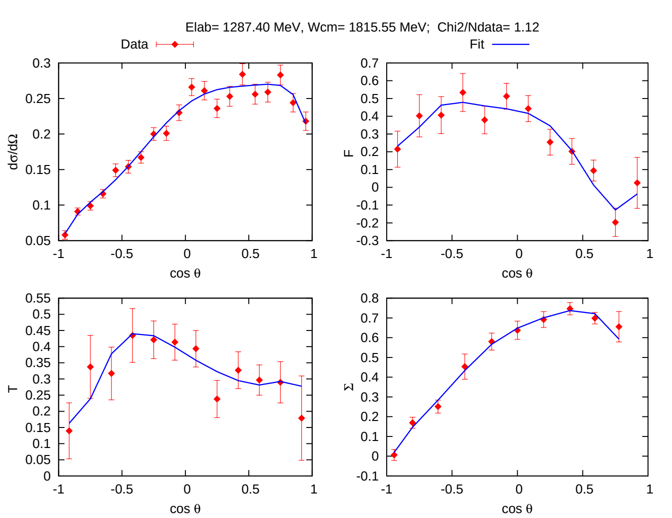

Elab= 739.70 MeV, Wcm= 1506.13 MeV; Chi2/Ndata= 0.54
Data
Fit
dσ/dΩ
F
T
Σ
cos θ
1.25
1.2
1.15
1.1
1.05
1
0.95
0.1
0.05
0
-0.05
-0.1
-0.15
0.12
0.08
0.06
0.04
0.02
-0.02
-0.04
0.18
0.16
0.14
0.12
-1
-0.5
0.5

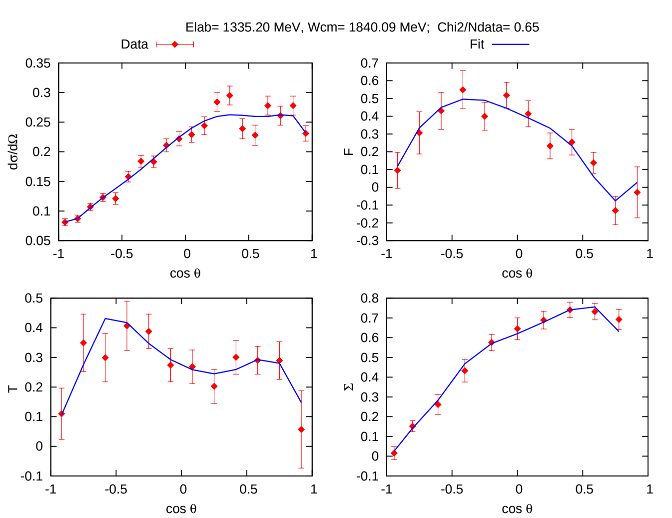

Elab= 1002.60 MeV, Wcm= 1661.86 MeV; Chi2/Ndata= 0.66
Data
Fit
dσ/dΩ
F
T
Σ
cos θ
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0.6
0.5
0
-0.1
0.7
-1
-0.5
1

SE PWA - Exp.data



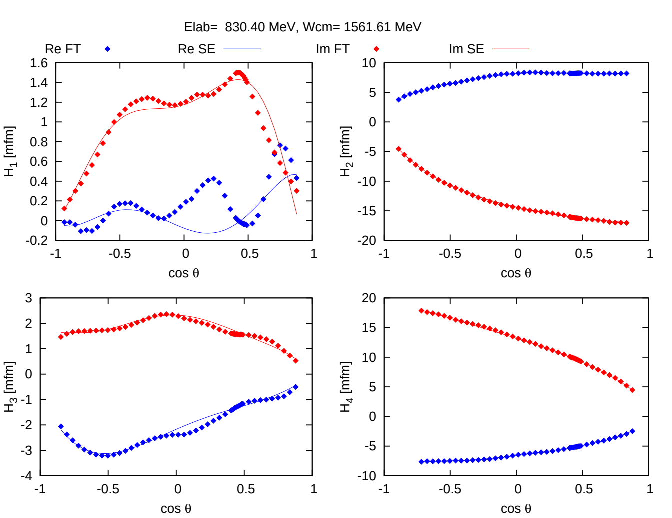
SE PWA - Exp.data



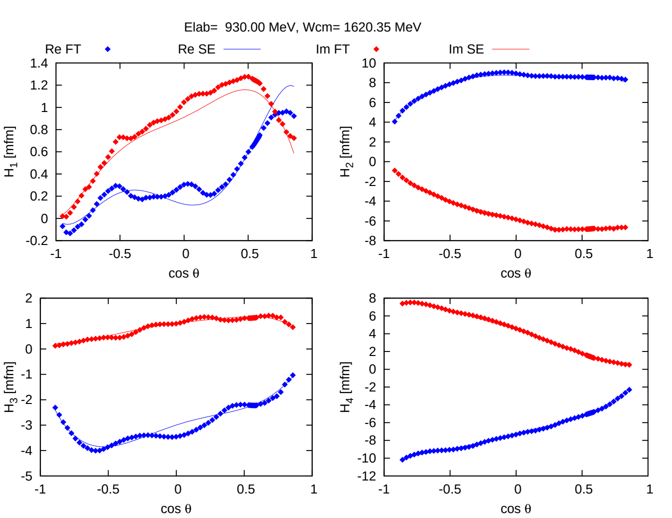


Elab= 1335.20 MeV, Wcm= 1840.09 MeV; Chi2/Ndata= 0.65
Data
Fit
dσ/dΩ
F
T
Σ
cos θ
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.8
-1
-0.5
0.5
1

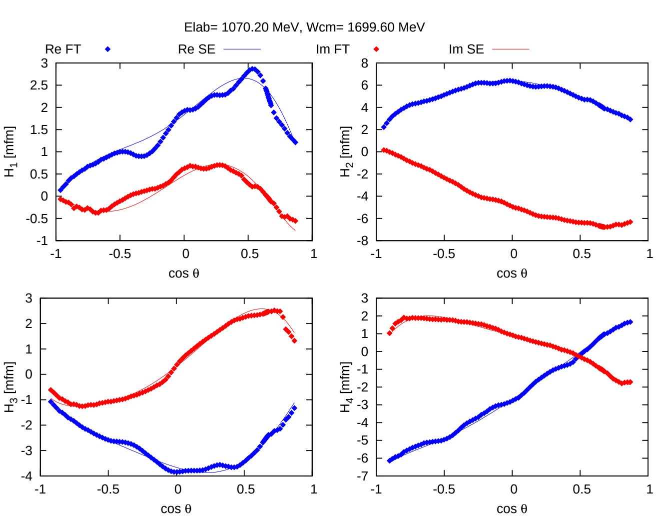
Helicity amplitudes



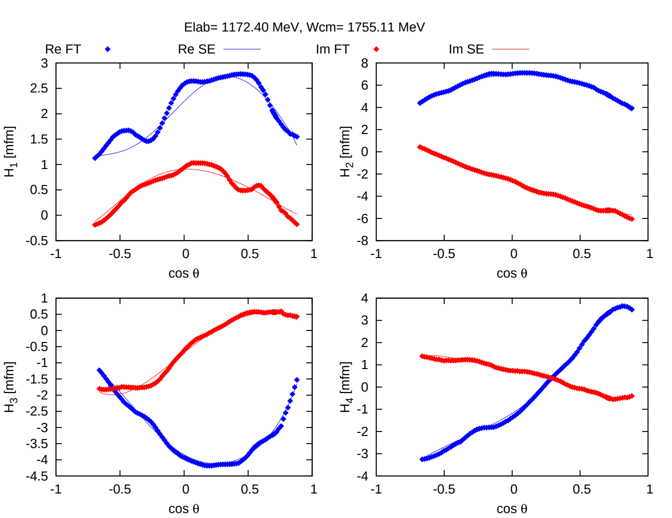
Helicity amplitudes



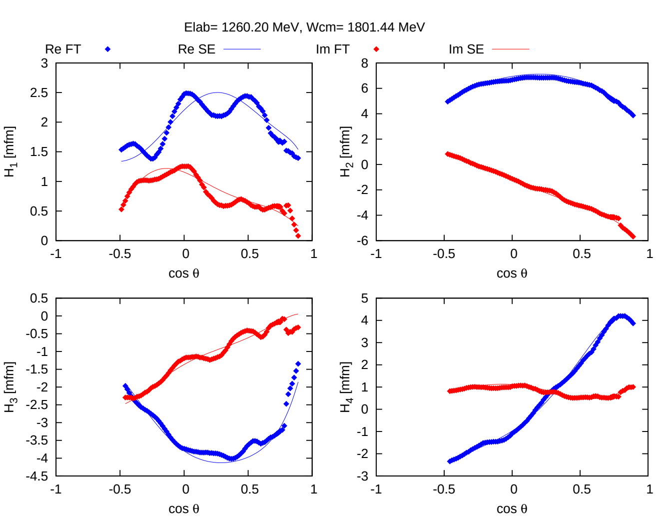
Helicity amplitudes



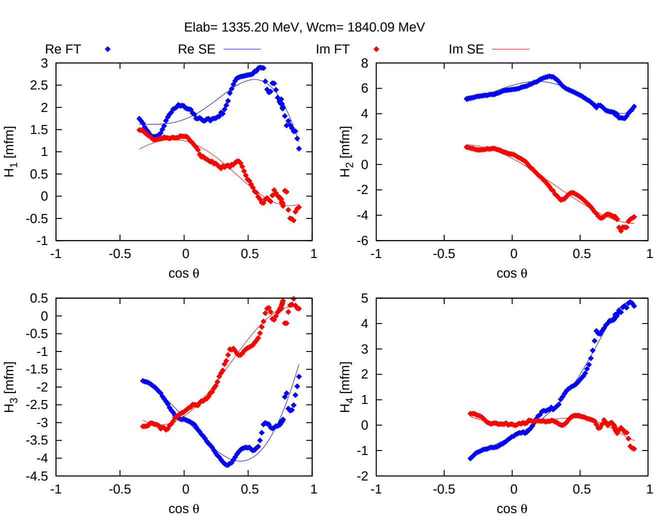
Helicity amplitudes



Helicity amplitudes



Helicity amplitudes



SE PWA BG - initial solution - Constrained fit

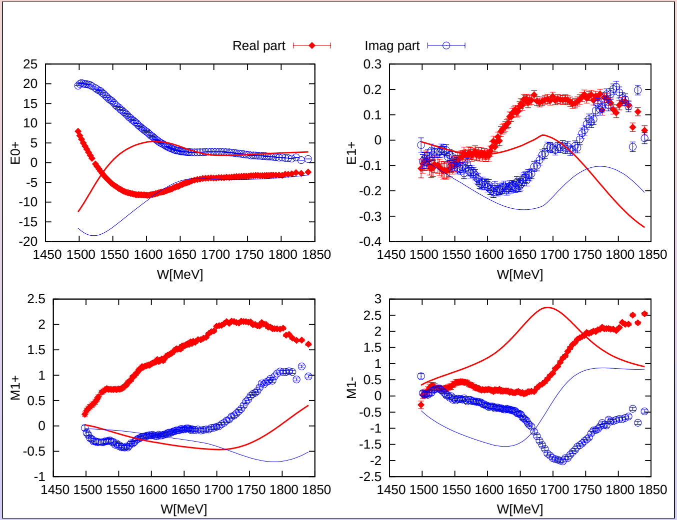


Figure:

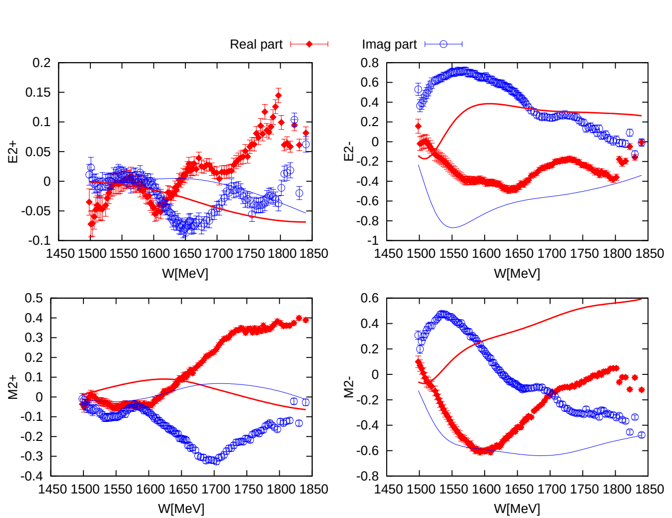


Figure: Red and blue solid lines-initial solution Bonn-Gatchina

SE PWA BG - initial solution - Constrained fit

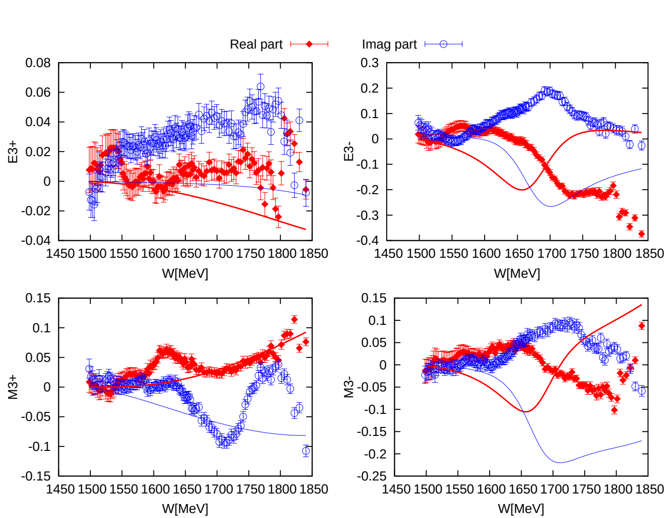


Figure: Red and blue solid lines-initial solution Bonn-Gatchina

SE PWA BG - initial solution - Constrained fit

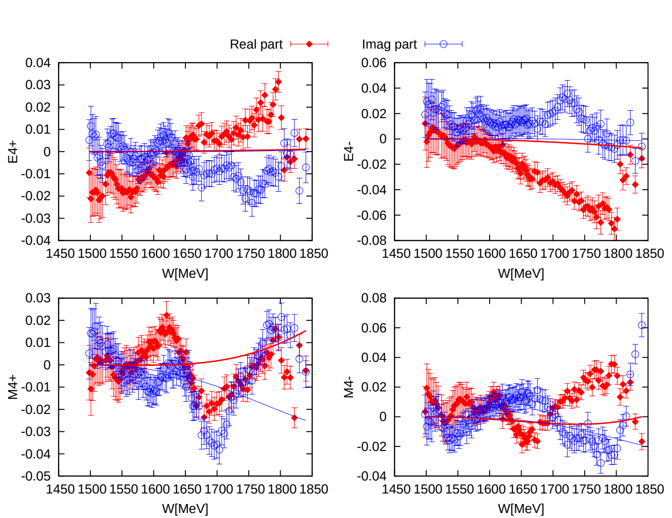


Figure: Red and blue solid lines-initial solution Bonn-Gatchina

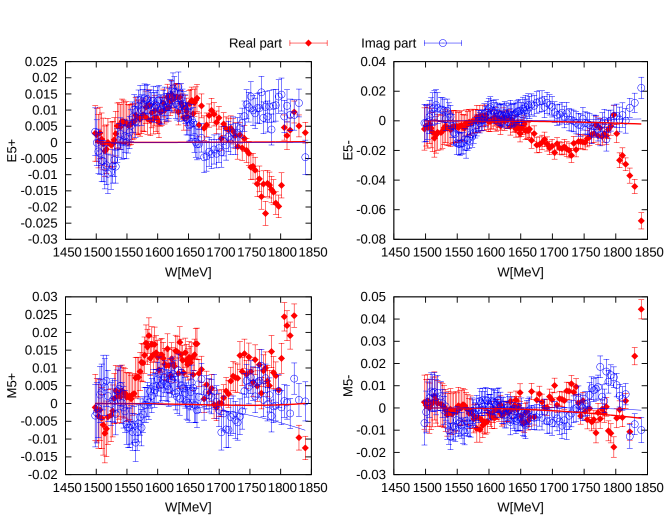


Figure: Red and blue solid lines-initial solution Bonn-Gatchina

Conclusions

- Applied procedure is model independent
- PWA with fixed-t constraint produce multipoles which are cosistent with crossing symmetry and fixed-t analyticity
- Helicity amplitudes from fixed-t show a good consistency with fixed- s analyticity. It implies that our amplitudes are consistent with both- fixed-t and fixed-s analyticity as well.
- Even a weak fixed-t constraint makes solution from SE PWA independent of starting solution.