

Christmas Homework Numerics of PDE, WS 2014/15

Let $\Omega \subseteq \mathbb{R}^2$ be a bounded and simply connected domain and let \mathbb{T} be a triangulation of Ω . Furthermore let V_h^H be the approximation of the solution space using cubic Hermite-Elements and V_h^L the one using cubic Lagrange-Elements, respectively.

- a) Show: $V_h^H \subset V_h^L$.
- b) Calculate the dimensions $\dim(V_h^H)$ and $\dim(V_h^L)$.
- c) Show: $\forall \mathbb{T}$ with $|\mathbb{T}| \geq 2$: $\dim(V_h^H) < \dim(V_h^L)$.
Hint: Use Euler's formula (see: http://en.wikipedia.org/wiki/Euler_characteristic), here number of faces means for number of triangles
- d) Explain why V_h^L has more DOFs.



Figure 1: Source: freechristmaswallpapers.net