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Christmas Homework
Analysis and Numerics of Conservation Laws, WS 2014/15

• Poiseuille Flow

Let $\Omega := \{(x,y) \,|\, 0 < x < L, 0 < y < 1\}$. Now at the upper and lower boundary of the channel Ω we have the solid-wall boundary condition, i.e $v = 0$, at the left boundary the Dirichlet and at the right boundary the "do-nothing" boundary condition (i.e. natural boundary condition).

a) Show that for all $c, \nu \in \mathbb{R}$

$$v(x,y) := \begin{pmatrix} y(1-y) \\ 0 \end{pmatrix}, \quad p(x,y) := c - 2\nu x,$$

is a weak solution of the Navier-Stokes equations.

b) Calculate all values of $c \in \mathbb{R}$ such that the natural boundary condition on the right boundary $x = L$ is valid.

Figure 1: freechristmaswallpapers.net