HW 1: Algebraische Geometrie II

Handing in: Hand in by November 2nd 2015. Grades are from 1 to 10. Keep a copy of your work and include an email address + home address to which we can return the graded exercises. You are strongly encouraged to write your solutions in TeX and hand in a printout. If you write your solutions by hand, write legibly! Parts that we cannot easily decipher will simply be ignored.

The rules of the game: Your solutions to these exercises should convince us that you have a good understanding of what has been discussed so far. This involves more than just getting the correct answers, which, anyway, might be obtained from various internet sources or the brain of a fellow student. You may use everything that has been discussed so far in the course. Give full details for your arguments, with precise references for the results you use. What you hand in should be your own, individual work.

Exercise 1. Let q be a prime power and $R = \mathbb{F}_q[x,y]/(xy-2)$. Compute $Z(R,t)$ and show that it is a rational function of t. (Be careful in characteristic two.)

Exercise 2. Let R be the ring $\mathbb{F}_2[x,y]/(y^2+y+x^3+1)$. Later in this course we will show that there exists an $\alpha \in \mathbb{C}$ with

$$Z(R,t) = \frac{(1-\alpha t)(1-\overline{\alpha}t)}{1-2t}.$$

In this exercise you may *assume* this. Denote the number of solutions of $y^2+y+x^3+1=0$ with x and y in the field $\mathbb{F}_{2^n}$ by v_n.

1. Show that $v_n = 2^n - \alpha^n - \overline{\alpha}^n$.
2. Compute v_1 and v_2 and use this to determine α.
3. Determine all the zeroes of $\zeta(R,s) = Z(R,2^{-s})$.

Exercise 3. Let k be an algebraically closed field. Let $Y \subset \mathbb{A}^2(k)$ be the zero set of $y - x^2$. Show that $\mathcal{O}(Y)$ is isomorphic to a polynomial ring in one variable.

Exercise 4. Let k be an algebraically closed field. Let $Y \subset \mathbb{A}^2(k)$ be the zero set of $xy-1$. Is $\mathcal{O}(Y)$ isomorphic to a polynomial ring in one variable?